A New Method for Identifying All Maximal Incomplete Sub Cubes of a Hypercube Under Processor Failure Model

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Abstract- Under processor failure model, a hypercube may operate in a gracefully degraded manner by supporting parallel algorithms in smaller fault-free cubes. In order to reduce execution slowdown in hypercube with a given faults, it is essential to identify the maximum healthy sub cubes (maximal incomplete sub cube) in the faulty hypercube. This paper proposes a new, general and efficient method to identify all the maximal incomplete sub cubes present in a faulty hypercube under its maximum fault tolerance level. The proposed method is recursive in nature and is well illustrated though an example. This method as such can be applicable to other important cube based topologies.

Keywords- Faulty hypercube, Maximal incomplete sub cube, Discarded region.

I. INTRODUCTION

In recent years, there has been considerable interest and increased efforts in developing large parallel computing systems. Parallel computers have been applied in real time environments and they are becoming increasingly popular for large commercial applications as well. An important component of parallel computer is its interconnection network. Out of many interconnection networks hypercube is treated as an important topology due to some of its important characteristics viz. regularity, symmetry, small diameter, strong connectivity, recursive construction and partitionability [1].

The n-dimensional hypercube is composed of 2^n nodes and has n- edges per node, n-bit binary addresses are assign to the nodes to the hypercube in such a way that an edge or link connects two nodes if and only if their binary addresses differ by a single bit [1].This Inter connection network supports large numbers of resources with small diameters. But the major drawbacks of the cube networks are the numbers of communication ports and channels per processors is the same as the logarithm of the total numbers of processors in the system. Therefore the number of communication ports and channels per processors increases by increasing the total number of processors in the system. This drawback seems to be waived in the case of incomplete cubes, which shows the emulation performance as the n/w scales up in size [2],[3][5]and[6].

The probability of fault in a larger system is given due importance. The faults may be either due to links or processors. When one or more processors fail in a hypercube, the hypercube is termed as incomplete hypercube. In order to maintain hypercube topology in the presence of faults, methods [2] and [3] are found in the literature. Sridhara et. al. [4] proposed a method for finding maximal sub cubes in a residual hypercube. But the study only limited to residual hypercube. Subsequently, Latifi [5] presented an algorithm for finding the sub cubes in a reliable hypercube. However, the method can't be used to find all the maximal incomplete hypercube of a hypercube. Chen et. al. [6] determines sub cubes in a faulty hypercube. Similar research works can be found in literature [7], [8] and [9]. So, finding all the incomplete sub cubes in a hypercube is quite a open problem. This motivates our study to propose a simple, general and recursive method for finding all the incomplete sub cubes of a hypercube.

This paper proposes an efficient distributed procedure for locating or identifying all maximal incomplete sub cube present in faulty hypercube .This procedure exhibit better empirically polynomial time complexity with respect to the system dimension and the number of faults. The concept of discarded regions eliminates those nodes impossible to be part of any fault free sub cube containing the given node. There by forming the maximal incomplete sub cube.

II. ARCHITECTURAL DETAIL OF HYPERCUBE INTERCONNECTION NETWORK:

An n-dimensional cube network of N processing elements (PEs) is specified by the following routing functions:

$$C_i\ (a_{n-1}\cdots a_1 a_0) = a_{n-1}\cdots a_{i+1}\bar{a}_{i-1}\cdots a_1 a_0\ ,\quad for\ i = 0,1,2,\cdots n-1 \qquad (1)$$

In the n cube, each PE located at a corner is directly connected to n neighbors. The neighboring PEs differs in exactly one bit position. There are 2^n number of processing elements and $n.2^{n-1}$ number of links in an n-dimensional hypercube (HC_n) [15]. (Fig.1)

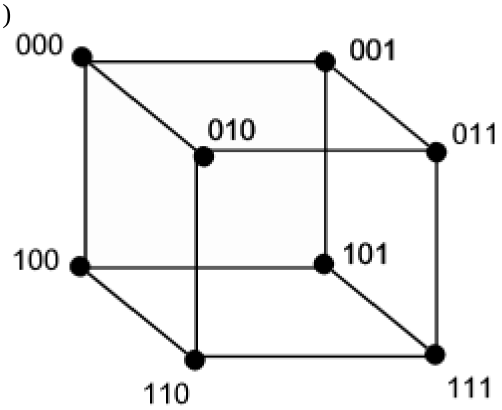


Fig.1: Hypercube (*N=8, n=3)*

III. PROPOSED METHOD FOR FINDING THE MAXIMAL INCOMPLETE SUB CUBES OF A HYPERCUBE:

A. **Notations:**

HCn Hypercube Interconnection n/w
S Source node
D Destination node
$\otimes$ *Discarding operation*
n System dimension
N numbes of nodes in hypercube
u,v,w Adjacent nodes of source node
$\bar{v}$, $\bar{w}$ *Antipodal nodes of v, w*

B. **Assumptions:**

1. The system is degradable.
2. Node faults are statistically independent with each other

C. **Proposed Mathematical Model:**

Definition 1: A maximal incomplete sub cube is obtained when link is properly added to a maximal sub cube of n-1 dimension so that the destination node D is reached.

Unlike complete hypercube an incomplete hypercube can be constructed with any numbers of nodes to avoid the practical restriction of cube topology on the numbers of nodes in a system must be a power of 2. A proper incomplete sub cube in a faulty hypercube refers to a fault free incomplete sub cube which is not contained entirely in any of the fault free sub cube.

Definition 2: A discarded region in an interconnection network is the smallest sub cube comprises of a faulty node and the antipodal nodes of the (n-1) fault free adjacent nodes.

For a faulty hypercube HC_n and a given source node S. It is possible to identify systematically every fault free sub cube which involves the source node S. This can be done by determining the region which never contribute to any fault free sub cube containing the node S. Each fault results in one such regions known as discarded region which is the smallest sub cube involving both the faulty and the antipodal nodes of adjacent (n-1) nodes. A discarded region is addressed by performing $\otimes$ operation on the labels of the faulty node and the antipodal node where $\otimes$ is the bit operation defined as: it yields 0 (or 1) if the two corresponding bits are "0" (or "1") and it is $*$ if the two corresponding bits differ.

Theorem-1:- ***The no. of discarded complete sub cube in an HCn of dimension 'n' is equal to the number of nodes present in a fault free maximal in complete sub cube.***

Proof: For an inter connection network HCn of dimension n , there are exactly n numbers of nodes adjacent to a given source node S. It can be straight forward from the properties of interconnection cube n/w that HC_{n-1} can be obtained by removing 2^{n-1} nodes from HCn. The process can be repeated such that HC_2 can be obtained which shows clearly that HCn is having a hierarchical structure. So , for a given source node S, one out of n adjacent nodes can be faulty so that , an interconnection network of lower dimension can be obtained while preserving the hierarchical and regular properties of the n/w. Thus for a given source node(S) and destination node(D) (which can never be faulty) $2^{n-1}+1$ nodes can not be faulty. Further, choosing a node N ϵ Adj(s) to be faulty. Finding the antipodal nodes Y_i' of X ϵ {Adj(s)},

i =1……(| adj(s)| -1) and carrying out $Y \otimes_i$ '{Adj(s)} leads to n faulty nodes and

$2^{n-1}+1$ discarded regions, which proves the theorem.

Theorem-2 : ***A maximal sub cube or maximal incomplete sub cube contains no nodes which are present in the discarded region***

Proof:- Let a n-dimensional hypercube is represented by HC_n, in which A is a faulty node without loss of property of n-cube the label of node A is $(O^{n-I}.1^i)$ and adjacent node is O^n. The discarded region involving node A is

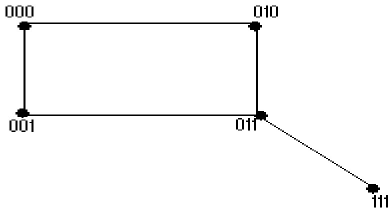
$(*^{n-i}.1^i)$.Also a node B which is in discarded region can be labeled as $(*^{n-i}.1^i)$.This is a contradiction to definition of a maximal sub cube or maximal incomplete sub cube.

Lemma 1: ***For an n-dimensional hypercube if one of n adjacent nodes of the source node faults then, the resulting maximal sub cube has a dimension of n-1.***

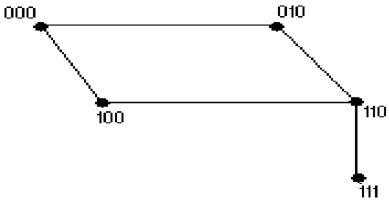
Proof: Consider an n-dimensional hypercube HCn in which one of the n-adjacent node to the source node S is faulty. This leads to disconnection of link to faulty node. Without loss of generality and obeying the symmetry of n-cube. The faulty node must present in HC_n, which contradicts definition of a maximal complete sub cube.

IV. ILLUSTRATION

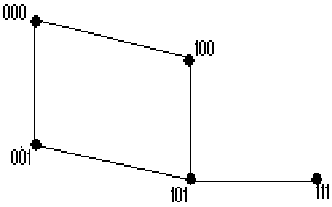
Each node in HC_n is labeled by a n-bit string. For a given source node S, there exists a numbers of adjacent nodes, out of which at least one node is faulty. Otherwise it will destroy the regularity property of HC_n. The addresses of the adjacent nodes are differ in exactly one bit. Assume 'u' be the faulty node, 'v' and 'w' be the non-faulty nodes. Where 'u', 'v' , 'w' are represented as binary strings [12]. $\bar{v}$ and $\bar{w}$ be the antipodal nodes of v and w. Taking bit operation $u \otimes \bar{v}$ and $u \otimes \bar{w}$ results n discarded regions. This leads to formation of an incomplete interconnection network i.e. hypercube. HC_{n-1}^{m}, m numbers of nodes in fault free incomplete sub cube with dimension of n-1.



2 (a) 100 taken as faulty



2 (b) 001 taken as faulty



2(c) 010 taken as faulty

Fig.2: 2(a), (b) and (c) represents incomplete maximal sub cubes of a 3 Dimensional hypercube

Consider a 3-D Hyper cube HC_3 as shown in fig.1 having the source and destination nodes labeled as 000 and 111 respectively. Out of the three adjacent nodes of source node let cube node 001 is faulty, Then the antipodal nodes of the two other adjacent nodes are 011 and 101.A discarded region is addressed simply by performing operation $\otimes$ on the labels of the faulty node and the antipodal nodes. In fig.1(a) faulty node is 001,antipodal node of 100 is 011 and $001 \otimes 011 = 0{*}1$ and $001 \otimes 101 = {*}01$.After removing this two discarded regions a maximal incomplete sub cube results which is shown in fig.2(a).The same operation can be performed by taking 100 and 010 cube nodes as faulty nodes. This results in the two other maximal incomplete sub cube as shown in fig.2(b) & 2(c).

V. CONCLUSIONS

A fast, simple and effective procedure has been introduced for identifying all maximal fault free incomplete sub cube from a faulty hypercube, taking maximum fault tolerance capacity of the interconnection system is equal to the system dimension. In this process every fault free node is required to participate in the identification process. The complete hypercube restricts the sizes of systems that can be constructed but system based on incomplete hypercube can be built with any number of computing nodes. The empirically polynomial time complexity with respect to system dimension for this system is found to be (n^2).With this low time complexity, this sub cube identification procedure could be useful for system design to operate in a gracefully degraded manner after faults occur. This procedure is suitable for a hypercube with arbitrary node failure.

VI. REFERENCES:

[1]. Y. Saad and M. H. Schultz, Topological properties of Hypercubes, *IEEE Trans. Comput.*, vol. 37, no. 7, pp. 86-88, 1988.

[2]. H. P. Katseff, “Incomplete hypercube”, IEEE Trans. On Computers, vol. 37, no. 5, 1988.

[3]. N.F.Tzeng, H.L.chen and P.J.chuang,, “Embeddings in incomplete hyper cubes”. *In Proc. Int. Conf. Parallel processing*, vol-1, pp.335-339, 1990.

[4]. M.A.Sridhar and C.S Raghavendra, “On finding maximal sub cubes in Residual Hyper cubes” *Proc. of IEEE Symp. on Parallel and distributed processing* pp 870-873, 1990.

[5]. S. Latifi, “Distributed Sub cube identification Algorithms for Reliable hypercubes,” *Information processing letters*, vol.38, pp.315-321, 1991.

[6]. H.L Chen and N.F Tieng , “Distributed Identification in faulty hyper cube “, *IEEE Trans. on computers*, vol 46, no 8, pp 87-89, 1997.

[7]. J.S.Fu, “Longest fault free paths in hyper cubes with vertex faults”, Inf. Sci. vol. 176, no. 7, pp.759-771, 2006.

[8]. J.M.Xu, M.J.Ma, and Z.Z.Du. “Edge–fault tolerant properties of hyper cubes and folded hyper cubes”, *Australian J. Combinatorics*, vol. 35, no. 1, pp. 7-16, 2006.

[9]. W. Wang and X. Chen, “A fault-free Hamiltonian cycle passing through prescribed edges in a hypercube with faulty edges”, *J. Information Processing Letters, vol.* 107, pp.205–210, 2008.