

## TeachSpin's Torsional Oscillator as a 'Watt Balance'

TeachSpin's Torsional Oscillator has always been known as **the** first-class system for exploring every facet of that most basic physical phenomenon – *simple harmonic motion*, with precision, accuracy, and reliability. But in this issue of Relaxation Times we feature a new-found use for this device. It can be made to function as a 'Watt Balance', a device able to illustrate the '**electronic kilogram**', the new technique that scientists are perfecting to *replace the present SI standard of mass*.

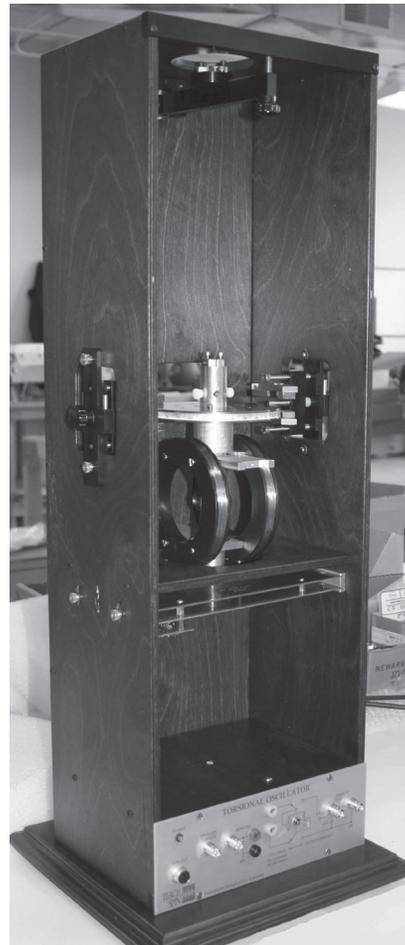


Figure 1: Two 'Watt balances': can you tell which one is TeachSpin's? The other is at NIST.

Since 1889, the world's standard of inertial mass has been the 'artifact kilogram', a cylinder of platinum-iridium alloy carefully preserved in France. All measurements of force, energy, power, even electrical quantities such as voltage, have depended on this kg standard (as seen in  $N = \text{kg m/s}^2$ ,  $J = \text{N}\cdot\text{m}$ ,  $W = \text{J/s}$ ,  $V = \text{J/C}$ , etc.). But there have been continuing worries about the vulnerability and the stability of the kg standard – the sole SI base unit still depending on an object. So the various national standards labs are on a campaign to replace, perhaps as soon as in 2018, the artifact kilogram with an 'electronic kilogram'. (More details are found in the cover story of July 2014's [Physics Today](#).)

The development of the electronic kilogram requires crucial innovations in the definitions of two key quantities:

- first, the *redefinition of electrical standards* for voltage, resistance, and/or current in terms of invariant physical quantities. (They are already being *maintained* this way, via the Josephson effect and  $2e/h$ , and the von Klitzing quantized resistance  $h/e^2$ ),
- second, the *redefinition of the kilogram* by requiring the mechanical Watt ( $= 1 \text{ kg m}^2/\text{s}^3$ ) to be equal to the electrical Watt ( $= 1 \text{ V}\cdot\text{A}$ ).

But translating these innovations into laboratory operations requires an apparatus that can assign a mass to some candidate object, without any reference to the soon-to-be-retired kg artifact, by an actual experiment involving that mass and electrical measurements. Such experiments are conducted in ‘Watt balances’, built to create *electrically* the force required to balance a gravitational force  $mg$  acting on an unknown mass  $m$ .

But how does TeachSpin come into this? We started by thinking about how instructors will *teach* the new kg standard, and we wondered if we ought to build a demonstration Watt balance. After a bit of designing, we realized that our Torsional Oscillator can already do the job, as shipped, with zero extra parts required!

So, project yourself into the near future, when your laboratory measurements of voltage and current will be anchored to the post-2018 SI (quantum-based) electrical standards. In such a lab, you will no longer have a connection to the now-retired artifact kilogram. To assign a mass-value  $m$  to some brass object, you’ll need to make *electrical* measurements – but of what?

In the case of using our Torsional Oscillator, you can suspend *two* such masses by strings and pulleys, and arrange them to create a net static torque of magnitude  $2\cdot mg\cdot R$  (and a net force of zero) on the rotor of our instrument. Here  $g$  is the local free-fall acceleration (which translates mass  $m$  to force  $mg$ ), and  $R$  is the radius of the hub on which the torque is exerted. Next, you can *balance out* that mechanical torque by a magnetic torque  $|\boldsymbol{\mu} \times \mathbf{B}| = \mu\cdot k i_{bal}$ . Here  $\mu$  is the magnetic moment of the permanent magnet mounted on the Torsional Oscillator’s rotor,  $k$  is the coil constant of the coil which produces the magnetic

field  $B$ , and  $i_{bal}$  is the dc electrical current needed in those coils to achieve balance. At static balance, then, you have adjusted the current  $i$  to a value where

$$2 mg R = \mu k i_{bal}.$$

How do you get the necessary value of the product  $\mu k$ ? Just as in a professional Watt balance, you get it via a second, and this time a *dynamic*, use of the same magnet-and-coil system. Here you can remove the masses and the torques, and just turn the rotor and its magnet, so it moves at angular velocity  $\omega$  around its axis. The rotating magnet then induces in the coil a Faraday’s-Law emf  $\varepsilon$ , whose value comes out to be  $\varepsilon = \mu k \cdot \omega$ , involving the *same*  $\mu k$ -constant as in torque production,  $\tau = \mu k \cdot i$ . That is to say, the *emf per unit angular velocity* (in this ‘generator’ mode) is equal to the *torque per unit current* (in the previous ‘motor’ mode), according to a crucial reciprocity theorem of electromagnetism.

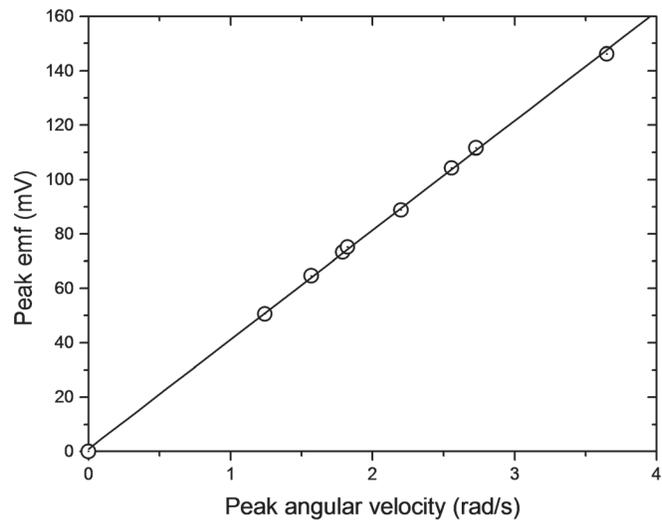


Figure 2: Emf  $\varepsilon$  generated as a function of instantaneous angular velocity

So if you were to turn the rotor at instantaneous angular velocity  $\omega$ , you’d generate an emf given by

$$\varepsilon_{gen} = (\mu k) \omega = \left( \frac{2 mg R}{i_{bal}} \right) \omega ,$$

so that

$$\varepsilon_{gen} \cdot i_{bal} = 2 mg \omega R .$$

On the left-hand side is a voltage·current product giving an electrical power (in Watts), and on the right-hand side is a force·velocity product giving a mechanical power (in Watts). Hence the name ‘Watt balance’ for this two-fold use of a single magnet-and-coil system.

Solving the second equation gives the result for  $m$ :

$$m = \frac{\varepsilon_{gen} i_{bal}}{2 g R \omega} .$$

The numerator is purely electrical, and (after 2018) will be traceable back to standards defined via assigned values for  $e$  and  $h$ . The denominator is purely mechanical, and depends only on standards of length and time. So the value of  $m$  that can be assigned through this two-step use of the Torsional Oscillator can fairly be said to be in ‘electronic kilograms’.

We’ve *done* just such a measurement at TeachSpin, using techniques already described in the Manual for our Torsional Oscillator. (If you have one of our instruments, check the Manual’s sections 1.4 and 2.1.) For ‘unknowns’, we used two brass objects, each of mass  $m$  in the range 250-290 g (with their mass ‘blinded’ to the investigator), and we elected to use an oscillating-rotor rather than a constant-velocity method for finding the  $\varepsilon/\omega$  quotient. Our results for the unknown masses  $m$  were

$$i_{bal} = (1.691 \pm 0.007) \text{ A}$$

$$\text{and } \varepsilon/\omega = (40.9 \pm 0.2) \text{ mV}/(\text{rad/s}) .$$

Using our local  $g$ -value of  $(9.804 \pm 0.001) \text{ m/s}^2$ , and the hub radius of  $R = (12.78 \pm 0.03) \text{ mm}$ , we deduce for each brass object a mass

$$m = (0.276 \pm 0.002) \text{ kg, ie. } (276 \pm 2) \text{ g} .$$

This represents a measured mass-value which depends only on electrical units, and on the units of length and time. But since it’s not yet 2018, we could still (after the fact!) weigh those brass masses on an ordinary balance, and we found a value  $(275.9 \pm 0.2) \text{ g}$ , ultimately traceable to the still-standard artifact kilogram.

As you can see we have achieved a precision, and an accuracy, of about  $\pm 1\%$  in this first try at an ‘electronic kilogram’. By contrast, the national standards labs aim to reach a precision and accuracy of a few parts in  $10^8$  before they’ll be content. But even at our 1% level, there is a wealth of investigatory expertise to be cultivated:

- Students can deal with an honest **unknown**: make them commit themselves to a measured  $m$ -value, in a lab emptied of all mass standards, scales, and balances!
- They can find out what level of **resolution** they can attain: how small an added  $\Delta m$  can they reliably detect?
- They can understand an ‘**uncertainty budget**’: which factor in their use of the  $m$ -equation has the greatest fractional uncertainty?
- They can come to understand **reproducibility**: what results do different groups get using the same method, on the same instrument, with the same unknowns?
- They can even have a friendly **competition**: which group can achieve the smallest uncertainty? or the smallest discrepancy, between results measured the old way and the new way?

Alternatively, during the 2015-2018 interval, you can confront students with another outlook: suppose they conduct an ‘electronic weighing’ and a ‘regular weighing’ of the same object. Then their procedure can become a check on the electrical measurement tools that lie behind the values assigned to  $\varepsilon_{gen}$  and  $i_{bal}$ . (Here at TeachSpin we caught a 1.8% systematic error in one of our DMMs-as-ammeter from just this sort of comparison. The results above came using electrical measurements *not* subject to that error.)

We are still a bit amazed that the TeachSpin Torsional Oscillator, using quite basic ideas from mechanics and electromagnetism, can be used to make this upcoming definition of the kilogram eminently understandable. And, if your students understand the ideas behind this definition, and verify it for themselves, they will NEVER forget it. When the formal changeover is made in 2018 or later, they will be telling their students, and anyone else who will listen, that THEY knew all about it ‘way back in 2015, and they even ‘proved’ it in their physics lab!



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## What's a 'Watt Balance'?

and how's it related to the  
**'Electronic Kilogram'?**

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