17 May 2013
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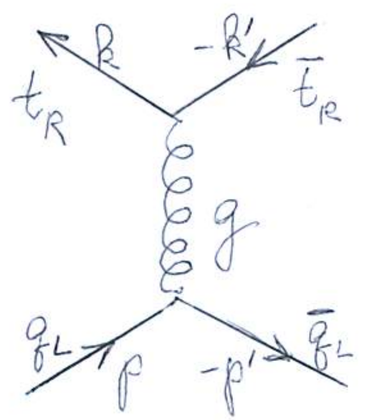
The CDF Top-Antitop Asymmetry

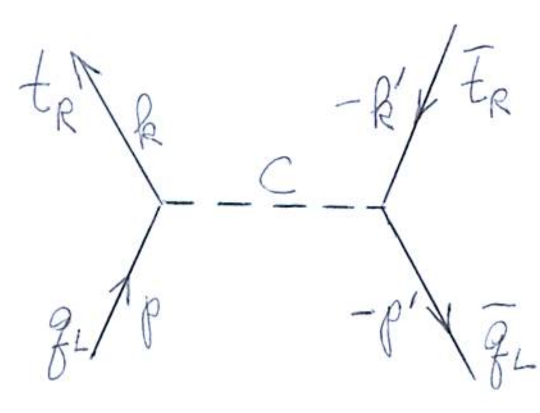
I. Introduction

In previous notes I have considered an extension of the Higgs sector which puts the vanilla Higgs boson and its three Goldstone companions into an electroweak-doublet family nonet. Amongst the members of this nonet are "coset" bosons C containing one and only one third-generation index. They have the same transformation properties under family and electroweak symmetry transformations as $t_R \overline{q_L}$, where q lives in the first or second generation. Of the sundry new states, these coset particles are the most copiously produced at hadron colliders (the mechanism is $q + g \rightarrow C + t$, via t-channel top-quark exchange). But even if their masses are not far above the top quark mass, they are not easy to find at the LHC, because their final-state signature is top-antitop plus quark jet, for which there is a very large background (c.f. arXiv 1209.6593).

These coset bosons do influence top-quark pair production via quark-antiquark annihilation, and produce a forward-backward angular asymmetry. The purpose of this note is to estimate the magnitude of this effect. We shall find that the expectations appear to fit the Fermilab observations quite well provided the masses of the coset states are not far above the top quark mass. Such a scenario is not ruled out by LHC data, although the limits on coupling parameters are not far away from what is given by the family-nonet model.

The Feynman diagrams for the process of interest are shown below.





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It turns out that it is easiest to understand what is going on in the limit of small top quark mass. In Section II we sketch out the arguments appropriate to this limit. In Section III, we carry out the honest calculation. In Section IV we plot the results, and discuss the implications. Section V contains some cross-checks of factors of 2. Section VI is devoted to concluding comments, especially what needs to be done at a more professional level.

II. Simplified Kinematics

In the limit of vanishing top quark mass, and in the absence of the new C-exchange term, the standard model angular distribution has the familiar form

$$\frac{d\sigma}{d\Omega} \sim \left(1 + \cos^2\theta\right) = \left(1 + \cos\theta\right)^2 + \left(1 - \cos\theta\right)^2$$

We have separated the distribution into the contribution of the two distinct helicity amplitudes.

Assuming a left-handed initial-state quark, forward production is dominated by a left-handed final-state top quark. However, the top quark in the C-exchange graph must be right-handed, according to the precepts of the nonet model. Therefore it only interferes with the helicity amplitude which dominates in the backward direction. Furthermore, it turns out that the interference is always destructive. And, as we shall see, at all center of mass energies larger than, or of the order of, the mass of the exchanged C boson, the new amplitude never dominates the primary gluon-annihilation amplitude. Indeed at very high energies, the two amplitudes have the same energy dependence.

Let us now write this out. The invariant amplitudes have the form

$$\mathcal{M}_g = \left(\bar{t}_R \gamma_\mu \lambda_\alpha t_R\right)\left(\bar{q}_L \gamma^\mu \lambda_\alpha q_L\right)\frac{e_s^2}{s}$$

$$\mathcal{M}_C = \left(\bar{t}_R q_L\right)\left(\bar{q}_L t_R\right)\frac{g_C^2}{(t - m_C^2)}$$

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The coupling constants are

$$e_s^2 = 4\pi\alpha_s \qquad g_C^2 \simeq 2\left(\frac{m_t}{v}\right)^2$$

Note that the coupling of the C to the quarks is defined by the model. It is not a free parameter. The kinematic parameters s and t, along with others that we will subsequently need, are

$$p = E(1,1,0,0) \qquad k = E(1, \beta\cos\theta, \beta\sin\theta, 0) \qquad s = (p+p')^2 = 4E^2 = 4m_t^2(1-\beta^2)$$

$$p' = E(1,-1,0,0) \qquad k' = E(1, -\beta\cos\theta, -\beta\sin\theta, 0) \qquad t = (p-k)^2 = m_t^2 - 2E^2(1-\beta\cos\theta)$$

Without troubling ourselves with the details of the color and spinor factors (which will be addressed more professionally in the next section), we can see that the amplitude will be of the form

$$|\mathcal{M}|^2 = (K.F.)\left|\frac{4\pi\alpha_s}{s} + \left(\frac{m_t}{v}\right)^2\frac{1}{(t - m_C^2)}\right|^2 = (K.F_0)\left|1 - \frac{\epsilon s}{(m_C^2 - t)}\right|^2$$

"K.F." stands for "kinematic factors" and is a catchall for all the miscellaneous overall factors which we choose not to write out. Please note that 2 x (K.F.) = (K.F.).

We have also introduced the parameter

$$\epsilon = \frac{g_C^2}{e_S^2} = \frac{2}{4\pi\alpha_S}\left(\frac{m_t}{V}\right)^2 \approx 0.7$$

We also used hindsight (i.e. the calculations of Section III) to set the relative normalization and, very importantly, the relative sign of the two terms in the amplitude. Evidently the parameter expresses the relative importance of the new C-exchange term to the standard-model gluon-exchange term.

It follows that, upon squaring the amplitude and summing over spins and colors, the result must be of the form

$$|\mathcal{M}|^2 = (K.F.)\left[2(1+\cos\theta)^2 + (1-\cos\theta)^2 + (1-\cos\theta)\left|1-\frac{\epsilon s}{(m_C^2-t)}\right|^2 + \frac{1}{8}(1-\cos\theta)^2\frac{\epsilon^2 s^2}{(m_C^2-t)^2}\right]$$

$$= (K.F.)\left[(1+\cos^2\theta) - \frac{\epsilon s}{2(m_C^2-t)}(1-\cos\theta)^2 + \frac{9}{32}\frac{\epsilon^2 s^2}{(m_C^2-t)^2}(1-\cos\theta)^2\right]$$

We have separated out non-interfering contributions associated with initial-state right-handed quarks and final-state left-handed top quarks. In addition, when the quark-antiquark is in a color-singlet initial state, the gluon-annihilation amplitude vanishes, while the new C-exchange amplitude does not. This provides the final term in the above expression.

Therefore, we see that the modifications always decrease the cross section. And when the cms energy is small compared to the mass of the C, the effects are evidently very small. In the other extreme, we find that the effective perturbation parameter is

$$(1-\cos\theta)\frac{\epsilon s}{(m_C^2-t)} \rightarrow 2\epsilon$$

The cross section in the opposite, high-energy limit, $s >> m_C^2$, is very simple:

$$|\mathcal{M}|^2 = (K.F.)\left[(1-\epsilon+\frac{9}{8}\epsilon^2) + \epsilon\cos\theta + \cos^2\theta\right]$$

Given our value of $\epsilon \approx 0.7$, in this limit we find that the forward and 90 degree cross sections are slightly enhanced, while the 180 degree cross section is depleted by a factor of about 0.6. These numbers are not far away from what is needed to explain the asymmetry of about 0.2 observed at Fermilab.

III. The Detailed Calculation

With these preliminaries, we turn to the full computation. The square of the gluon-annihilation graph, summed over colors and spins, takes the form

$$\sum{}' |\mathcal{M}_g|^2 = \left(\frac{4\pi\alpha_s}{s}\right)^2 \left[\mathrm{Tr}\,(-\not k' + m_t)\gamma_\mu \lambda_\alpha (\not k + m_t)\gamma_\nu \lambda_\beta\right]\left[\mathrm{Tr}\,(-\not p')\gamma^\mu \lambda_\alpha \not p\, \gamma^\nu \lambda_\beta\right]$$

The interference term is

$$\mathrm{Re}\sum{}' \mathcal{M}_g \mathcal{M}_c^* = \left(\frac{4\pi\alpha_s}{s}\right)\left(\frac{m_t^2}{V^2}\right)\frac{(-1)^3}{(m_c^2-t)}\left[\mathrm{Tr}\,(-\not p')\gamma^\mu \lambda_\alpha \not p (\not k + m_t)\gamma_\mu \lambda_\alpha (-\not k' + m_t)\left(\frac{1-\gamma_5}{2}\right)\right]$$

Finally, the C-exchange term is

$$\sum{}' |\mathcal{M}_c|^2 = \left(\frac{m_t^2}{V^2}\right)^2 \frac{1}{(m_c^2-t)^2}\left[\mathrm{Tr}\,(\not k + m_t)\not p \left(\frac{1+\gamma_5}{2}\right)\right]\left[\mathrm{Tr}\,(-\not k' + m_t)(-\not p')\left(\frac{1+\gamma_5}{2}\right)\right]$$

In the above expressions, the symbol Tr denotes a trace operation over both color and spin degrees of freedom the rules for the color trace are from Peskin and Schroeder:

$$\mathrm{Tr}\,\lambda_\alpha \lambda_\beta = \frac{1}{2}\delta_{\alpha\beta}\,\mathrm{tr} \qquad \mathrm{Tr}\,1 = 3\,\mathrm{tr}$$

Here the symbol tr denotes the trace only over Dirac matrices.

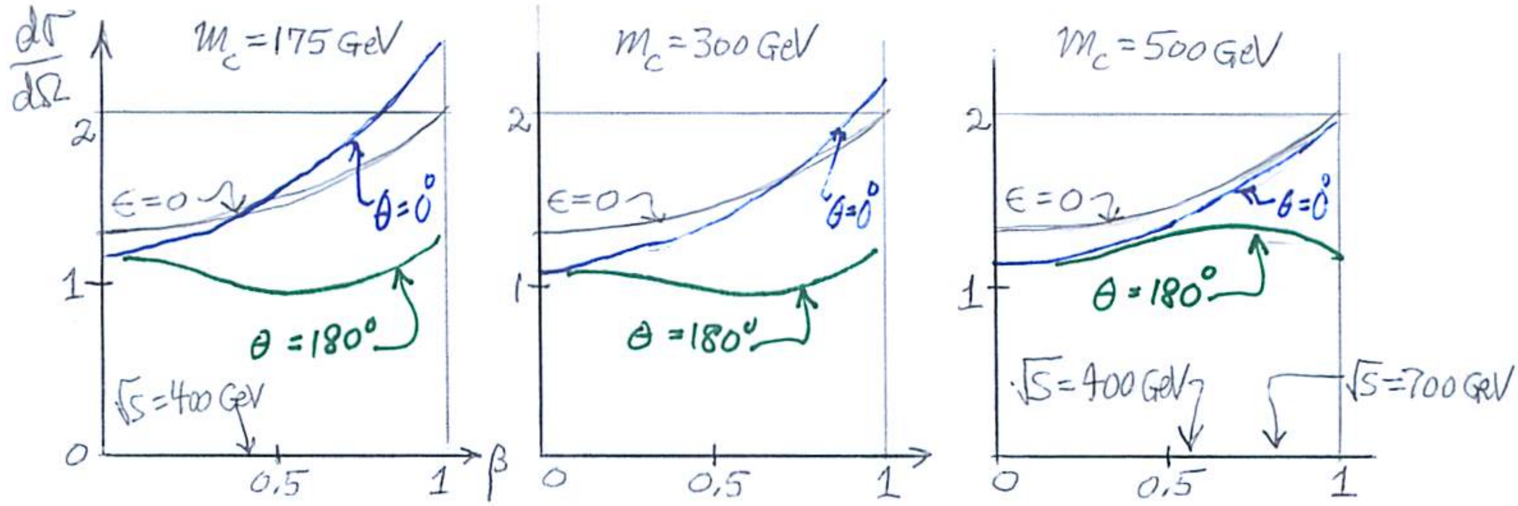
Crucial is the sign of the interference term, relative to the others. There are three minus signs. One is that the gluon propagator has opposite sign to the C propagator, in our standardized Bjorken-Drell notation. Another is that t is spacelike, while s is timelike. Finally there is a Fermi-statistics minus sign (-1 per closed loop). After carrying out the standard trace calculations, we obtain

$$\sum |\mathcal{M}_g + \mathcal{M}_c|^2 = (\mathrm{K.F.})\left\{\begin{array}{l} \left[k\cdot p\; k'\cdot p' + k\cdot p'\; k'\cdot p + m_t^2\, p\cdot p'\right] \\ -\dfrac{\epsilon s}{(m_c^2-t)}\left[k\cdot p\; k'\cdot p' + \dfrac{m_t^2}{2}\, p\cdot p'\right] \\ +\dfrac{9}{16}\dfrac{\epsilon^2 s^2}{(m_c^2-t)^2}\left[k\cdot p\; k'\cdot p'\right] \end{array}\right\}$$

$$= (\mathrm{K.F.})\left\{\begin{array}{l} [(1+\beta^2\cos^2\theta) + \frac{1}{4}(1-\beta^2) \\ -\dfrac{\epsilon(1-\beta\cos\theta)^2}{\left[(1-\beta\cos\theta) + \frac{(m_c^2-m_t^2)}{2m_t^2}(1-\beta^2)\right]} + \dfrac{9}{8}\dfrac{\epsilon^2(1-\beta\cos\theta)^2}{\left[(1-\beta\cos\theta)+\frac{(m_c^2-m_t^2)}{2m_t^2}(1-\beta^2)\right]^2} \end{array}\right\}$$

IV. Results

We plot below the zero-degree and the 180-degree cross sections as function of β. Also shown on the horizontal axis are typical values of the center-of-mass energy $\sqrt{s}$. We have chosen three choices for the mass of the exchanged C boson.



We see that, for this mass range for the C, the asymmetry effect is robust and quite insensitive to the detailed choice of cms energy or mass of the C. Furthermore, as already mentioned in the introduction, the new amplitude never dominates the gluon-annihilation amplitude. This makes detection of the asymmetry at the higher LHC energies difficult, because of competing background processes.

V. The Yukawa Coupling

Because the effect is so sensitive to getting all factors of two correct, in this section we check to make sure the value of the coupling constant of C to quarks was computed correctly.

In the limit we have taken, the Yukawa term couples the right-handed top quark to all the left-handed quarks. In what follows, we will only need three of the six, namely u, t, and b:

$$\mathcal{L} = f\,\bar{q}_L \Phi M q_R + h.c. \simeq f\left\{ \left[\bar{u}_L C + \bar{t}_L\left(\frac{v+h+i\omega_3}{\sqrt{2}}\right) + \bar{b}_L\left(\frac{\omega_1 - i\omega_2}{\sqrt{2}}\right) + \cdots\right] m_t t_R \right\} + \cdots$$

Note that the C and the Goldstone bosons $w^{\pm}$ have canonical normalizations:

$$C = \frac{(H_1^3 + i\omega_1^3)}{\sqrt{2}} \qquad H_1^3 = (H_1^3)^\dagger = H_3^1 \qquad \omega^{\pm} \equiv \frac{(\omega_1 \pm i\omega_2)}{\sqrt{2}}$$

$$\omega_1^3 = (\omega_1^3)^\dagger = \omega_3^1$$

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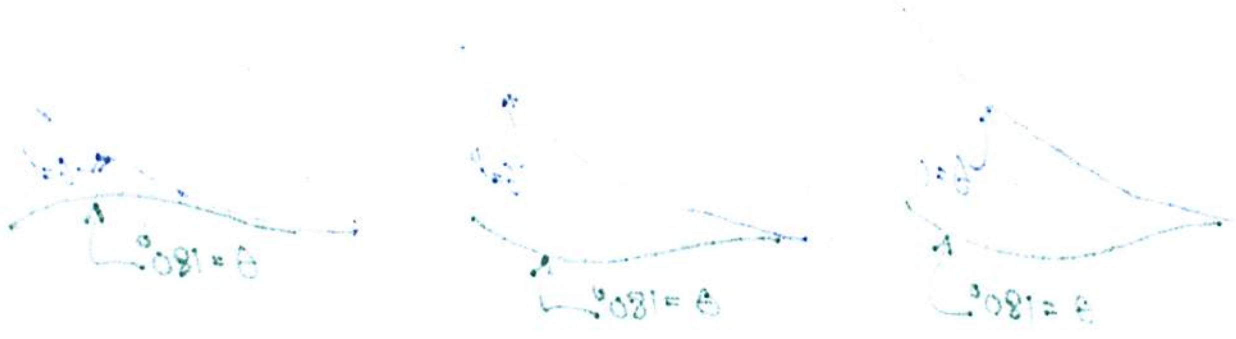
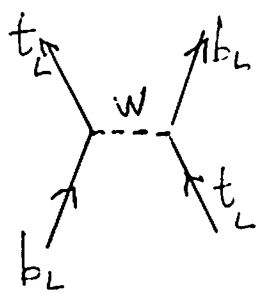
The condition determining the mass of the top quark is simply

$$\frac{fv}{\sqrt{2}} = 1$$



To check the normalization of the vev v, we look at W-exchange in the gaugeless limit (and with the b-quark mass set to zero):



$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\, \bar{t}\gamma_\mu(1-\gamma_5)b_L \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_W^2}\right)\left(\frac{m_W^2}{m_W^2 - q^2}\right)\bar{b}_L\gamma^\nu(1-\gamma_5)t$$

$$\xrightarrow[(q\to 0;\, m_W \to 0)]{} \frac{4G_F}{\sqrt{2}}(\bar{t}\not{q}b_L)\frac{1}{q^2}(\bar{b}_L\not{q}t) = \frac{4G_F m_t^2}{\sqrt{2}}(\bar{t}b_L)\frac{1}{q^2}(\bar{b}_L t)$$

In the gaugeless limit, $m_W \to 0$, and the exchange is of the Goldstone boson w^-. Therefore

$$2\sqrt{2}G_F m_t^2 = f^2 m_t^2$$

Since

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \qquad (v = 246\ \mathrm{GeV})$$

it follows that

$$f = \frac{\sqrt{2}}{v} \qquad g_c \cong 1$$

This agrees with the convention used above, and in the previous sections of this note. The value of $g_c \cong 1$ is in marginal agreement with the LHC data cited earlier (cf. arXiv 1212.1718 and 1107.4364).

VI. Concluding Comments

The success of this scenario in dealing with the Fermilab top-antitop asymmetry would seem to intensify the need to search for the C at the LHC. Its mass is loosely constrained to be 350 ± 150 GeV. The existing limit as quoted earlier is close to the discovery level. Given the specificity of the properties of the C, this may allow more focused strategies for removing some of the copious background and isolating a signal.

Meanwhile, it must be remembered that the basic idea is not yet fully viable from a theoretical point of view. A scenario that includes all the masses and mixings in a consistent way is important to construct. I am cautiously optimistic that this can be done. However, even if this is successfully done, it does not follow that the results will be consistent with the precision electroweak constraints and the limits on rare family-symmetry-violating processes.

VII.Erratum

I have worked on this material for months. But only upon doing the final edit of this pdf did I discover a grievous error. I have assumed only one member of the Higgs nonet is exchanged between the quarks, when actually there are two distinct bosons with distinct masses which are exchanged. The first one, denoted C, is described by the field H_1^3, while the second one, denoted in previous notes as G, is described by the field $(w_3)_1^3 \equiv z_1^3$. The claim on p. 5 that $H_1^3 = H_3^1$ is simply wrong. Upon sorting out the associated errors, the bottom line is that the starting expression on p. 2 is modified as follows:

$$\frac{g_C^2}{(t-m_C^2)} = \frac{2\left(\frac{m_t}{v}\right)^2}{(t-m_C^2)} \Rightarrow \left(\frac{m_t}{v}\right)^2\left[\frac{1}{(t-m_C^2)}+\frac{1}{(t-m_G^2)}\right]$$

Therefore the damage done by this stupid error is quite limited. If the masses of C and G are near each other, the conclusions of this note are essentially unchanged. If the mass of one of the two is very large, there is clearly dilution. So, while the parameter determination is more involved, the bottom line---that a Higgs family nonet can account for the Fermilab top-antitop angular asymmetry----appears to remain intact.

Furthermore, the search for the C and the G becomes a bit more difficult. The independent cross sections for production of either C or G are evidently decreased. And even if the C and G are near-degenerate in mass, the search algorithms will evidently be affected in a serious way.

Finally, I offer apologies for not repairing this entire note. I am frankly both demoralized and fatigued by the discovery of this error. I hope to regain enough strength in the future to get the repairs done.