

Elmwood Press
Core Mathematics C3
Paper E
(Mark Scheme)

**All exam papers are issued free to students for education purpose only.
Mr.S.V.Swarnaraja (Marking Examiner, Team Leader & Author)
www.swanash.com, Mobile: +94777304755 , email: swa@swanash.com**

Worked Solutions

Edexcel C3 Paper E

$$1. \quad \frac{(2x-3)(x+1)}{(x+1)} + \frac{(x-2)(x+2)}{(x+2)} = 3x-5 \Rightarrow A=3, B=-5 \quad (4)$$

$$3x-5 = x^2-9 \Rightarrow x^2-3x-4=0$$

$$(x-4)(x+1)=0$$

$$\Rightarrow x=+4 \text{ or } x=-1 \quad (2)$$

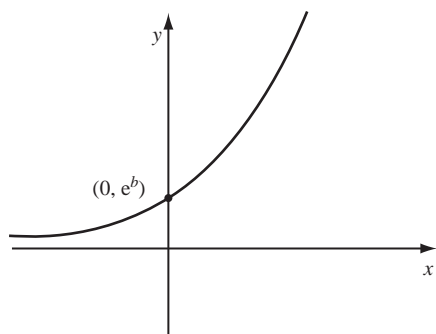
$$2. \quad (a) \quad \because \text{domain of } \sin^{-1}(x) \quad \frac{-\pi}{2} < x < 0 \quad \sin x = -\frac{3}{5}$$

$$\cos^2 x + \sin^2 x = 1$$

$$\therefore \cos^2 x + \frac{9}{25} = 1 \quad \therefore \cos^2 x = \frac{16}{25} \Rightarrow \cos x = +\frac{4}{5} \quad (4)$$

$$(b) \quad \cos 2x = 1 - 2\sin^2 x = 1 - \frac{18}{25} = \frac{7}{25} \quad (2)$$

3. (a)



(2)

$$(b) \quad e^b = 4 \quad \therefore b = \ln 4 \quad (1)$$

$$(c) \quad \frac{dy}{dx} = a \cdot e^{ax+b} = a \cdot e^{ax} \cdot e^b = 4ae^{ax}$$

$$x=2, \quad 10e^5 = 4ae^{2a} \quad a = 2\frac{1}{2} \quad (5)$$

$$4. \quad (a) \quad \frac{d}{dx} (\cos^2 x) = 2 \cos x (-\sin x)$$

$$= -2 \cos x \sin x. \quad (3)$$

$$(b) \quad \frac{d}{dx} \left(\frac{\ln x}{x} \right) = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$$

$$= \frac{1 - \ln x}{x^2} \quad (3)$$

$$(c) \quad \frac{d}{dx} (x^2 e^x) = x^2 \cdot e^x + e^x \cdot 2x \quad (4)$$

$$5. \quad (a) \quad \sin x + \frac{\cos^2 x}{\sin x} \equiv \frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$= \frac{1}{\sin x} = \operatorname{cosec} x. \quad (3)$$

$$(b) \quad \operatorname{cosec} x - \sin x = 3$$

$$1 - \sin^2 x = 3 \sin x$$

$$\sin^2 x + 3 \sin x - 1 = 0$$

$$\left[\begin{array}{l} \text{OR } \frac{\cos^2 x}{\sin x} = 3 \\ 1 - \sin^2 x = 3 \sin x \end{array} \right]$$

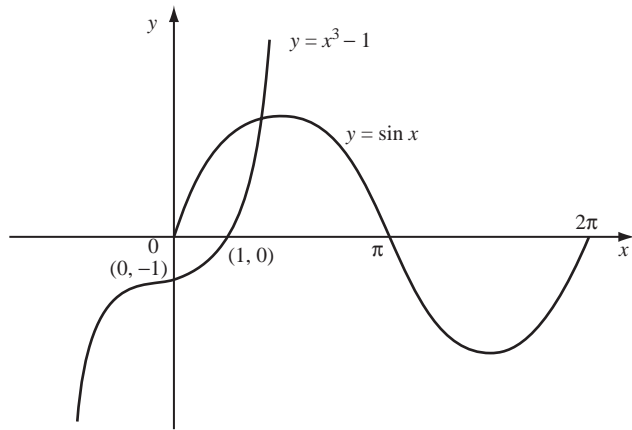
$$\sin x = \frac{-3 \pm \sqrt{13}}{2}$$

$$\sin x = \frac{-3 + \sqrt{13}}{2}$$

$$= 0.30277 \dots$$

$$x = 17.6^\circ, 162.4^\circ \quad (4)$$

6.



(a) curves cross once

(b) $f(x) = \sin x - x^3 + 1$

$$\left. \begin{array}{l} f(0) = 0 - 0 + 1 \\ f\left(\frac{\pi}{2}\right) = -1.875 \end{array} \right\} \text{change in sign indicates root in interval}$$

(c) $x_1 = 1.2257,$

$x_2 = 1.2474,$

$x_3 = 1.2489,$

$x_4 = 1.2490,$

$\alpha = 1.2491$

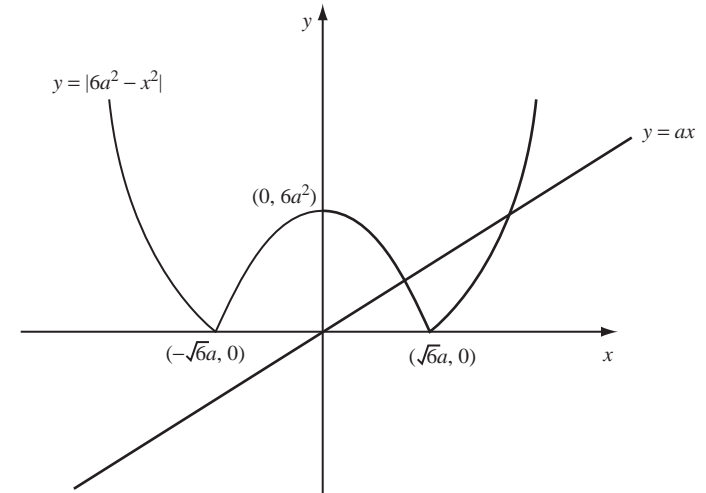
(4)

(1)

(2)

(3)

7.



(a) $x = 2a, y = |6a^2 - 4a^2| = 2a^2$

$x = 2a, y = a \cdot 2a = 2a^2$

(b) $-(6a^2 - x^2) = ax$

(c) $x^2 - ax - 6a^2 = 0$

$(x + 2a)(x - 3a) = 0$

$x = -2a$ and $x = 3a$

second point: $x = 3a, y = 3a^2$

(4)

(1)

(2)

(4)

8. (a) $\sqrt{3} \sin x + \cos x = 2 \left(\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \right)$

$$R = 2, \alpha = \frac{\pi}{3}$$

(b) $2 \cos \left(x - \frac{\pi}{3} \right) = \sqrt{2}$

$$\cos \left(x - \frac{\pi}{3} \right) = \frac{1}{\sqrt{2}}$$

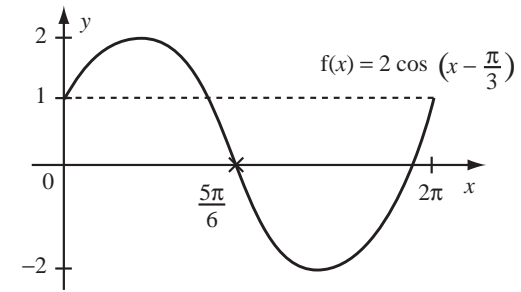
$$x - \frac{\pi}{3} = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{7\pi}{4},$$

$$x = \frac{\pi}{12}, \frac{7\pi}{12}$$

$$f(x) = 2 \cos \left(x - \frac{\pi}{3} \right)$$

(4)

(c)



(4)

(d) $2f(x) + 1 = 4 \cos \left(x - \frac{\pi}{3} \right) + 1$

$$\text{maximum value} = 5$$

$$\text{minimum value} = -3$$

(6)

(3)