The Uncovered Set and Its Applications

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"Politics is the art of the possible" -- Otto von Bismarck

1 Introduction

Virtually all democracies use majority rule in parliamentary decision-making. Throughout business and society, decisions involve voting almost always use some form of majority rule. Many equate majority rule with democracy itself and with decent decision-making rules more broadly. While plurality rule, run-off elections and other methods of voting are discussed in other chapters of this book, some variant of pairwise majority rule stands at the heart of most voting procedures aimed at aggregating individual preferences into social decisions about policy.

Social choice theorists have paid considerable attention to *the majority rule program*. That asks: given a set of alternatives and voters or legislators who have preferences over those alternatives, what outcomes may ensue given majority rule? Of course, outcomes are shaped by procedures that determine the set of alternatives under consideration. However, these constraints are typically endogenous and themselves subject to majority vote. Knowing what decision-makers want, and assuming their control of procedure, what end result we should expect? How does majority rule influence political outcomes of interest, from party influence to electoral reforms and the stability of democracy itself?

This chapter summarizes the contribution of a group of scholars, that includes the authors of this chapter but also, Jacob Bauer-Bir, Ivan JeliaskovJ, Nicholas D'Amico, Gyung-Ho Jeong, Michael Lynch and Gary Miller[1] to the majority rule program using technique for estimating the uncovered set given ideal points embedded in a two-dimensional space where the location of the ideal points may be empirically estimated. We start with a briefly review of the path, from Black's Median Voter Theorem to the McKelvey-Schofield "chaos theorems" that set the stage for the primacy of applying the uncovered set as a solution concept and its application.

[1] Please consult the reference list for the exact contribution of each member of this group to the overall effort.

2. Majority Rule: What We Don't Know And Why It Matters

While real-world legislatures vary in many ways, the use of pairwise majority rule is widely accepted to both determine procedures and select policy outcomes. Thus, an understanding of how majority rule works is essential to address the most fundamental question about legislatures: what outcomes are possible? In the study of real-world legislatures, the set of possible outcomes provides a baseline for assessing the impact of behaviors such as agenda setting, strategic voting, bargaining, and party organization. While legislative scholars believe that these behaviors occur and have a significant influence on outcomes. , it is difficult to verify such claims or to attribute particular outcomes to the use of such behaviors without a characterization of the baseline.

Consider the debate over party organization in the U.S. Congress. One side (e.g., Aldrich and Rohde 2001) argues that the majority party can largely determine the outcome of legislative proceedings through agenda control. The other side (e.g., Krehbiel 1999) argues that agenda control conveys no special power and that the majority party's apparent influence stems from the fact that by definition it has more members (and votes) than the minority. Suppose we see a policy outcome that favors the majority party. One inference is that party leaders used agenda control to produce this outcome – and that otherwise a substantially different outcome would have emerged. To evaluate this assertion, we need to know what would have happened under unconstrained majority rule. It may be that this inference is true, but it is also possible that this same outcome would have resulted if party leaders simply let the legislative process play out its way without restriction. After all, their party has the majority on its side.

This dispute embodies a fundamental question about legislative decision-making, namely, how much do parties matter? That is, if we are trying to explain why a particular proposal was enacted, defeated, or never even brought up for debate, must we consider agenda-setting efforts of majority party leaders as a potential explanatory variable? Alternatively, are legislative outcomes fully explained by what individual legislators are willing to vote for, with party leaders having no influence beyond the votes they cast as members of the chamber, or is it all about party organization and leadership?

The two sides of this debate imply different predictions about relationships between legislators' preferences and legislative rules on the one hand and policy outcomes on the other. If agenda control conveys an advantage to the leadership of the majority party, a change in party control can be expected to alter outcomes, even if the distribution of legislators' preferences in the chamber stays the same. Under the agenda control scenario, outcomes will also be sensitive to changes in the preferences held by majority-party leaders, changes in their agenda power, and changes in the internal structures of parties and the way they conduct business. If, on the other hand, agenda control is irrelevant, a change in majority-party status will by itself have little if any effect on legislative outcomes. Rather, outcomes will be sensitive to changes in the preferences of the legislature as a whole.. Thus, explanation of legislative outcomes in the contemporary Congress, or any other legislature, requires a resolution of the debate over the role that party organizations and party leaders play in shaping these outcomes, which in turn requires an understanding of majority rule.

More generally, a central claim of both theoretical (Shepsle, 1979) and empirical (Binder, 2006) legislative studies is that institutions matter – that outcomes are shaped by factors such as the organization of committees and the rules governing floor proceedings. While there is no doubt that all legislatures and most real-world groups make decisions using a dense set of formal and informal procedures and rules, it is not clear whether these restrictions generate stability, advantage, or any effect at all. The reason is the lack of a benchmark. If we do not know what would have happened in the absence of these conjectured institutional effects, it is impossible to attribute any outcome to their selection. The persistence or stability of an outcome across legislative sessions may reflect a compromise between legislators holding different preferences across time and legislative sessions, the effect of formal or informal rules, the fact that few outcomes are enactable given legislators' preferences, or simply a trivial consequence of sheer luck. In this chapter we argue that the persistence of some outcomes and the short lives of others is a reflection of much more profound attribute of majority rule: the uncovered set that it implies as its most fundamental solution concept.

3. The Majority Rule Program

Concerns about the predictability or the normative characteristics of majority rule are not new. Early work on group decision-making, including that by Condorcet, Dodgson and others, aimed at exactly the question discussed here: when a group of individuals use a voting rule to select an alternative from a set, which alternative or alternatives will they pick? With a few notable exceptions, these analyses focused on majority rule.

For example, in Black and Newing's (1951) seminal work on voting, the use of majority rule is assumed in the very first sentence. And while more general analyses such as Arrow (1951) prove results for entire classes of preference aggregation functions, the overwhelming majority of the examples and results are illustrated using majority rule. In fact, aside from a few references to scoring rules such as Borda (see Chapter 5.2), majority rule is the only voting procedure listed in Arrow's index. We say that alternative *x beats y* if more voters prefer *x* to *y* than prefer *y* to *x*. A *Condorcet winner* is an alternative that beats every other alternative, but a configuration of alternatives and voter preferences may fail to have such a winner because of the cycling problem discussed in Chapters 3.1 and 4.1.

The best-known result concerning majority rule of research is Black's (1948) Median Voter Theorem (MVT), discussed in (Hotelling 1929; Black 1948; Downs 1957; cf. Chapter 4.1: *if all possible outcomes can be characterized as points on a one-dimensional space, individuals have single-peaked preferences, and the agenda is open, i.e., any individual can make any proposal, the expected outcome of majority rule voting is the ideal point of the median voter.* A Condorcet winner is a point or a policy position that beats, in a pairwise contest, any other policy position in the set of all possible policy positions. The ideal point of the median voter is the Condorcet winner in a one-dimensional environment. As suggested by several other chapters in this volume, the MVT has been applied in a variety of electoral and legislative settings to predict voting and outcomes and to analyze the impact of a variety of institutions from agenda setters to expert committees.

While the MVT has many implications, its applicability is limited to the case of a single policy dimension, which eliminates many of the complexities inherent in collective choices that involve tradeoffs across different dimensions and typically produce majority cycles. For example, budget constraints imply that spending more on one program requires a reduction in spending on others, and choices regarding a new entitlement such as guaranteed health care require decision-makers to consider both costs and access. The MVT cannot account for how such complexities shape final outcomes.

Accordingly, a principal task in the majority rule program in the wake of the MVT was to generalize those results to cases where preferences and outcomes are better characterized by two or more policy dimensions, allowing more complex and more relevant real-world situations. The fact that the MVT does not generalize to multiple dimensions implies that outcomes in these situations are sensitive to agendas, voting rules, and other constraints. The so-called 'Chaos Theorems' of McKelvey (1976 and 1979) and Schofield (1978) state that majority rule almost always exhibits global cycles and therefore majority-based decision making, unchecked by institutions, can go "from anywhere to anywhere," rendering the ultimate outcome of legislative action indeterminate in the absence of institutional constraints.

4. The Uncovered Set and Its Estimation

Given a non-spatial environment such as that discussed in Chapter 3.1, Miller (1980) proposed the 'uncovered set' a social choice set of interest. Alternative *x covers y* if *x* beats *y* and *x* also beats every alternative beaten by *y*. The *uncovered set* (UCS) is the set of alternatives not covered by any other alternatives. Miller showed that the UCS always exists, is equal to the

Condorcet winner if one exists, is a subset of the Pareto set[1], and that an uncovered alternative beats every other alternative in at most two steps, i.e., if *x* is uncovered, for any other alternative *y*, either *x* beats *y* or *x* beats some *z* that beats *y*. Miller (1980) further showed that if voters look ahead and consider the consequences of their earlier votes on final outcomes (that is, if they vote in a 'sophisticated' fashion), outcomes under standard amendment procedure all belong to UCS, and also that electoral competition between two 'office-seeking' candidates or political parties likewise produces outcomes in the UCS. Miller made some conjectures about the size and central location of the UCS in a two-dimensional spatial context when voters have Euclidean preferences (see Chapter 4.1), which were subsequently demonstrated formally by McKelvey (1986).

In the electoral context, the attractiveness of the uncovered set as a solution concept lies in the fact, as Cox (1987: 419) puts it, "If one accepts the . . . assumption that candidates will not adopt a spatial strategy Y if there is another available strategy X which is at least as good as Y against any strategy…, and is better against some of the opponent's possible strategies, then one can conclude that candidates will confine themselves to strategies in the uncovered set."

While Cox's argument focuses on candidates and electoral politics, its logic applies to legislatures and legislation: Outcomes that lie outside the uncovered set are unlikely to be seriously considered by sophisticated decision makers, who know that such proposals are unlikely to survive given other proposals that are likely to be made. Strategic legislators should therefore eliminate covered points from voting agendas. Instead of promoting outcomes that are quite likely to be defeated later in the game, sophisticated legislators should promote points in the uncovered set that may survive the voting process. Regardless of where the 'status quo' is when voting begins, there is a simple two-step (amendment) agenda that yields some point in the uncovered set as its final

1 An alternative *x* belongs to the *Pareto set* if, for every other alternative *y*, at least one voter strictly prefers *x* to *y*.

outcome. Thus supporters of outcomes in the uncovered set can secure these outcomes using relatively simple agendas and, moreover, can defend them against attempts to overturn them by opponents who propose outcomes outside the uncovered set. This logic suggests that the set of enactable proposals that may be chosen by legislative bodies is restricted to the uncovered set. Thus, if we know which outcomes are in the uncovered set, we know what is possible in a legislative setting—what might happen when proposals are offered and voted on.

Unfortunately, in all but the simplest spatial settings (e.g. Epstein, 1997; Feld et al. 1987), it was hard to determine the size, shape, or the location of the uncovered set, even if the preferences of decision-makers could be measured with precision. In fact, for a while, the calculations were believed to be computationally intractable (Nurmi, 1995).

Absent an applicable general solution concept to serve as an explanation and a predictive tool for majority rule, attention turned to explaining majority rule outcomes in terms of the institutions under which voting takes place. The work of many scholars shows that institutions such as agenda control, committee systems and germaneness can combine to force complex multidimensional issues into a series of one-dimensional, single-issue votes (Shepsle 1979; Cox and McCubbins 2005). In this way, the search for explanations of legislative outcomes in the U.S. Congress and elsewhere has focused on institutions such as party organizations, leaving the most fundamental institution, majority rule itself, largely unexplored.

We believe that our work, as described in the following section has 'salvaged' the uncovered set as a very potent solution concept by marrying advances in computational technology with a technique that was hiding in plain sight. In his seminal article on the uncovered set, Richard McKelvey (1986: 291) noted the difficulty of demarcating the uncovered set, but he pointed out that the two-step principle 'gives a potential "brute force" method for computing the size, shape, and location of the uncovered set to any desired degree of accuracy. One could simply check whether a point is beaten in one or two steps by all other points] on some fine-enough grid." At the time McKelvey's work was published, his observation was purely theoretical – very few scholars had access to computing capacities needed for such grid-search procedures. However, by the late-1990's, with Pentium-based desktop computers available at low cost, computing power was no longer a constraint. Our work has simply implemented McKelvey's insight with this more recent technology.

Our technique for estimating the uncovered set treats the policy space as a collection of discrete points rather than a continuous space. It recovers an approximation of the uncovered set in continuous space, and the approximation converges on the true uncovered set as the resolution of the grid increases indefinitely (see Bianco et al 2004). For the cases treated here, the ideal points and outcomes are located in a two-dimensional space, but the algorithm as subsequently developed can deal with higher dimensionality just as well.

To begin to apply McKelvey's intuition, we start with a configuration of ideal points in a two-dimensional space and compare points across a coarse grid to determine the general location of the uncovered set. For each point x in the coarse grid, we compare x to every other point y in the grid. For each point y that beats x, we iterate again to find all points z that beat y. If x beats z, then x is not covered by y. However, if every such z beats x, then x is covered by y and we mark x for removal from the set of potentially uncovered points. With the set of potentially uncovered set roughly demarcated, we then check whether points within it are covered in a finer grid, and so forth (Bianco et al. 2004).

4. Validating the Uncovered Set: Predictive Power in Experimental Settings

With a technique in hand to estimate uncovered sets, we first set ourselves to establish that the theoretical attractiveness of the uncovered set is matched by predictive power of real-world behavior. For starters, we undertook to analyze data from classic majority-rule voting experiments conducted by others (Bianco et al. 2006) and later we analyzed data from experiments we conducted ourselves (Bianco et al., 2008).

Experiments are useful in underlining generic aspects of social processes. In this case we have two competing hypothesis at stake. In a famous quote, Riker (1981, 447) concluded, based on the "chaos theorems" and related results that: "Politics is *the* dismal science because we have learned from it that there are no equilibria to predict. In the absence of equilibria we cannot know much about the future at all." Ironically, Riker wrote this just around this time, Miller (1980) and McKelvey (1986) advanced the uncovered set concept as a self-regulating mechanism that turns out to be a by-product of the use of majority rule by sophisticated voters (Cox, 1987). While our experiments were set up specifically to see if the UCS had a significant predictive effect on individuals engaged in the use of majority rule in a collective choice environment, the classic experiments predating the UCS had a very similar design.

A seminal paper by Fiorina and Plott (1978) reports on a series of classic voting experiments. It describes 16 theories (solution concepts) that make competing predictions about what outcomes will be chosen by a committee under various settings. Fiorina and Plott ran a series of three committee-voting experiments in controlled laboratory conditions, which led the authors to reject twelve of the 16 competing theories. Such critical tests would be extremely difficult to do with real-world committee data.

Our analysis of the Fiorina-Plott experiments along with a dozen others (Bianco et al 2006) found that the uncovered set was an extraordinarily good predictor of experimental outcomes. Consider the Fiorina and Plott experiment. Outcomes from their Series 3 are shown in Figure 1 with the uncovered set in grey. Fiorina and Plott (590) were unsure how to interpret the results of these experiments, as none of their sixteen solution concepts exhibited much predictive power, but they observed that "the pattern of experimental findings does not explode [as suggested by the Chaos Theorems], a fact which makes us wonder whether some unidentified theory is waiting to be discovered and used."

*** Figure 1 About Here ***

Our analysis of this experimental data proceeded as follows. First, given a set of ideal points and experimental outcomes, we calculate and plot the estimated uncovered set, overlay the outcomes, and assess whether these outcomes are contained within the estimated uncovered set. As the figure shows, most (twelve of fifteen) of Fiorina and Plott's committees chose final policies located inside the uncovered set and one chose a policy very close to the boundary of the uncovered set. The uncovered set turns out to be a much better predictor of policy outcomes than any of the sixteen theories tested by Fiorina and Plott.

In our experiments (Bianco 2008), each participant was assigned a unique ideal point, told its location in a two-dimensional policy space and that his or her monetary payoffs would decline the more distant the final outcome chosen by the committee was from this ideal point, inducing Euclidean preferences. To avoid any status quo related issues, voters started at a quo point outside the Pareto set. Voting then proceeded using an open agenda and random recognition procedure. The recognized participant proposed a point on the space that could be discussed if participants so desired. At the conclusion of the discussion, participants voted openly to accept or reject the proposal using majority rule. Participants could then choose to vote whether to continue or adjourn, again by majority rule. If they voted to continue, another participant was recognized and the process repeated. If the participants voted to adjourn, the last proposal receiving majority support was the outcome. These experiments were designed to test the fundamental intuition of the uncovered set: given majority rule with the starkest institutional constraint, we expect outcomes to be constrained by the boundary of the uncovered set rather than spread over the entire policy space. These experiments involved both small-n and large-n designs.

The small-n design entailed 5-player computerized experiments that allowed for full player communication through the use of an unlimited (but anonymous) messaging system and a computer-mediated system of randomly assigned recognition of agenda setters.

The top two panels of Figure 2 show the two small-n configurations, S1 and S2, in this design, with player ideal points denoted as diamonds and the uncovered set as a gray shape. The ideal points of 4 players remained constant across the two configurations. The only difference between the two configurations is a shift in the location of one ideal point that causes the location and size of the uncovered set to shift dramatically with the movement of this one pivotal player. In S1, the uncovered set is large, covering approximately 64% of the Pareto set. In S2, the pivotal player's ideal point is inside the Pareto set and the uncovered set shrinks dramatically to 18% of the Pareto set. In each configuration the UCS is pulled the direction of the larger cluster of ideal points.

*** Figure 2 about here ***

The large-n design was a 35-participant, paper-and-pencil format, in which communications was restricted/prohibited. There were two configurations, L1 and L2, as shown in the bottom two panels of Figure 2 along with the associated uncovered sets. Similar to the ideal points of members in Congress estimated by procedures such as those described in Chapter 6.1, the ideal points in the L1 and L2 are divided into two "partisan" clusters. The difference between L1 and L2 results from the switch of a few voters from one cluster to another, as if a few competitive seats had changed partisan hands and produced resulting in a shift in majority control from one cluster to another. The two designs share the characteristic that the uncovered set shifts location in like manner. This variation provides a test of the hypothesis that experimental outcomes will lie within uncovered set. The working hypothesis was that outcomes selected would be within the uncovered set in both configurations. Our analysis accounts for outcomes that are inside the uncovered set as well as close misses—outcomes that are within one grid unit of the uncovered set calculated for each configuration. Table 1 summarizes the results of the 103 sessions we undertook for both types of experiments.

Table One Here

The rates of success of the uncovered set at predicting the final outcomes are impressive. That the Pareto set appears to be a better predictor is misleading, since it is much larger. In a series of statistical and likelihood calculations, Bianco et al., (2008) show that the uncovered set is a more efficient predictor than the Pareto set in that it predicts more points correctly with much smaller predictive sets. Purely theoretical solution concepts are very rarely tested against any data and, when they are tested, they usually don't do very well (Bianco et al 2006). So it is quite remarkable to find such a resounding empirical support for a solution concept that is so deeply engrained in the dominant theory of legislative behavior.

Bauer-Bir et al. (2015) conducted a critical test between the UCS and the "strong point." In the spatial context, the *strong point* (SP) is the point that has the smallest win set, where the *win set* of x is the set of points that beat x (see Chapter 4.1). The SP always lies within the UCS. The size of a win set is measured in Euclidian terms or, in our analysis, in terms of the number of grid points it contains. Godfrey, Grofman, and Feld (2011) show that every spatial voting game with Euclidean preferences has a unique SP and that win set size increases with distance from the SP. These factors predict that in a majority-rule spatial voting game, the probability that a particular point is a final outcome is a function of its distance to the SP – the closer the point, the higher the probability.

The appeal of the SP is that it builds on a well-known and easily described concept, the *win set*, which embodies the majority preference across outcomes. Precisely, given Euclidean preferences and the win set of any point, the win set of any other point can be determined. Moreover, the idea that outcomes should cluster around the SP is consistent with the well-known concept of transaction costs: decision-makers would settle on the SP or a nearby outcome, because such points have relatively small and highly fractured win sets (Miller, 2007), with many separate small "leaves" pointing in many directions, making it difficult to locate another outcome that is majority-preferred by enough to offset the transaction costs expected by legislators if they labored to move there. Alternatively, the SP can be thought of as the least contentious point – the outcome with the fewest majority-preferred competitors and the highest transaction cost associated with negotiations among legislators. Figure 3 provides a hypothetical example of the two solution concepts to illustrate the differences between the SP and UCS hypotheses.

Figure 3 Here

In the figure, the grey shape is the UCS, while the SP is labeled. The SP provides a singleton location as the expected outcome of majority rule voting, and suggests the hypothesis that outcomes are more likely to be closer to the SP rather than farther away. Most notably, the SP logic implies that a point that lies outside the UCS could nonetheless be realized as a final outcome, if it is quite close to the SP (e.g., in the northwest region of the figure). In contrast, the UCS logic implies that outcomes will always be inside the UCS. Thus, the SP logic implies that the distribution of outcomes should not vary with the size of the UCS, while the logic of the UCS implies that the support of the distribution of outcomes will positively correlated with the size of the UCS.

The critical test between the UCS and the SP boils down to a question about the distribution of experimental outcomes: are outcomes clustered in the uncovered set, and is this clustering centered on the strong point?

The foundation of our critical test is the Quadrant Method (Shiode 2008). This method divides a space into a series of squares to count the number of items in each square, with the goal of finding squares with a disproportionate number of items, implying that these squares contain clusters. Bauer-Bir et al (2015) begin by dividing the outcome space for each experiment into four pie slice-shaped regions centered on the SP, as shown in Figure 3. For each pie slice, they count the number of experimental outcomes, and determine the percentage of the UCS that overlaps the pie slice. In Figure 3, a large percentage of the UCS is contained in the NE quadrant, with lower percentages in the SE, SW, and NW quadrants.. The SP theory predicts an equal percentage of outcomes (25%) in each quadrant. By the UCS theory, the percentage of outcomes should be correlated with the percentage of the UCS points in each quadrant. Given the small number of outcomes in the experiments, the data across all experiments was aggregated to provide a single comparison across all them.

The results of our quadrant test on the aggregated dataset of experimental outcomes are statistically significant at better than .01 and show clear evidence for the UCS over the SP. If the SP was driving outcomes in these experiments, the percentage of outcomes in each quadrant would be equal, with about 25% of outcomes in each quadrant. But the data shows that the percentage of outcomes in a quadrant is significantly sensitive to the amount of the UCS points that are in the quadrant, which is what we would expect given the UCS hypothesis. (For further details and minute reports of the methods and results see Bauer-Bir et al. 2015).

5. From Lab Experiments to 'Natural Experiments.'

Ultimately, the most important test of any theory is against real-world data. In this section we report of two rather striking application of our methodology to two very detailed analyses of two very different legislative environments: the U. S. Congress and the Israeli Parliament.

Using diverse heuristics to identify the two most relevant issue dimensions in three debate environments in the U.S. Senate, Jeong et al. (2009, 2011, 2014) have estimated the ideal points of the Senators in these decision making spaces, the spatial locations of bills and amendments and the uncovered set. Having traced the sequences of proposals and amendments in each instance, they found that these sequences followed very much the logic and dynamics we observed at the lab. Starting at the status quo point, voting quickly moves into the uncovered set, and then deliberation and voting continue until a deal is struck that invariably lies within the uncovered set.

As an example, Jeong et al. (2009) consider voting on civil rights and federal aid to education reauthorization in the mid-1970s. In 1965 President Johnson signed the historic Elementary and Secondary Education Act, which authorized unprecedented federal funding for local education. However, by 1974, a backlash against the civil rights gains of the1960s had settled in. What makes this case interesting is that, while Nixon won the Presidency for the second time with a "law and order" campaign, the Democrats still controlled Congress. Weakened by Watergate, Nixon hoped to rally Republicans by his continued opposition to busing. By this time, Republicans had become more homogeneously conservative and Democrats more homogeneously liberal. Two controversial components in the reauthorization of the education bill were the scope of funding and busing. The estimated ideal points of Senators in Figure 4 show a low correlation between the two issue dimensions.

Figure Four Here

"SQ" represents the location of the current state at the time. The original bill (labeled A on the figure) submitted to the floor reflected an initiative by the Democratic leadership that was very liberal with respect to both race and scope. Given Democratic control of the Senate, one might expect this version of the bill to do well but this was not to be. In Figure 4, the contours of the UCS are drawn in different shades of grey, denoting the uncertainty of the estimates due to the uncertainty in the estimates of the ideal points of the Senators. As the figure shows, the uncovered set of the Senate in 1974 was small and its location suggests that only far more racially conservative legislation could pass. Intense amendment activity produced successful changes from "A" to 2" to "6", to "7", and then a final outcome of at "10". It is striking that even in this highly charged anti-busing environment of the early 1970s the UCS still 'exercises' the power of constraining the set of enactable outcomes.

Much has been made of the supermajority rule requirement in the U.S. Senate. To avoid filibusters, the Senate tends to pass unanimous consent agreements to bring cloture and avoid filibusters down the line (Binder et al, 2007; Binder and Smith 1998). Cloture requires a 60%

supermajority to pass, and (as noted in Chapter 4.1) a supermajority core (SMC) typically exists and may be rather large in a two-dimensional space. Jeong et al. (2009) offer a direct test of three rival hypotheses: (1) final outcomes should fall within the uncovered set of the majority party, as the party control hypothesis of Aldrich and Rohde (2001) would suggest; (2) final outcomes should fall within the uncovered set of the whole Senate as our present argument suggests; and (3) final outcomes should fall within the SMC Krehbiel, (1999) would suggest. Jeong et al (2009) computed the supermajority core, shown by contours of dotted lines in Figure 4, and the UCS of the majority party, shown by the contours of continuous lines in Figure 4.

As shown in Figure 4, the final outcome in this case lies outside of the majority party UCS. It is harder to adjudicate the case of the UCS against the SMC as they overlap substantially. However, by the very definition of a core, if the SMC were the appropriate solution concept, there would be little if any amendment activity on the floor. Even if a supermajority vote is required to consider the bill or for its final passage, amendments require only a simple majority vote. The intense amendment activity on the floor militates against the SMC as the solution concept.

A second case that we have analyzed comes from the Israeli legislature, the Knesset, in the 1990s. It is a particularly interesting case as it allows us to apply the theory to a clear and rather clean case of a natural experiment in institutional reform.[2] Electoral reforms are actually more common than most think. But they are just as controversial. Electoral rules are complex, and each aspect of any electoral rule interacts with other elements in the political arena so as to create a rather complex jigsaw puzzle. Israel's experience with electoral reform illustrates the challenge of electoral reform. A two-ballot system took effect in the Israeli elections in 1996 that provided for direct election of the prime minister in combination with the existing system of electing all 120 member of the Knesset from a single nationwide by proportional representation. The system was designed to strengthen the

[2] For a detailed account of the case that we only briefly review here, see Nachmias and Sened 1999.

power of the prime minister by giving him a popular mandate. Moreover, given that there could be only one winner of the election for prime minister, it was expected that voters would concentrate their votes on the leading candidates, presumably the leaders of the larger parties, and it was hoped that voters would then tend vote for these larger and more moderate parties in the parliamentary election as well. The result, in our language, would be a relatively small and centrist UCS.

Critics argued that the reforms would have the opposite effect because, having cast one "responsible" ballot for prime minister, many voters would feel comfortable casting the second ballot to the party of their choice, regardless of the extremism of its ideology or its likelihood of meeting the (then) 1.5% threshold needed to win seats in parliament, with the result that the split ballot would advantage relatively small and extreme parties in the parliamentary election (Nachmias and Sened 1999). If so, the two-ballot system would reduce stability, because extreme parties with an empty center would yield a large uncovered set and result in policy uncertainty reducing the governability and the power of the prime minister that the reform was supposed to enhance. Our ability to calculate uncovered sets allows a direct test of these pro- and anti-reform arguments by comparing the size of uncovered set in the Israeli Knesset immediately before and after the 1996 reform.

Figures 5 (pre-reform) and 6 (post-reform) summarize our evidence pertaining to this 'natural experiment'. The tiny dots denote the ideal points of a representative sample of voters estimated on the basis of a survey with a significant number of attitudinal questions relevant to the two most salient dimensions of security and religion. The ideological positions of the parties that entered the two respective parliaments were estimated using the same survey questions asked of experts, asked to answer the questions 'as if' they were the leaders / representatives of the respective parties. The estimated positions of the parties are included in the figures with the relevant legend on the left side. (For a detailed analysis of the surveys and data that served as a basis for these estimates, see Schofield and Sened, 2006: 70-100). Based on these data, collected for a completely different project, we computed the electoral uncovered set, based on the ideal points in the voter sample, and the parliamentary uncovered set, based on the estimated party ideal points, both before and after 1996. Comparing the figures, it is evident that positions of the voters changed only slightly and the electoral uncovered set did not change much either. What did change was the parliamentary uncovered set of parliament, which we can attribute, at least in part, to the institutional change in the voting rule. Comparing the two figures, one can appreciate how the institutional change caused significant changes in the distribution of the positions of parties in parliament allowing minor parties to secure seats in parliament they would not have under the old rule and changes in the relative seat strength of the parliamentary parties. These changes caused, in turn, an enlargement of the uncovered set of the Israeli parliament, as evident in Figure 7, which made it so dysfunctional that the new rule had to be rescinded. A return to the old electoral rule was approved by a large majority of parliament, shortly after the 2000 elections.

Figure 5/6 Here

Here again, the analysis of Israeli electoral reform confirms the usefulness of the UCS as a prediction of baseline outcomes and as a tool for analyzing institutional changes.

6. Conclusions

Ten years ago, we succeeded in devising a computational method to calculate the size and location of the uncovered set for any set of ideal points. This was important given the expectation of McKelvey and others that various majority-rule institutions lead to outcomes in the UCS. Since then we, and so many others have been able to use our method to analyze many phenomena regarding the use of majority rule in diverse environments.

We feel very lucky to have served our beloved mentor William H. Riker almost 'against his will.' As we noted earlier, Riker (1980: 447) was overly impressed with the 'Chaos Theorems' and concluded that "politics is *the* dismal science" because it lacks equilibria. We believe that Riker was too quick to give up on political science. Over the last decade we have labored to show that the UCS offers a less devastating view of our science.   Our work shows a central regularity in real-world majority rule: policy choices are constrained to the UCS. The fact that final outcomes converge to the UCS in so many different institutional decision making environments clearly support our claim that the uncovered set should be the solution concept of choice for the spatial theory of legislative behavior and for majority rule more generally.

Figure 1.
The Fiorina – Plott Experiment

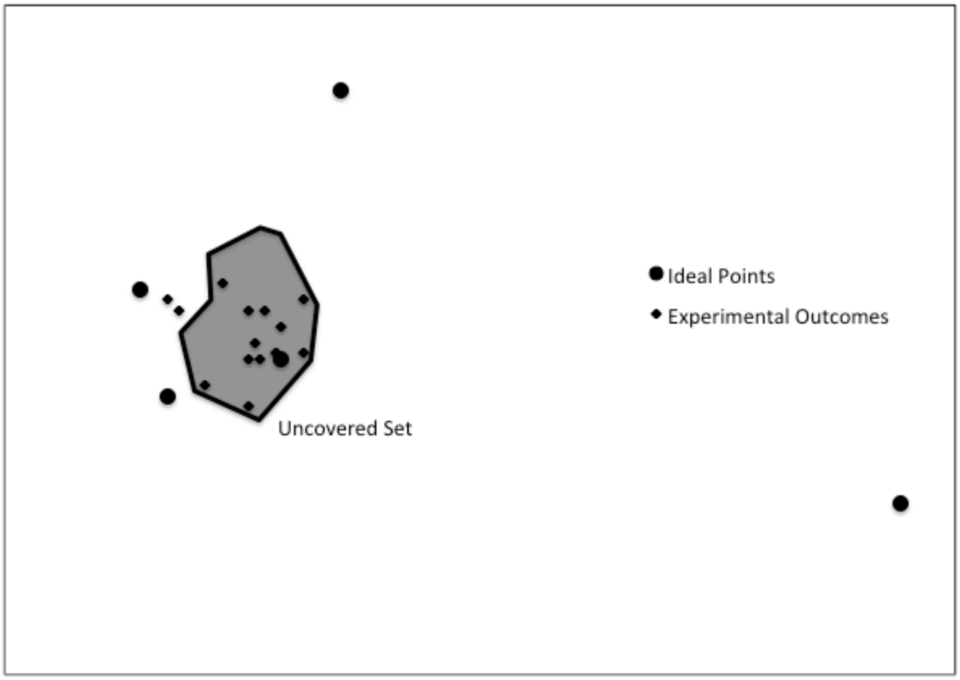

Figure 2.
Large-N and Small-N Experiments

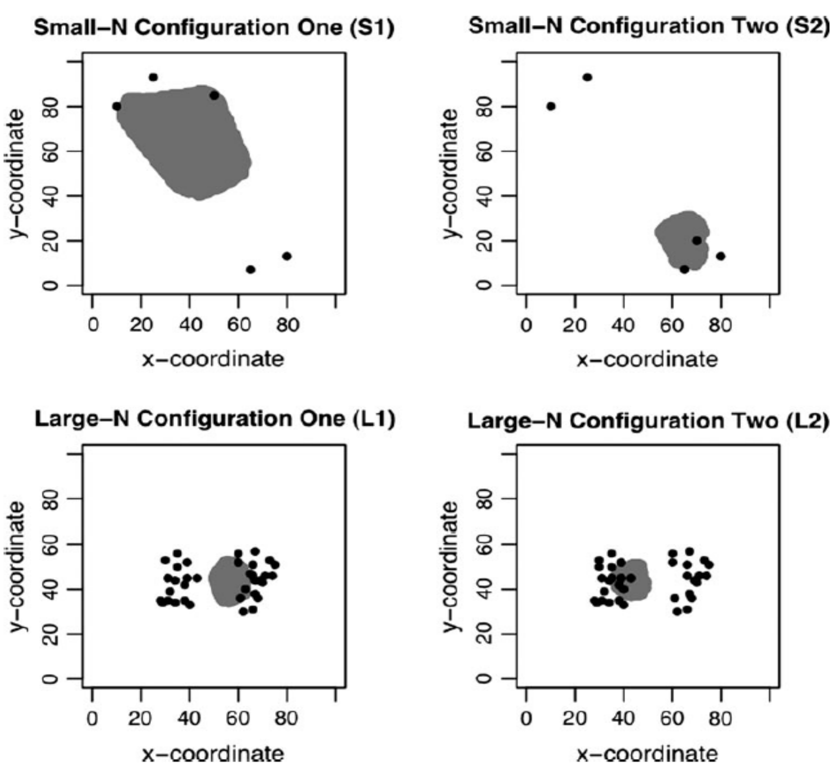

Figure 3.
Hypothetical Strong Point and Uncovered Set with four quadrants

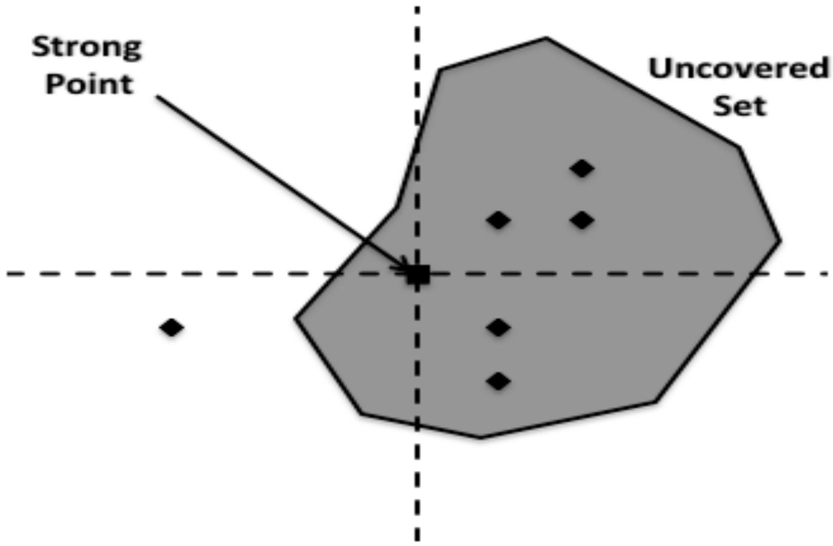

Figure 4.
The Education Amendments of 1974.

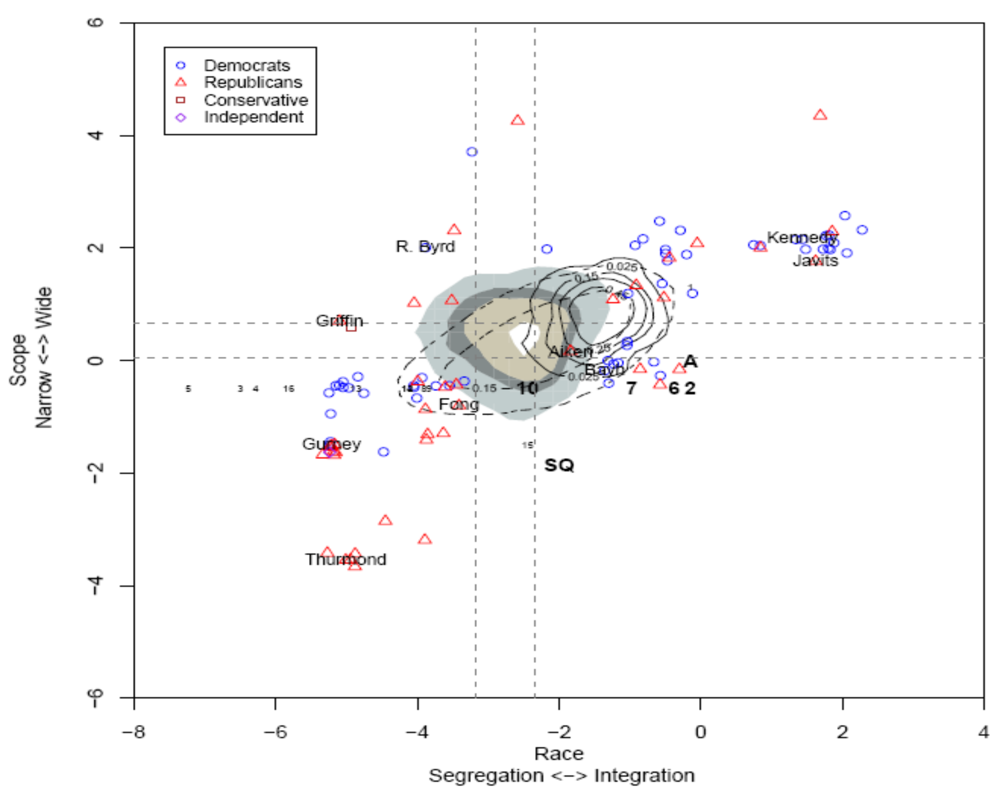

Figure 5: The 1992 Elections prior to Dual Ballot Reform Election

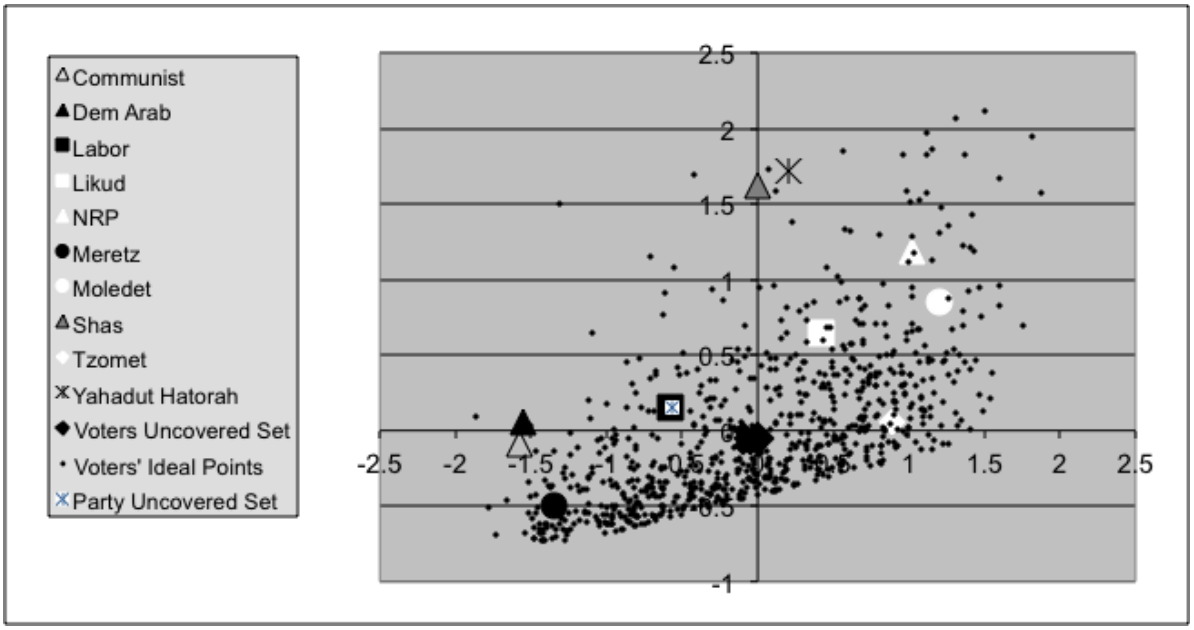

Figure 6: The 1996 Election – The First Dual Ballots Reform Elections

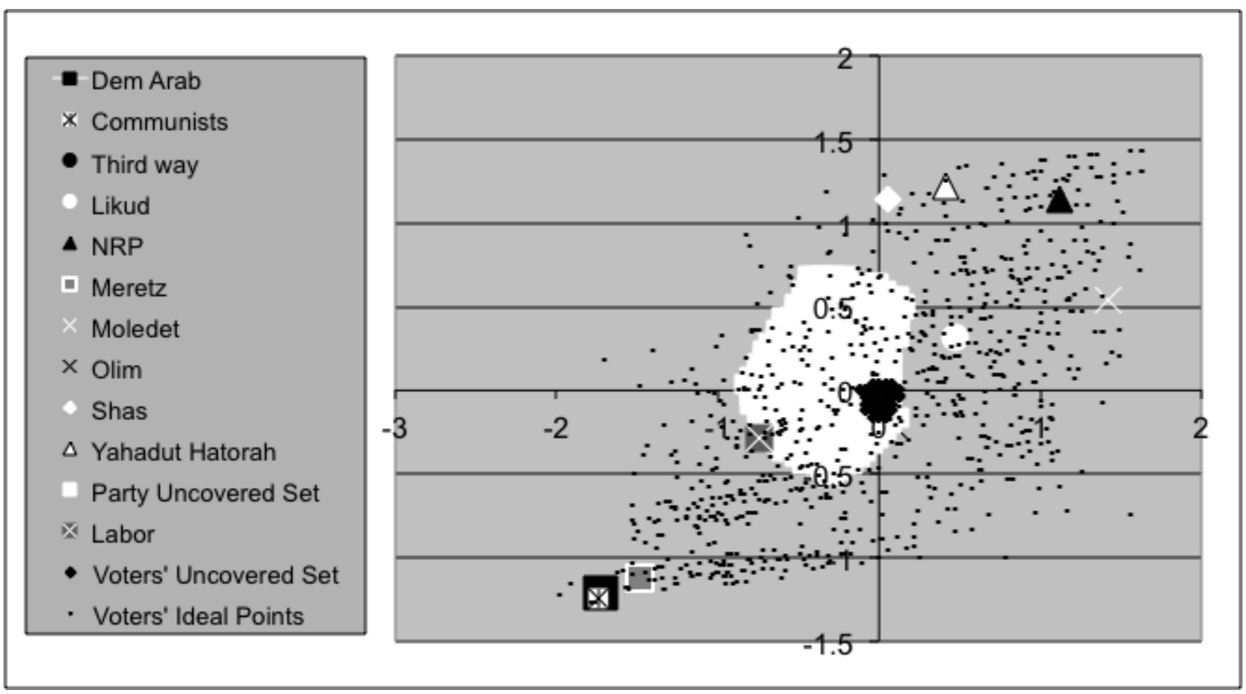

Table One: Measuring Predictive Power (1): Hits and Misses.

	Small-N Design		Large-N Design	
	S1	S2	L1	L2
Final Outcomes in Pareto Set	28 (100.0%)	26 (92.9%)	25 (100.0%)	22 (100.0%)
(including close misses)	28 (100.0%)	26 (92.9%)	25 (100.0%)	22 (100.0%)
Final Outcomes in Uncovered Set	28 (100.0%)	17 (60.7%)	10 (40.0%)	10 (45.5%)
(including close misses)	28 (100.0%)	21 (75.0%)	16 (64.0%)	13 (59.1%)

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