Math 1497 - Calc 2

Tests for $\sum_{n=1}^{\infty} a_n$

(1) n^{th} term

if $\lim_{n\to\infty} a_n \neq 0$ the series diverges

Ex1 $\sum_{n=1}^{\infty} \frac{2^n-1}{2^n+1}$ $\lim_{n\to\infty} \frac{2^n-1}{2^n+1} \overset{LH}{=} \lim_{n\to\infty} \frac{2^n \ln 2}{2^n \ln 2} = 1 \neq 0$

the series diverges by the n^{th} term test

(2) **Integral Test**

we ~~compare~~ consider $f(n) = a_n$ & $\int_1^{\infty} f(n)\,dn$

we need the condition

(1) $f > 0$ (2) f cont^s (3) f dec ($f' < 0$)

if $\int_1^{\infty} f(n)\,dn$ conv (div) $\sum a_n$ conv (div)

ex2 $\sum_{n=1}^{\infty} n e^{-n}$ $f(n) = n e^{-n}$ $f > 0$ ✓ cont^s ✓

$f' = e^{-n} - n e^{-n} = (1-n) e^{-n} < 0$

for $n > 1$ ✓ so dec

Consider $\int_2^{\infty} n e^{-n}\,dn = \lim_{b\to\infty} \int_2^b n e^{-n}\,dn$

$= \lim_{b\to\infty} -(n+1)e^{-n}\Big|_2^b = \lim_{b\to\infty} -(b+1)e^{-b} + 3e^{-2} \to 3e^{-2}$ conv

Now $\lim_{b\to\infty} \frac{b+1}{e^b} \overset{LH}{=} \lim_{b\to\infty} \frac{1}{e^b} = 0$ so

so by $\int$ test the series conv.

8) LCT

we compare $\sum_{n=1}^{\infty} a_n$ $\sum_{n=1}^{\infty} b_n$ typically $\sum a_n$ is given we come up with $\sum b_n$

if $\lim_{n\to\infty} \frac{a_n}{b_n} = \#$ (not zero)

both series do the same

Ex 3 $\sum_{n=1}^{\infty} \frac{2n^2+1}{3n^3+n}$ compare w/ $\sum_{n=1}^{\infty} \frac{1}{n}$ div

$$\lim_{n\to\infty} \frac{2n^2+1}{3n^3+n} \Big/ \frac{1}{n} = \lim_{n\to\infty} \frac{2n^3+n}{3n^3+n^2} = 2/3 \quad (\#)$$

$\because \sum \frac{1}{n}$ div by LCT our series div.

<u>Test 4</u> Direct Comparison Test (DCT)

Consider $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

would we agree that

$$\frac{1}{n^2+1} < \frac{1}{n^2}$$

Aside

$$\frac{1}{n^2+1} \overset{?}{<} \frac{1}{n^2}$$

$$n^2 \overset{?}{<} n^2+1$$

$$0 \overset{?}{<} 1 \checkmark$$

so $\sum_{n=1}^{\infty} \frac{1}{n^2+1} < \sum_{n=1}^{\infty} \frac{1}{n^2}$

this converges meaning

$$\sum \frac{1}{n^2} \to L$$

so $\sum_{n=1}^{\infty} \frac{1}{n^2+1} < L$ so the series must conv.

Yes by $\int$ test & LCT w/ $\sum \frac{1}{n^2}$

The Test DCT

if $a_n \le b_n$ & $\sum b_n$ conv then $\sum a_n$ conv

if $\sum a_n$ diverges then $\sum b_n$ div

Ex 4 $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n+n}$

do we think conv or div?

If conv then

$$\frac{1}{2^n+n} < ? \leftarrow$$ and this conv

If div then

$$\rightarrow ? < \frac{1}{2^n+n}$$

this diverges

we think converge

2^n most dominates

$$\frac{1}{2^n+n} \overset{?}{<} \frac{1}{2^n}$$

$2^n \overset{?}{<} 2^n + n \iff 0 \overset{?}{<} n$ ✓ yes

Since $\sum \frac{1}{2^n}$ conv $(r=\frac{1}{2})$ by the DCT our

series conv

<u>Ex 5</u> $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

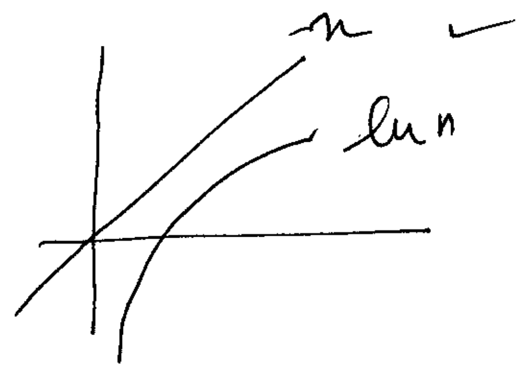
if conv $\quad \frac{1}{\ln n} < ?$ ← this conv

if div $\quad ? < \frac{1}{\ln n}$ ← I think this

Try $\quad \frac{1}{n} \overset{?}{<} \frac{1}{\ln n}$

$\ln n \overset{?}{<} n$

Yes



$\therefore \sum \frac{1}{n}$ div

then $\sum \frac{1}{\ln n}$ div by DCT.

Consider $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+1}$

we know it div (1) $\int$ test (2) LCT $\sum \frac{1}{n}$

$$\frac{1}{n} \overset{?}{<} \frac{1}{n+1}$$

$$n+1 \overset{?}{<} n \quad \text{or} \quad 1 \overset{?}{<} 0 \quad \text{Well no!}$$

so here we can't compare with $\sum \frac{1}{n}$

so we would try another test

<u>Note</u> if $a_n < b_n$

& $\sum b_n$ diverges we can't say anything about $\sum a_n$

similarly if $\sum a_n$ conv nothing can be said about $\sum b_n$