CAP 5993/CAP 4993
Game Theory

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HW1

- Out today due 1/26
- HW policy:
  - You can discuss general concepts with other students, but must work on the problems individually.
  - List out all resources consulted.
  - Two late days, then 50% credit, then 0%.
  - Homework due at start of class (3:30 PM). Can be emailed.

Strategic-form games

- A game in **strategic form (**or in **normal form**) is an ordered triple G = (N, (S_i) I in N, (u_i) i in N), in which:
  - N = {1,2,…,n} is a finite set of players.
  - Si is the set of strategies of player i, for every player i in N. Denote the set of all vectors of strategies by S = S_1 x S_2 x … x S_n.
  - u_i : S → R is a function associating each vector of strategies s = (s_i), i in N, with the **payoff (utility)** u_i(s) to player i, for every player i in N.

Strategic-form game examples

- Chicken
- Security game
- Rock-paper-scissors
- Prisoner’s dilemma
- Battle of the sexes

- We saw von Neumann’s theorem in the special case of two players and three possible outcomes: victory for White, a draw, or victory for Black.
- Central question of game theory: what “will happen” in a given game?

Central question of game theory

1. An empirical, descriptive interpretation: How do players, in fact, play in a given game?
2. A normative interpretation: How "should" players play in a given game?
3. A theoretical interpretation: What can we predict will happen in a game given certain assumptions regarding "reasonable" or "rational" behavior on the part of the players?

- For each of the five example games we discussed:
  - How will real players act?
  - How “should” players act?
  - How would theoretically perfectly rational players act?
- Golden Balls: Split or Steal?
  https://www.youtube.com/watch?v=S0qjK3TWZE8

Prisoner’s dilemma experiments

- Frank, Gilovich, and Regan (1993) conducted an experimental study of the prisoner’s dilemma. The subjects were students in their first and final years of undergraduate economics, and undergraduates in other disciplines. Subjects were paired, placed in a typical game scenario, then asked to choose either to "cooperate" or to "defect." Pairs of subjects were told that if they both chose "defect" the payoff for each would be 1. If both cooperated, the payoff for each would be 2. If one defected and the other cooperated, the payoff would be 3 for the defector and 0 for the cooperator. Each subject in a pair made his choice without knowing what the other member of the pair chose.

- First year economics students, and students doing disciplines other than economics, overwhelmingly chose to cooperate. But 4th year students in economics tended to not cooperate. Frank et al. concluded, that with "an eye toward both the social good and the well-being of their own students, economists may wish to stress a broader view of human motivation in their teaching."

Rock-paper-scissors competition?

- http://www.wimp.com/behold-the-epic-finale-of-a-japanese-rock-paper-scissors-competition/

- It is impossible to gain an advantage over a truly random opponent. However, by exploiting the weaknesses of nonrandom opponents, it is possible to gain a significant advantage. Indeed, human players tend to be nonrandom. As such, there have been programming competitions for algorithms that play rock–paper–scissors.
- In tournament play, some players employ tactics to confuse or trick the other player into making an illegal move, resulting in a loss. One such tactic is to shout the name of one move before throwing another, in order to misdirect and confuse their opponent. During tournaments, players often prepare their sequence of three gestures prior to the tournament's commencement.

- Iocaine Powder, which won the First International RoShamBo Programming Competition in 1999, uses a heuristically designed compilation of strategies. For each strategy it employs, it also has six metastrategies which defeat second-guessing, triple-guessing, as well as second-guessing the opponent, and so on. The optimal strategy or metastrategy is chosen based on past performance.

- The main strategies it employs are history matching, frequency analysis, and random guessing. Its strongest strategy, history matching, searches for a sequence in the past that matches the last few moves in order to predict the next move of the algorithm. In frequency analysis, the program simply identifies the most frequently played move. The random guess is a fallback method that is used to prevent a devastating loss in the event that the other strategies fail. More than ten years later, the top performing strategies on an ongoing rock–paper–scissors programming competition similarly use metastrategies. However, there have been some innovations, such as using multiple history matching schemes that each match a different aspect of the history – for example, the opponent's moves, the program's own moves, or a combination of both. There have also been other algorithms based on Markov chains.

- Researchers at the University of Tokyo have created a robot hand that has a 100% winning rate playing rock–paper–scissors. Using a high-speed camera, the robot recognizes within one millisecond which shape the human hand is making, then produces the corresponding winning shape.

2/3 the average?

- Real number 0 to 100 (inclusive)

- This game is a common demonstration in game theory classes, where even economics graduate students fail to guess 0. When performed among ordinary people it is usually found that the winner guess is much higher than 0, e.g., 21.6 was the winning value in a large internet-based competition organized by the Danish newspaper Politiken. This included 19,196 people and with a prize of 5000 Danish kroner.
- Creativity Games has an online version of the game where you play against the last 100 visitors.

Ultimatum game

- The first player (the proposer) receives a sum of money and proposes how to divide the sum between the proposer and the other player. The second player (the responder) chooses to either accept or reject this proposal. If the second player accepts, the money is split according to the proposal. If the second player rejects, neither player receives any money. The game is typically played only once so that reciprocation is not an issue.

- When carried out between members of a shared social group (e.g., a village, a tribe, a nation, humanity) people offer "fair" (i.e., 50:50) splits, and offers of less than 30% are often rejected.

- One limited study on twins claims that genetic variation can have an effect on reactions to unfair offers, though the study failed to employ actual controls for environmental differences. It has also been found that delaying the responder's decision makes people accept "unfair" offers more often. Common chimpanzees behaved similarly to human by proposing fair offers in one version of the ultimatum game involving direct interaction between the chimpanzees. However, another study also published in November 2012 showed that both kinds of chimpanzees (common chimpanzees and bonobos) did not reject unfair offers, using a mechanical apparatus. As of February 2015, bonobos have not been studied using the protocol involving direct interaction.

Notation

- Let $N = \{1,\ldots,n\}$ be a finite set, and for each i in N let X_i be any set.
- Let X denote the cross product of the X_i, and for each i in N define X_{-i} to be the Cartesian product over all X_j for j != i.
- An element in X_{-i} will be denoted
- $x_{-i} = (x_1,\ldots, x_{i-1}, x_{i+1},\ldots, x_n)$

Domination

	L	M	R
T	1, 0	1, 2	0, 1
B	0, 3	0, 1	2, 0

- A strategy s_i of player i is **strictly dominated** if there exists another strategy t_i of player i such that for each strategy vector s_{-i} in S_{-i} of the other players, $u_i(s_i, s_{-i}) < u_i(t_i, s_{-i})$

- Assumption: A rational player will not choose a strictly dominated strategy.

- Assumption: All players in a game are rational.

- Can a strictly dominated strategy be eliminated under these two assumptions?

- Not necessarily. If player 2 is rational he will not choose strategy R. But if player 1 does not know that player 2 is rational, he is liable to believe that player 2 may choose strategy R, in which case it would be in player 1's interest to play strategy B.
- So need player 2 is rational, AND player 1 knows that player 2 is rational …

- Player 2 knows that player 1 knows that player 2 is rational …
- Otherwise, player 2 would need to consider the possibility that player 1 may play B, considering R to be a strategy contemplated by player 2, in which case player 2 may be tempted to play L.
- Need: player 1 knows that player 2 knows that player 1 knows that player 2 is rational
- https://www.youtube.com/watch?v=LUN2YN0bOi8

- A fact is **common knowledge** among the players of a game if for any finite chain of players i_i, i_2, …, i_k the following holds: player i_i knows that player i_2 knows that player i_3 knows … that player i_k knows the fact.

- Assumption: The fact that all players are rational is common knowledge among the players.

- Given the assumptions, we can eliminate R.

	L	M	R
T	1, 0	1, 2	0, 1
B	0, 3	0, 1	2, 0

→

	L	M
T	1, 0	1, 2
B	0, 3	0, 1

	L	M	R
T	1, 0	1, 2	0, 1
B	0, 3	0, 1	2, 0

→

	L	M
T	1, 0	1, 2
B	0, 3	0, 1

→

	L	M
T	1, 0	1, 2

- Procedure called **iterated elimination of strictly dominated strategies**. When this process yields a single strategy vector (one strategy per player), then under the assumptions that is the strategy vector that we will obtain, and it may be regarded as the **solution** of the game.
- Special case: if every player has a strategy that strictly dominates all of his other strategies, that is, a **strictly dominant** strategy. Then elimination leaves each player with only one strategy. We say that the game has a solution in **strictly dominated strategies**.

- 2/3 average game?
- Ultimatum game?
- Chicken?
- Battle of the sexes?
- Rock-Paper-Scissors?
- Security game?
- Prisoner's dilemma?

Prisoner's dilemma

	C	**D**
C	4, 4	0, 5
D	5, 0	1, 1

- Does it matter if we eliminate player 1's C or player 2's C first?
- Theorem: Whenever iterated elimination of strictly dominated strategies leads to a single strategy vector, that outcome is independent of the order of elimination.
  - Even if it yields a set of strategies, that set does not depend on the order of elimination.

	L	**R**
T	1, 2	2, 3
B	2, 2	2, 0

Weakly dominated strategies

- Strategy s_i of player i is **weakly dominated** if there exists another strategy t_i of player i satisfying the following two conditions:
  1. For every strategy vector s_{-i} in S_{-i} of the other players, $u_i(s_i, s_{-i}) <= u_i(t_i, s_{-i})$
  2. There exists a strategy vector t_{-i} in S_{-i} of the other players such that $u_i(t_i, s_{-i}) < u_i(t_i, t_{-i})$
- In this case we say that strategy s_i is **weakly dominated** by strategy t_i, and that strategy t_i **weakly dominates** strategy s_i

- Clearly strict domination implies weak domination.
- We will use the term “domination” to mean “weak domination.”
- New assumption: A rational player does not use a dominated strategy.
- The process of iterated elimination of weakly dominated strategies is called **rationalizability**.
- A strategy vector s in S is **rational** if it is the unique result of a process of iterative elimination of weakly dominated strategies.

	L	**R**
T	1, 2	2, 3
B	2, 2	2, 0

- Assumption: A rational player will not choose a strictly dominated strategy.
- New assumption: A rational player does not use a (weakly) dominated strategy.
- **Trembling hand principle**: suppose every single strategy available to a player may be used with positive probability, which may be extremely small.
  - May happen by mistake, irrationality, or miscalculations.

Trembling hand principle

- Suppose player 2 chooses L and R with probabilities x and 1-x respectively, where $0 < x < 1$.
- The expected payoff to player 1 if he chooses T is:
  - $x + 2(1-x) = 2-x$
- The expected payoff to player 1 if he chooses B is 2.
- So strategy B gives him a strictly higher payoff than T, so that a rational player 1 facing player 2 who has a ``trembling hand'' will choose B and not T; i.e., he will not choose the weakly dominated strategy.

- The fact that a strategy s_i of player i (weakly or strictly) dominates t_i depends only on player i's payoff function, and is independent of the payoff functions of the other players.
- Therefore, a player can eliminate his dominated strategies even when he does not know the payoff functions of other players.
- For rationalizability, eliminate dominated strategies one by one. For player i to remove a strategy s_i after s_j of player j, need to assume that player i believes that player j will not implement s_j. This is reasonable only if player i knows j's payoff function.
- Iterative elimination of (weakly) dominated strategies can be justified only if the payoff functions of the players are common knowledge among them.

Homework for next class

- HW1 out today due 1/26
- Chapter 5 from Maschler textbook