THE EQUATIONS OF MOTION AND THEIR EFFECT ON ROLL

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Introduction

In order to understand the factors causing a torpedo to roll, it is convenient to study the equations of motion. These equations are obtained by equating the net force and net moment acting on the torpedo to zero in each of the three principal planes. All three planes must generally be considered when non-uniform motion exists because of the interactions causing roll. These interactions will be discussed in this paper, together with methods for estimating some of the more understandable and predictable terms. It is convenient to begin the analysis of roll motion by studying the simplified equations of motion in the roll and yaw planes.

Simplified Roll Moment Equation

In general, a torpedo will operate under conditions of constant velocity, small angles of attack, moderate turn rates and a low heel angle. During certain portions of a run, the torpedo may accelerate or decelerate, reach large angles of attack or heel, and in some instances, may be designed to spin during flight. Factors affecting roll in the second case will be discussed later. The simplified equations of motion considered in this section will pertain only to those torpedoes that operate under uniform speed conditions where the forces and moments are linear and no secondary interactions exist.

Standard torpedo nomenclature is used in the following analysis, and is listed at the end of this paper. The sign convention is shown in Figs. 1 to 3.

The simplified roll moment equation is obtained from Ref. (1) and is written as follows:

ΣK = Sum of the hydrodynamic and static roll moments
= Sum of the inertial roll moments

(1)
$$\Sigma K = \overbrace{K_\beta \beta + K_r r}^{\text{Unequal vertical fins}} + \overbrace{K_{\delta_r} \delta_r}^{\text{Unequal rudders}} + \overbrace{K_p \dot{\phi}}^{\text{Fin damping}}$$
$$\overbrace{- W z_g \phi}^{\text{Pull-around}} + \overbrace{K_o \pm \Delta K}^{\text{Misalignments, etc.}} = \overbrace{J_x \ddot{\phi}}^{\text{Angular accel.}} + \overbrace{m z_g V r}^{\text{Centrif. force}}$$

Even this simplified expression appears somewhat complex, since it shows that in the linear range the roll moment is a function of the constant hydrodynamic coefficients (K_β, K_r, K_{δ_r}, K_p), physical parameters (W, z_g, K_o, $\pm \Delta K$, J_x, m), the yaw angle of attack (β), yaw turn rate (r),

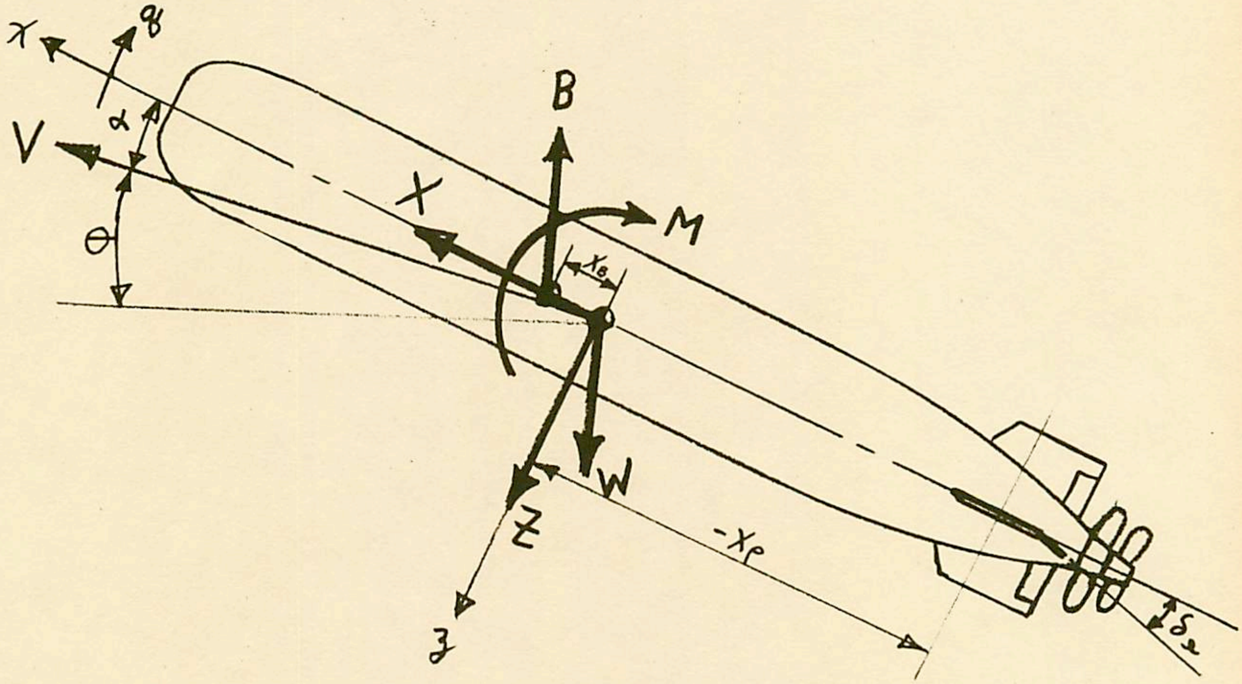

FIG. 1. PITCH PLANE

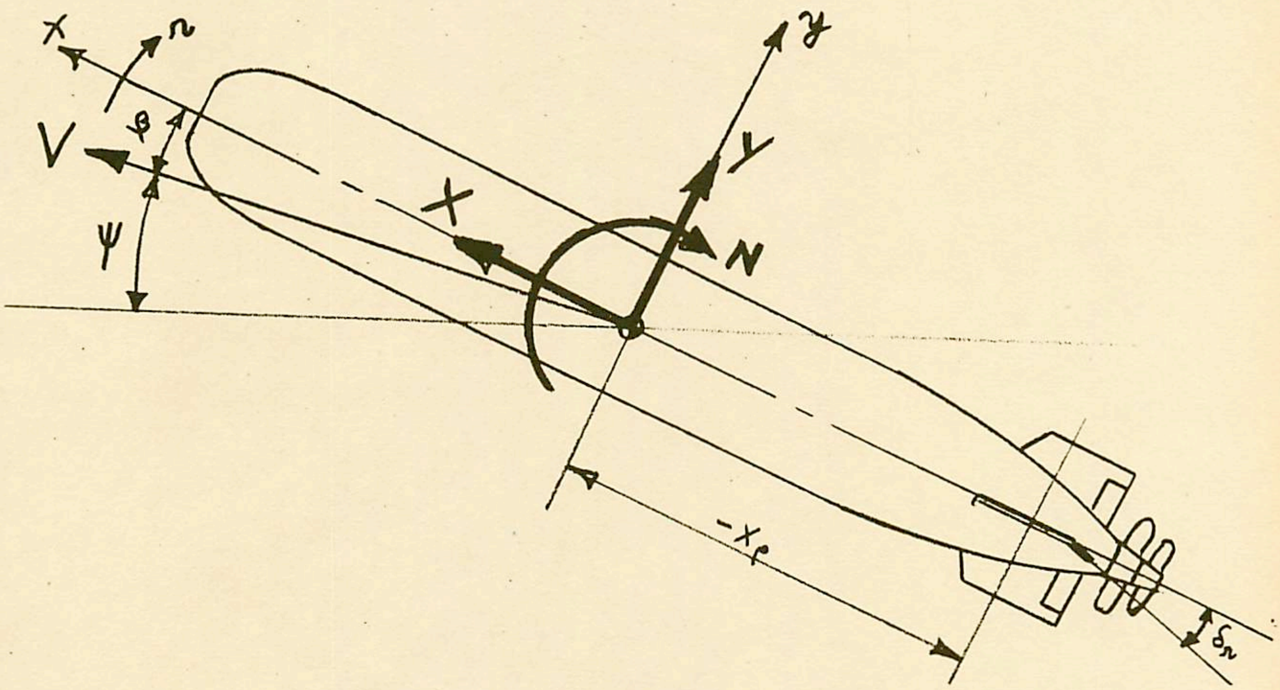

FIG. 2 YAW PLANE

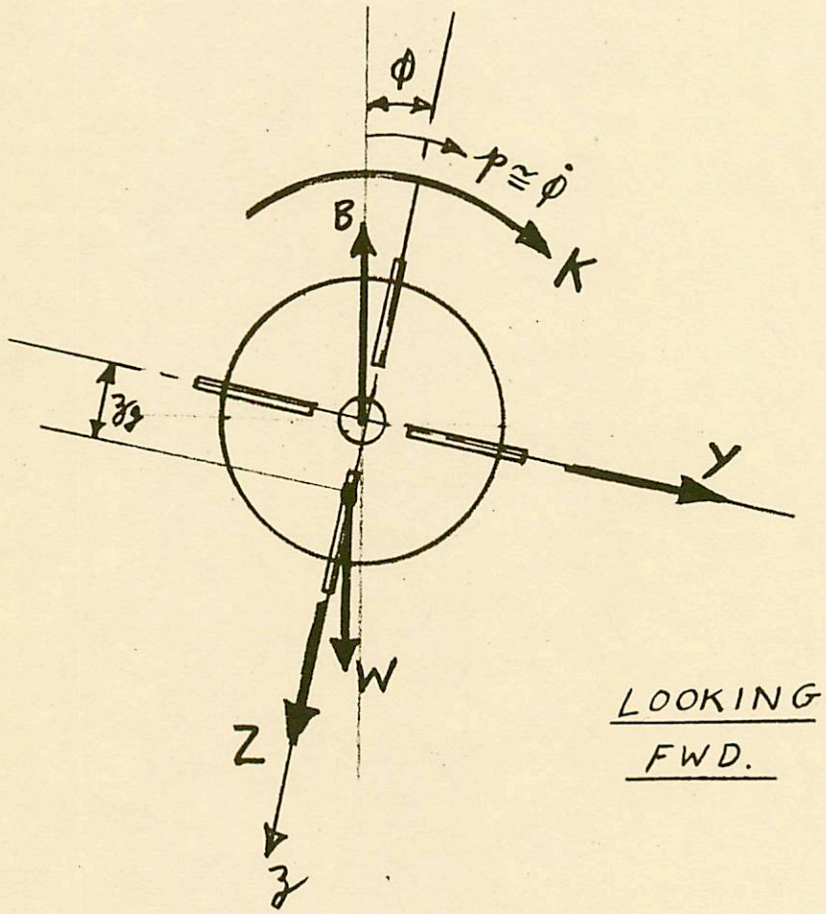

FIG. 3 ROLL PLANE

rudder deflection (δ_r), roll rate ($\dot{\varphi} = \frac{d\varphi}{dt}$), roll angular acceleration ($\ddot{\varphi} = \frac{d^2\varphi}{dt^2}$), roll angle ($\varphi$), and velocity (V). Eq. (1) can be rewritten by collecting terms containing φ as follows:

$$(2) \quad J_x \ddot{\varphi} - K_p \dot{\varphi} + W Z_g \varphi = K_o \pm \Delta K + K_\beta \beta + K_r r + K_{\delta_r} \delta_r - m Z_g V r$$

By considering the special case of a straight running torpedo, it is possible to establish a criteria for determining the correct amount of pull-around. During a straight run $\ddot{\varphi} = 0$, $\dot{\varphi} = 0$, $\beta = 0$, r = 0, and $\delta_r = 0$. Eq. (2) then reduces to:

$$(3) \quad \varphi_{(STRAIGHT)} = \frac{K_o \pm \Delta K}{W Z_g}$$

Since W = torpedo weight (lb), Z_g = vertical distance (ft) of the center of gravity below the center of buoyancy, the term WZ_g is the pull-around moment (or the righting moment if the torpedo was rotated 90°).

K_o is the average roll moment acting on the torpedo due to fin or stator misalignment, splay, control surface misalignment, propeller shaft bearing and seal friction, propeller torque unbalance, off-center location of internal components, etc. $\pm\Delta K$ is the variation in roll moment from the average that might be expected due to the manufacturing tolerances plus changes that might occur with changes in depth or speed. In any torpedo configuration, the average roll moment, K_o, can be reduced to zero by setting the correct splay in the fixed fins or control surfaces, or by locating the center of gravity off to one side. Consequently, the roll angle variation in a straight run can be reduced to:

$$(4) \quad \varphi_{(STRAIGHT)} = \pm \frac{\Delta K}{W Z_g}$$

This shows that the roll angle spread can be reduced either by reducing the manufacturing tolerances or by increasing the pull-around moment. Consequently, if the roll moment fluctuations are known, Eq. (4) can be used to calculate the pull-around moment required to keep the roll angle within given limits.

The next step in analyzing the roll problem is to consider a steady turn when $\dot{r} = 0$, $\ddot{\varphi} = 0$, and $\dot{\varphi} = 0$. Using the subscript "t" to refer to the steady turn conditions, Eq. (2) and Eq. (3) can be combined as follows:

$$(5) \quad W Z_g (\varphi_{TURN} - \varphi_{STRAIGHT}) = K_\beta \beta_t + K_r r_t + K_{\delta_r} \delta_{r_t} - m Z_g V r$$

The term, $(\varphi_{turn} - \varphi_{straight})$ is the change in roll angle from a straight run to that in a turn. In an ideal torpedo design, the roll change will be zero.

The terms K_β , K_r, and K_{δ_r} are the roll moments per unit yaw angle of attack, yaw turn rate, and rudder deflection, respectively. The remaining term is the roll moment caused by the centrifugal force acting on the lowered center of gravity.

Both $K_\beta \beta_t$ and $K_r r_t$ are roll moments in a turn resulting from unequal vertical fins. If, for example, the lower vertical fin is larger than the upper, then a yaw angle of attack, β , would produce sideforces on the two fins that are unequal. The roll moment, $K_\beta \beta$, is the net roll moment caused by these unequal sideforces.

When the torpedo turns in its horizontal plane at a rate, r, a transverse velocity is induced at the tail fins equal to $(-X_e) r_t$. Since the fins are also moving forward at a velocity V, the net velocity vector makes an angle $\beta_i = \tan^{-1} \frac{(-X_e) r_t}{V}$ with respect to the fin centerline. Hence, a small angle of attack, $\beta_i \cong \frac{-X_e r_t}{V}$, is induced at the tail fins. The roll torque caused by this angle of attack is therefore $K_\beta \beta_i \cong K_\beta \frac{(-X_e) r_t}{V}$. This roll moment is also written as $K_r r_t$; hence,

$$K_r r_t = K_\beta \frac{(-X_e) r_t}{V}$$

Consequently, it is seen that K_r and K_β are related in the following manner:

(6) $$K_r = \frac{(-X_e)}{V} K_\beta$$

If the torpedo is correctly designed, the roll angle change $(\phi_{turn} - \phi_{straight})$ at all turn rates should be zero.

Substituting this condition and Eq. (6) into Eq. (5), the desired relationship betweek K_β , K_{δ_r} , and the pull-around is:

$$K_\beta \left[\beta_t + \frac{(-X_e) r_t}{V}\right] + K_{\delta_r} \delta_{r_t} = m z_g r_t$$

dividing by r_t ,

(7) $$K_\beta \left[\left(\frac{\beta}{r}\right)_t - \frac{X_e}{V}\right] + K_{\delta_r} \left(\frac{\delta_r}{r}\right)_t = m z_g$$

The terms $(\frac{\beta}{r})_t$ and $(\frac{\delta_r}{r})_t$ are constants and can be obtained from the yaw equation of motion for a uniform turn. K_{δ_r} is close to zero if the rudders are equal. Otherwise, K_{δ_r} is calculated by estimating the force per unit rudder deflection on each rudder, and then resolving the differential forces times their distance from the torpedo centerline to obtain the roll moment.

K_β is calculated in a similar manner. If the torpedo has a four-finned tail, as shown in Fig. 4, the following method may be used for evaluating K_β :

$$(8) \qquad K_\beta \beta = - L_{\beta u} \beta z_{pu} - L_{\beta l} \beta z_{pl}$$

where, $L_{\beta_{u,l}}$ = lift per radian of the upper or lower fin (lb)

$Z_{p_{u,l}}$ = distance from the centerline to the center of pressure of the upper or lower fin (ft), (+) down, (-) up

$a_{u,l}$ = radius of the body plus boundary layer momentum thickness at the upper or lower fin midpoint (ft)

$b_{u,l}$ = radius to the upper or lower fin tip (ft)

$c_{u,l}$ = average upper or lower fin chord length (ft)

From Ref. (2), the total sideforce of a body plus a pair of fins is:

$$(9) \qquad L_\beta \beta = C_{L_\alpha} \beta (2bc)(1 - a^2/b^2) \frac{\rho}{2} V^2$$

where $C_{L\alpha}$ = lift coefficient derivative for a rectangular fin having an over-all aspect ratio of $\frac{2b}{c}$.

This lift force, $L_\beta \beta$, is distributed over the body and fin. Since the lift force on the body is always directed toward the center, the only force contributing to a roll moment is that occurring on the fin. Ref. (2) also shows that the fin lift/total lift ratio is approximately proportional to their areas, or for rectangular fins, proportional to $\frac{b-a}{b} = 1 - a/b$ in the range $0 < a/b < .3$.

Hence, the net roll moment $K_\beta \beta$ due to unequal fins, obtained from Eqs. (8) and (9), is:

$$K_\beta \beta \cong C_{L\alpha u} \beta (2 b_u c_u)\left(1 - \frac{a_u^2}{b_u^2}\right)\frac{\rho}{2}V^2\left(\frac{1 - a_u/b_u}{2}\right)(-z_{pu})$$
$$- C_{L\alpha l} \beta (2 b_l c_l)\left(1 - \frac{a_l^2}{b_l^2}\right)\frac{\rho}{2}V^2\left(\frac{1 - a_l/b_l}{2}\right)(z_{pl})$$

or,

$$(10) \qquad K_\beta \cong \frac{\rho}{2} V^2 \left\{ C_{L\alpha u} b_u c_u (-z_{pu})\left(1 - \frac{a_u}{b_u}\right)^2\left(1 + \frac{a_u}{b_u}\right) - C_{L\alpha l} b_l c_l (z_{pl})\left(1 - \frac{a_l}{b_l}\right)^2\left(1 + \frac{a_l}{b_l}\right)\right\}$$

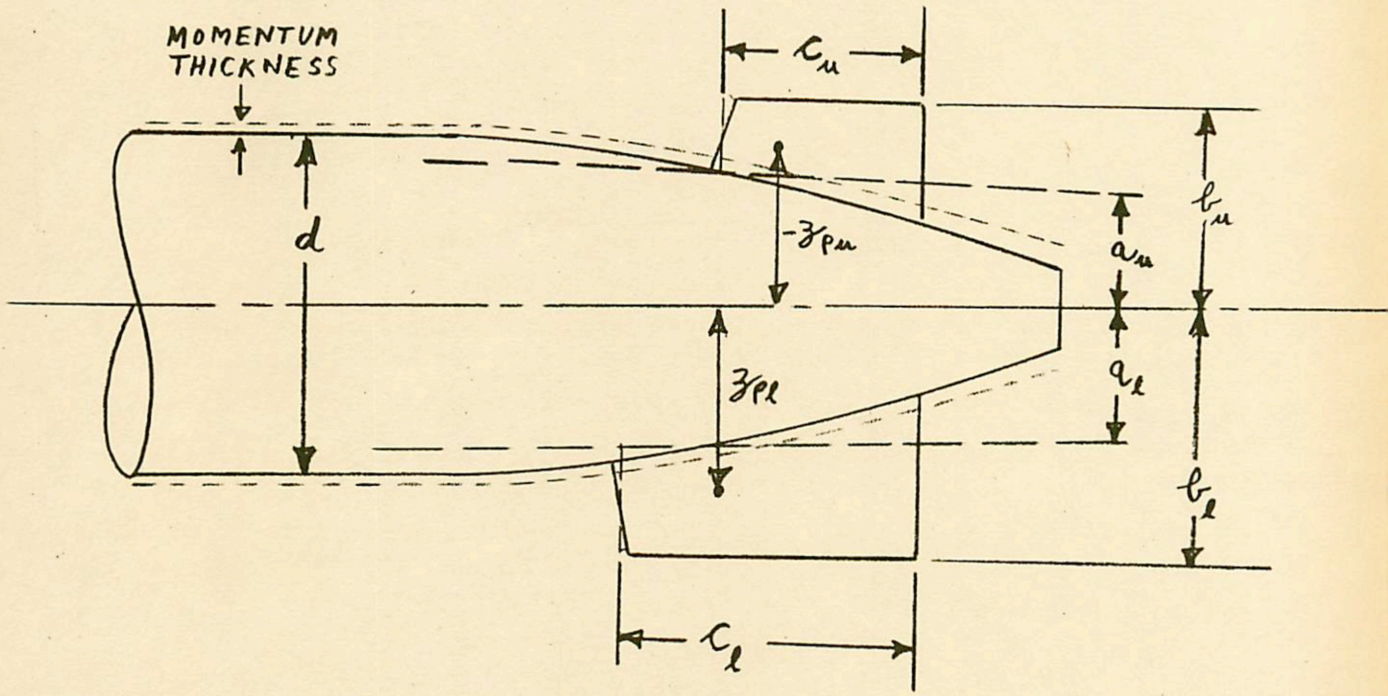

FIG. 4 EQUIVALENT TAIL

Hence, if the physical fin sizes are known, the roll moment, K_β, can be calculated.

In general, if the roll is to remain small, particularly during the transient period between a straight run and a uniform turn, computer solutions show that K_{δ_r} should be small or zero. This means that both rudders should be similar in size. In this case, Eq. (7) reduces to:

$$K_\beta \left[\left(\frac{\beta}{r}\right)_t - \frac{x_e}{V}\right] = m\, z_g$$

so, when $K_{\delta_r} = 0$, minimum heel is obtained when:

(11) $$K_\beta = \frac{m\, z_g}{\left(\frac{\beta}{r}\right)_t - \frac{x_e}{V}}$$

Consequently, a trial-and-error process can be used to compute the correct relative fin sizes from Eqs. (10) and (11). The term $-\left(\frac{\beta}{r}\right)_t$ is obtained from the yaw equation of motion, which will be discussed later.

Referring back to the basic roll moment equation, (Eq. 2), the roll damping moment, K_p, and the roll moment of inertia, J_x, remain to be evaluated. Using average dimensions for a, b, and c shown in Fig. 4, the roll damping moment of four fins is,

(12) $$K_{p\,(4\ \text{FINS})} \cong -\left[\frac{\pi \rho V b^4}{2\left(\sqrt{\frac{AR^2}{16}+1}+1\right)} - \frac{\pi \rho V a^4}{2}\right]$$

This expression for K_p is basically obtained from Ref. (3) but modified by including the body interference and doubled since four fins are now used instead of the single wing considered in Ref. (3). Some interference effects will occur between the four fins as the torpedo rolls and the damping effect is a little different for rectangular fins than for the elliptic wing considered in Ref. (3), but these effects are expected to be small.

The loss of roll damping moment by the presence of the body is $\frac{\pi \rho V a^4}{2}$ and is derived in the following manner. The mass flow lost by the presence of the body in the region of the fins is $m_a = \rho \pi a^2 V$ (slugs/sec). If the body were not present, this mass flow would enter the region of the fins with no angular rotation and leave the region rotating at a rate approximately equal to $\dot{\phi}$, the rotational rate of the fins. Hence, the change in angular momentum per second of this fluid mass is $\frac{1}{2} m_a a^2 \dot{\phi}$ or $K_{p\,(\text{body loss})} = -\frac{\rho \pi a^4 V \dot{\phi}}{2}$

Hence,

(13) $$K_{p\,(\text{BODY LOSS})} = -\frac{\rho \pi V a^4}{2} \quad (\text{ft. Lbs/RAD/SEC})$$

In a similar manner, the net roll damping moment of a pumpjet can be shown to be,

(14) $$K_{p_{(pumpjet)}} = \frac{-\rho Q}{2}\left(b^2 + a^2\right)$$

where: b = radius to the midpoint of the pumpjet shroud (ft)
Q = flow through the pumpjet (ft^3/sec)
$= \pi V_{ave}\left(b^2 - a^2\right)$
V_{ave} = average flow velocity through the pumpjet (ft/sec)
a = body radius at the pumpjet (ft)

The roll damping moment contributed by the torpedo body skin friction is small, so it is generally neglected in all calculations.

The remaining term to be evaluated in equation (2) is J_x, the moment of inertia about the x-axis. This term is the sum of the inertial (I_x) and the virtual (I_{o_x}) moments of inertia as follows:

(15) $$J_x = I_x + I_{ox}$$

A quick method for obtaining an approximate value for I_x, the moment of inertia of the torpedo in air, is to use the following expression,

(16) $$I_x \cong m d^2 \left(.156 - \frac{.0312}{C_p}\right)$$

where m = torpedo mass (slugs)
d = torpedo diameter (ft)
C_p = prismatic coefficient
$= \dfrac{\text{torpedo volume}}{\pi/4\, d^2 \ell}$
ℓ = torpedo length (ft)

The term I_{o_x} is the moment of inertia of the fluid captured by the tail as the torpedo rotates. Ref. (4) shows that the virtual moment of inertia of a wing having a span 2 b, and a chordlength c, is:

(17) $$I_{ox\,(WING)} = \frac{\pi\rho}{4.44}\,\frac{c^2 b^4}{(b + 1.3c)} \quad (\text{SLUG}\cdot\text{FT}^2)$$

The vitual moment of inertia of a four-finned tail is equivalent to that of two wings rotating about their center, times an interference factor. Since no experimental interference factors are known for the four-finned configuration, an interference factor of .9 is assumed.

Hence,

(18) $$I_{ox\,(4\ FINS)} \cong I_{ox\,(WING)}(2)(.9)$$

or combining (17) and (18),

(19) $$I_{ox\,(4\ FINS)} \cong \frac{\pi\rho}{2.46} \frac{c^2 b^4}{(b + 1.3c)}$$

The moment of inertia of the fluid displaced by a circular cylinder is:

(20) $$I_{ox\,(CYL)} = \tfrac{1}{2} m_a a^2 = \frac{\pi\rho c a^4}{2}$$

In the region $0 < \frac{a}{b} < .5$ the presence of the body reduces the virtual moment of inertia of four fins approximately by the amount shown in Eq. (20).

Hence, the net virtual moment of inertia of four fins, including the body interference, is:

(21) $$I_{ox\,(4\ FINS + INTERF.)} \cong \frac{\pi\rho}{2.46} \frac{c^2 b^4}{(b + 1.3c)} - \frac{\pi\rho c a^4}{2}$$

If a pumpjet is used instead of fins, the virtual moment of inertia is simply that of a cylinder of fluid of the same size.

Hence, from Eq. (20),

(22) $$I_{ox\,(PUMPJET)} = \frac{\pi\rho c}{2} (b^4 - a^4)$$

Since the roll moment depends upon the yaw angle of attack and turn rate, the simplified yaw equation of motion is considered next.

Simplified Yaw Equation of Motion

Assuming that the torpedo angles of attack (α and β) are small, the roll angle (ϕ) is small, the velocity (V) constant, the turn rate (r) and rudder deflection (δ_r) are moderate and in the linear range, the simplified yaw force and moment equations can be used. These are obtained from Ref. (1) as follows:

ΣY = Sum of the principal hydrodynamic yaw forces
= Sum of the principal inertial yaw forces

(23) $$\Sigma Y = Y_\beta \beta + Y_r r + Y_{\delta_r} \delta_r + Y_{\dot r} \dot r = -m_T V \dot\beta + m_L V r$$

ΣN = Sum of the principal hydrodynamic yaw moments
= Sum of the principal inertial yaw moments

(24) $$\Sigma N = N_\beta \beta + N_r r + N_{\delta_r} \delta_r + N_{\dot\beta} \dot\beta = J_x \dot r$$

In a steady turn, $\dot{r} = 0$ and $\dot{\beta} = 0$. Using the subscript "t" for steady turn conditions, Eqs. (23) and (24) become:

$$Y_\beta \beta_t + Y_r r_t + Y_{\delta_r} \delta_{r_t} = m_1 V r_t \tag{25}$$

$$N_\beta \beta_t + N_r r_t + N_{\delta_r} \delta_{r_t} = 0 \tag{26}$$

Eliminating δ_{r_t} and combining (25) and (26),

$$\left(\frac{\beta_t}{r_t}\right) = \left(\frac{\beta}{r}\right)_t = \frac{N_{\delta_r}(m_1 V - Y_r) + Y_{\delta_r} N_r}{Y_\beta N_{\delta_r} - N_\beta Y_{\delta_r}} \tag{27}$$

Therefore, $\left(\frac{\beta}{r}\right)_t$ can be obtained if the hydrodynamic coefficients are known from tests or estimates. Ref. (5) is one of a number of sources which describe methods for estimating the hydrodynamic coefficients.

If the torpedo turn rate is in the transient region between a straight run and a uniform turn, the β/r ratio will be somewhat different from $(\beta/r)_t$ in a steady turn. Consequently, the pull-around centrifugal torque will not be completely balanced by the hydrodynamic forces acting on the unequal vertical fins. During this period, the roll can be kept negligibly small if the roll damping moment is sufficiently large. Different torpedo configurations have been programmed on an analogue computer and the results show that the roll is kept to a minimum during this transient period if K_{δ_r} is close to zero. On those torpedoes having only one rudder and low roll damping, the transient roll angle is likely to be high.

Additional Factors Affecting Roll

There are many other factors affecting roll that were not included in the simplified equations of motion. Most of these are difficult to analyze or predict, so they will be qualitatively discussed. It is expected that other presentations at this meeting will treat these factors in more detail and mention others not listed here.

1. Roll and angle-of-attack interaction. If the plane of the angle-of-attack is unsymmetrical with respect to the torpedo tail fins, a roll moment is produced which is proportional to the angle of attack. This roll moment is explained in Ref. (6) as being caused by the boundary layer thickening on the leeward side of the torpedo and decreasing on the windward side. Therefore, a fin on one side of the tailcone would have effectively less area than a geometrically equal fin on the opposite side, so a roll moment is produced. It is interesting to note that an overturning moment is produced when the angle between the torpedo vertical axis and the angle-of-attack plane is close to 0°, 90°, 180°, and 270°; whereas a stabilizing roll moment is produced near angles of 45°, 135°, 225°, and 315°. Maximum

roll moments occur at angles midway between those listed. Consequently, a stable roll condition will occur at pitch angles of attack when the roll angle of a 4-finned torpedo is 45°. At large angles of attack this may be used as a feasible method for roll control.

2. Cavitation. At high speed and shallow depths the stabilizing surfaces, control surfaces, and propellers (or pumpjets) are susceptible to cavitation. The onset of cavitation is sometimes critical and is a function of the profile shape and the angle of attack. Consequently, small differences in the angle of attack of fins near the critical cavitation speed may cause one fin to cavitate before another and produce large changes in sideforce and roll moment.

3. Speed variation. Changes in speed produce changes in torpedo trim, propeller shaft bearing and seal friction, splay torque, angular momentum of the rotating elements, etc. A change in speed will occur at launching, during turns, during changes in depth, and possibly during the run as a function of time. If these effects can be estimated and programmed on a computer, the torpedo roll angle change can be obtained as a function of time and if it is excessive, steps can be taken to reduce the roll. In this case, all the equations of motion presented in Ref. (1) must generally be programmed.

It is interesting to note that Eqs. (5), (6), (10), and (27) can be combined to show that the heel change from a straight run to a steady turn is proportional to the velocity. This fact also indicates that if the heel change is zero at one speed it will be zero at all speeds.

4. Control surfaces. If the rudder deflection and elevator deflection produce changes in momentum of the water flow in the same region, interactions are likely to occur which may cause roll. Also, if an actuator for two control surfaces is located on one side of a yoke, elastic strain in this yoke will produce a differential control surface deflection that may cause roll. A roll moment may also be produced if two control surfaces are splayed and operate in the wake of a fin. In this case, small deflections may produce lift on only one surface that extends beyond the wake, producing a differential force. If either single or counter-rotating geared propellers are used, a control surface deflection may produce a roll moment due to the downwash entering the propellers. This effect is probably more marked when the control surfaces are unsymmetrical.

5. Center of gravity. If the center of gravity is located away from the center of buoyancy in either the vertical or horizontal planes, turn rates and angular accelerations will cause roll due to the inertia effects. As shown in the first part of the paper, some of these inertial forces can be balanced hydrodynamically. The torpedo may also roll if the center of gravity shifts during the run, or if the weight changes. These effects on roll can generally

be estimated and partially counter-balanced by allowing the pull-around to vary during the run.

6. Interactions between the pitch, yaw, and roll planes. Many additional interactions between the three principle planes are mentioned in Ref. (1) such as products of inertia, non-linear combinations of angles of attack and attitude, etc. In general, to fully understand a complex roll problem, it is necessary to identify all the contributing factors and develop methods for estimating their effects. To do this, a combination of the experimental and analytical approach is indispensable.

NOMENCLATURE

a	Radius of the body plus boundary layer momentum thickness at the fin midpoint (ft)
A	Cross-sectional area of the torpedo $= \frac{\pi}{4} d^2$ (ft^2)
AR	Aspect ratio of tail fins or wing, including the body $= \frac{4b^2}{\text{surface area}} = \frac{2b}{c}$ for rectangular fins.
b	Radius to the fin tip or to the midpoint of a shroud ring cross-section (ft)
c	Average chord-length of a fin (ft)
C_{L_α}	Lift coefficient derivative $= \frac{\text{lift or sideforce}}{\frac{1}{2}\rho(2bc)V^2\beta}$
C_p	Prismatic coefficient $= \frac{\text{Torpedo volume}}{A\ell}$
I_x	Moment of inertia of the torpedo in air about the x-axis. (slug · ft^2)
I_{o_x}	Virtual moment of inertia of the fins about the x-axis (slug · ft^2)
J_x	Total effective moment of inertia about the x-axis (slug · ft^2)
K	Roll moment. (+) clockwise looking forward (ft lb)
K_o	Roll moment due to fin splay, etc. (ft lb)
ΔK	Variable roll moment due to misalignments, etc. (ft lb)
K_β	Roll moment per unit yaw angle of attack (ft lb/rad)
K_r	Roll moment per unit yaw turn rate (ft lb/rad/sec)
K_{δ_r}	Roll moment per unit rudder deflection (ft lb/rad)
K_p	Roll moment per unit roll rate (ft lb/rad/sec)
ℓ	Length of torpedo (ft)
L_β	Lift or sideforce per unit angle of attack (lb/rad)
m	Mass of the torpedo $= \frac{W}{g}$ (slugs)
$\dot{m}_a$	Mass flow per second in a given region (slugs/sec)
m_T	Transverse torpedo mass including virtual mass (slugs)

m_L	Longitudinal torpedo mass including virtual mass (slugs)
N	Yaw moment about the C.G. (+) overturning. (ft lb)
N_β	Yaw moment per unit yaw angle of attack (ft lb/rad)
$N_{\dot\beta}$	Yaw moment per unit yaw attack change rate (ft lb/rad/sec)
N_r	Yaw moment per unit yaw turn rate (ft lb/rad/sec)
N_{δ_r}	Yaw moment per unit rudder deflection (ft lb/rad)
Q	Flow through the pumpjet (ft^3/sec)
$r, \dot r$	Turn rate in the torpedo yaw plane and its time derivative (rad/sec, rad/sec^2)
r_t	Turn rate during a steady turn (rad/sec)
V	Torpedo velocity (ft/sec)
X_p	Distance from the center of gravity to the center of pressure of the tail. (−) aft. (ft)
Y	Sideforce in the torpedo yaw plane (+) Starboard. (lbs)
Y_β	Sideforce per unit yaw angle of attack (lb/rad)
Y_r	Sideforce per unit yaw turn rate (lb/rad/sec)
$Y_{\dot r}$	Sideforce per unit turn rate acceleration (lb/rad/sec^2)
Y_{δ_r}	Sideforce per unit rudder deflection (lb/rad)
Z_g	Distance from the center of buoyancy to the center of gravity. (+) downward. (ft)
Z_p	Distance from the center of buoyancy to the center of pressure of a tail fin. (+) downward. (ft) Note: Z_{pu} is (−) while Z_{pl} is (+).
α	Pitch angle of attack (rad)
$\beta, \dot\beta$	Angle of attack and change rate in the torpedo yaw plane (rad, rad/sec)
β_i	Yaw angle of attack induced at the tail by a yaw turn rate (rad)
β_t	Yaw angle of attack in a steady turn (rad)
δ_r	Rudder deflection (rad)
δ_{r_t}	Rudder deflection in a steady turn (rad)

ρ Density of water = 2 (slugs/ft^3)

$\phi, \dot{\phi}, \ddot{\phi}$ Roll angle, rate, and acceleration (rad, rad/sec, rad/sec^2)

Subscripts

"t" Conditions in a steady turn

"u" Dimensions of the upper vertical fin

"l" Dimensions of the lower vertical fin

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