Math 2471 Calculus III – Sample Final Questions

1. Find the unit tangent and unit normal vector for $\vec{r}(t) = < t, \frac{1}{2}t^2 >$.

2. (i) Prove the limits either exist or doesn't exist

$$(i) \quad \lim_{(x,y)->(0,0)} \frac{x^2+2y^2}{x^2+y^2} \qquad\qquad (ii) \quad \lim_{(x,y)->(0,0)} \frac{x^2y^4}{x^2+y^2}$$

3. Find the equation of the tangent plane to the given surface at the specified point

$$x^2y + xz + yz^2 = 3, \quad P(1,2,-1)$$

4. Find the directional derivative of $z = x^2 + 3xy + y^2$ at $(1,1)$ in the direction of $< -3, 4 >$

5. Classify the critical points for $z = x^2y - x^2 + y^2 - 18y$.

6. Set up and evaluate an integral to calculate

(i) the volume bound by $z = 0, \quad z = 1 - x^2 - y^2$
(ii) the volume inside both $x^2 + y^2 + z^2 = 2, \quad x^2 + y^2 = 1$
(ii) the surface area of $x + 2y + 3z = 6, \quad x, y, z \geq 0$

7. Set of the triple integral $\int\int\int f(x,y,z)dV$ in both cylindrical and spherical coordinates for the volume inside the cone $z = \sqrt{x^2+y^2}$ bound by $z = 1$

8. Is the following vector field conservative?

$$\vec{F} =< y^2 + 3yz, 2xy + 3xz, 3xy > .$$

If so, determine the potential function f such that $\vec{F} = \vec{\nabla} f$. Use this to evaluate

$$\int_c (y^2 + 3yz)\, dx + (2xy + 3xz)\, dy + 3xy\, dz > .$$

where c is a curve that connects the points $(0,0,0)$ and $(1,2,3)$.

9. Evaluate the following line integrals:

$(i) \quad \int_c 2xy\, dx + (x+1)\, dy$ where c is the counterclockwise direction around the square with vertice $(0,0), (1,0), (1,1)$ and $(0,1)$

$(ii) \quad \int_c (x-y)\, dx + (x+y)\, dy$ where c is clockwise direction around a circle of radius 2.

10. Verify Green's Theorem

$$\oint_c P dx + Q dy = \iint_R \left(Q_x - P_y\right) dA$$

where $\vec{F} = < 3x^2y, x^3 + x >$ and c the curve enclosing the area given by $y = x$ and $y = x^2$.

11. Evaluate the following surface integrals

(i) $\quad \iint_S xy\, dS$ where S is the portion of the plane $2x + y + z = 6$ in the first octant.

(ii) $\quad \iint_S (x + z)\, dS$ where S is the portion of $y^2 + z^2 = 9$ in the first octant between $x = 0$ and $x = 4$.

12. Verify the divergence theorem

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_V \nabla \cdot \vec{F}\, dV$$

where $\vec{F} = < x + yz, y + xz, z + xy >$ and V is the volume of the tetrahedron bound by $x + y + z = 1$ and the planes $x = 0$, $y = 0$ and $z = 0$.

13. Verify Stokes's theorem for

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S \nabla \times \vec{F} \cdot \vec{n}\, dS$$

where $\vec{F} = < xz - y, yz, 1 >$ and S is the surface of the paraboloid $z = 1 - x^2 - y^2$ for $z \geq 0$ and C is the curve where the paraboloid intersects the plane $z = 0$.