Evidence-Based Instructional Practices for Mathematics

Secondary Leads

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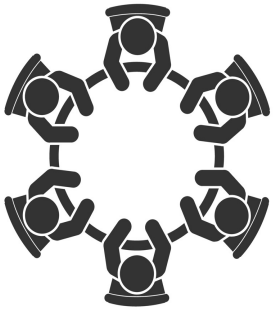

Introduce yourself.

Describe your strengths in supporting mathematics.

Describe an opportunity for growth.

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This Year

October 5th, 2023	IN PERSON with Leads Evidence-Based Instructional Practices for Mathematics
January 3rd, 2024	VIRTUAL with Teachers Evidence-Based Instructional Practices for Mathematics Sarah Powell (Grades K-5) Brad Witzel (Grades 6-12)
March 11th, 2024	IN PERSON with Leads Sarah Powell (Grades K-5) Brad Witzel (Grades 6-12)

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Mathematical Progressions

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Mathematical Progressions

Critical Content

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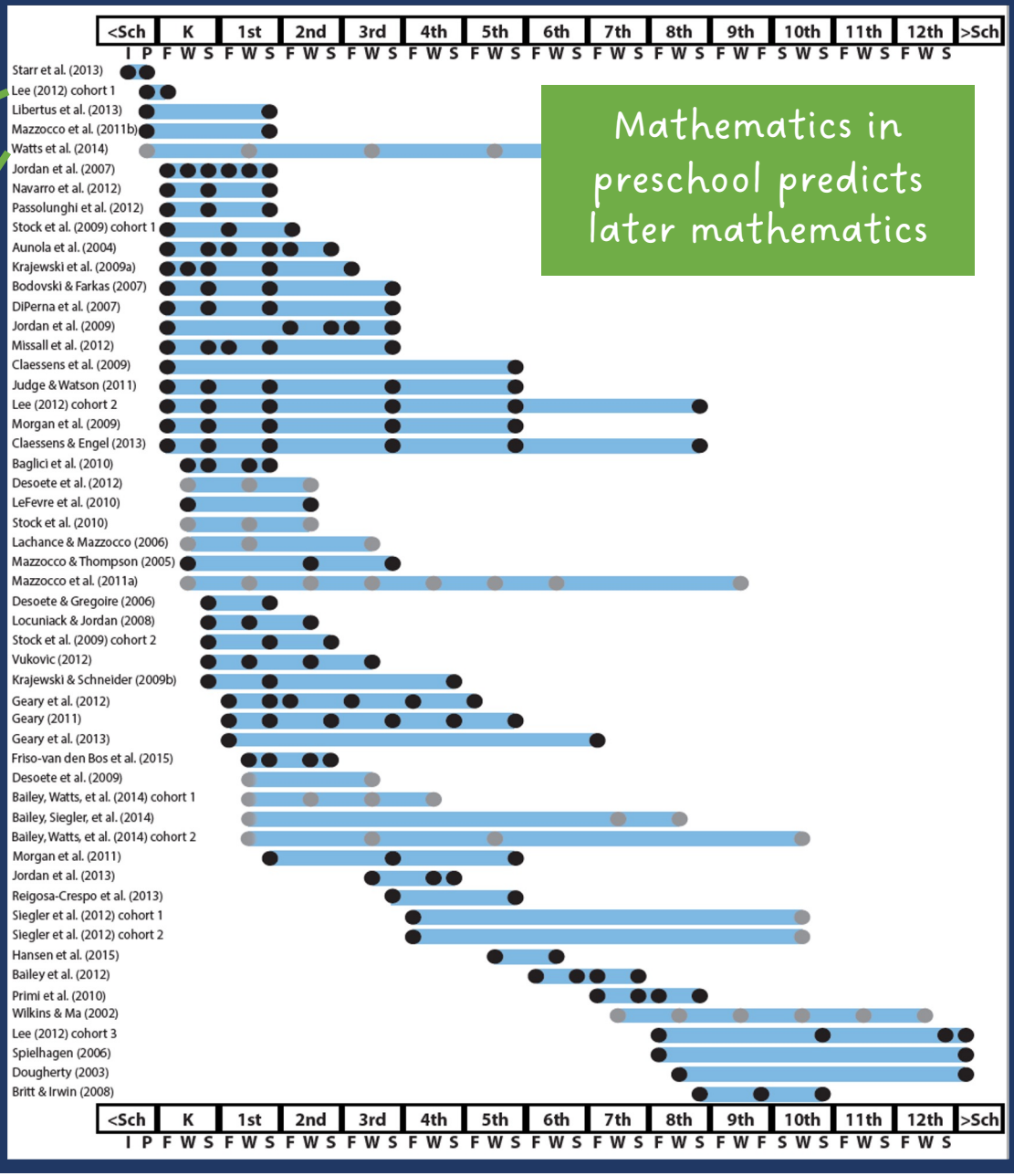
Broad math in preK predicted K broad math

Broad math in preK predicted grade 10 broad math



Mathematics in preschool predicts later mathematics

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

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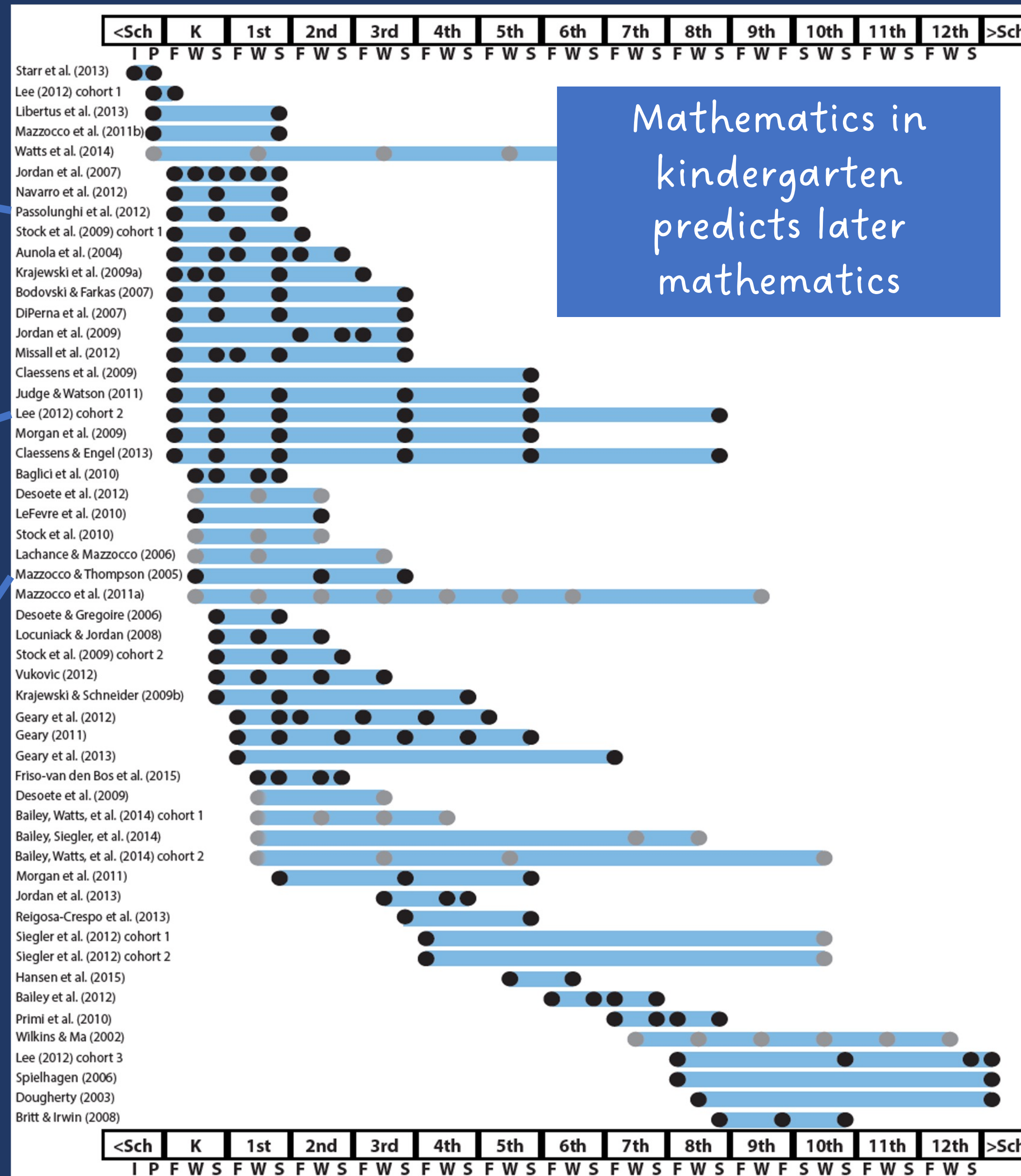
Counting in K predicted grade 1 broad math

Broad math in K predicted grade 8 broad math

K math accurately predicted math performance below 10th percentile in grades 2 and 3 with 84% correct classification

Mathematics in kindergarten predicts later mathematics



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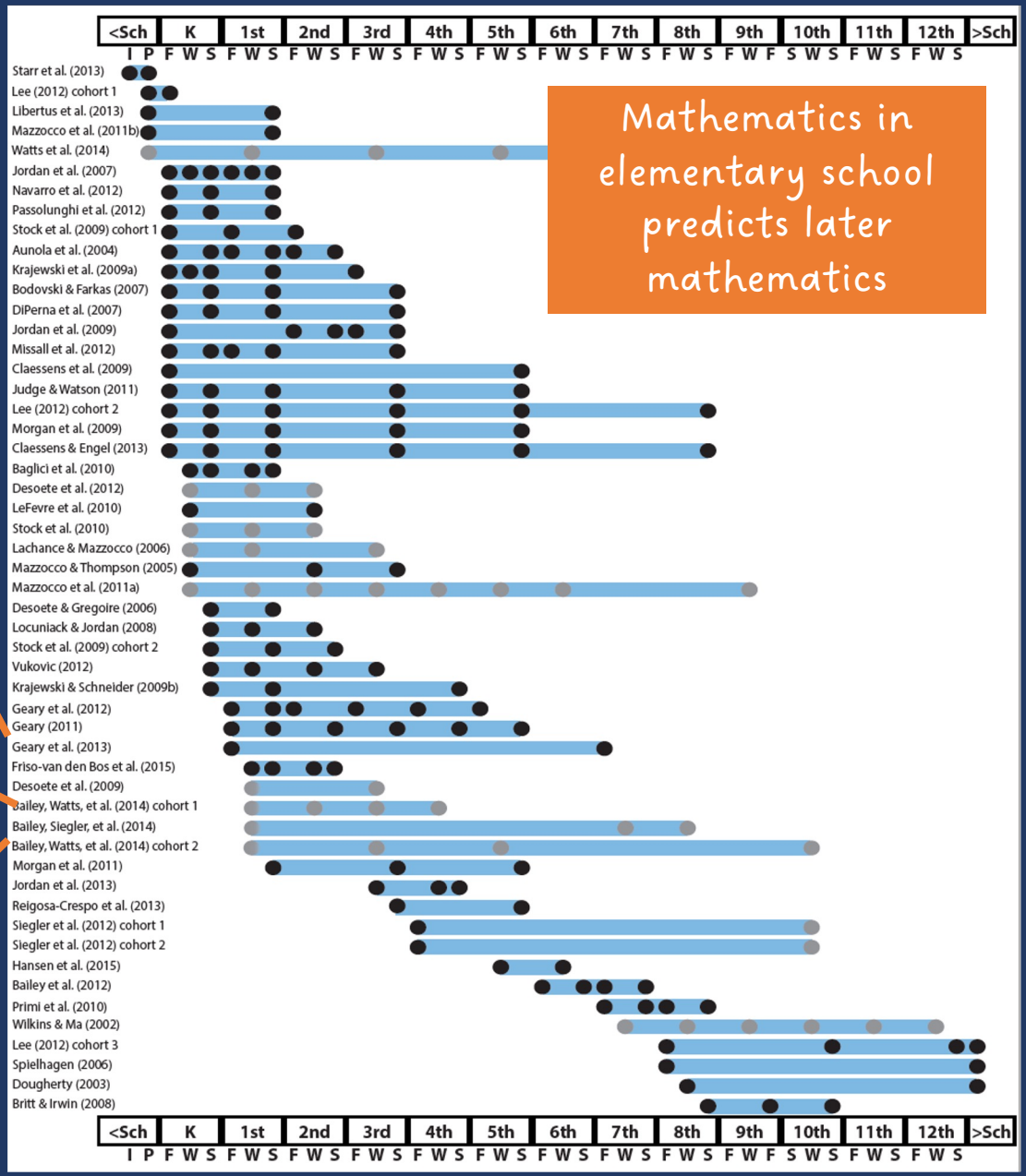
Addition influenced arithmetic with increasing importance from grades 1 to 5

Grade 1 arithmetic predicted arithmetic at grades 2, 3, and 4

Grade 1 broad math predicted broad math at grades 3, 5, and 10



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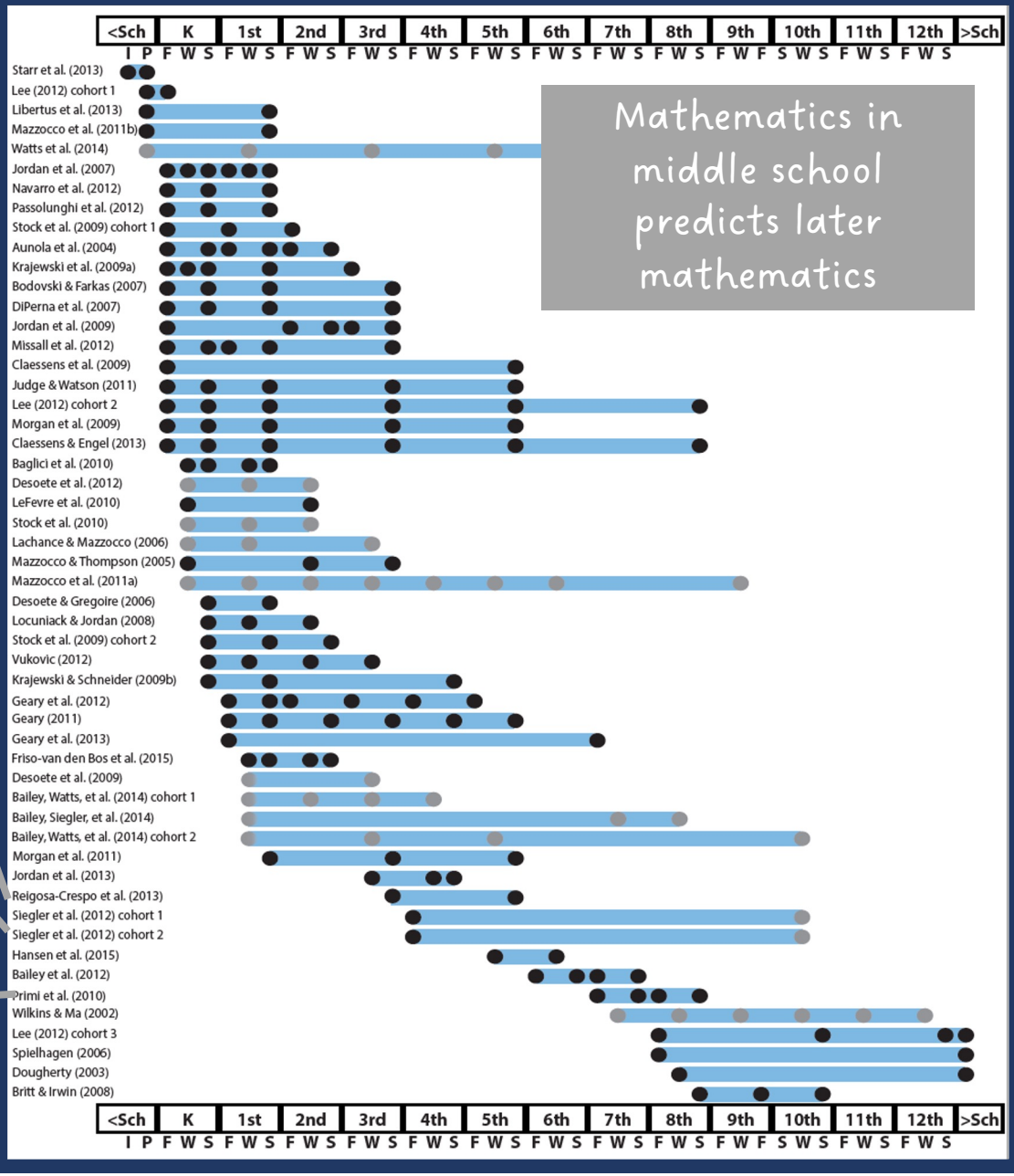
Counting and comparison in grades 2 or 4 predicted broad math 1 year later

Fractions at 10-12 years old predicted broad math 5 years later

Broad math in grade 7 predicted broad math in grade 8



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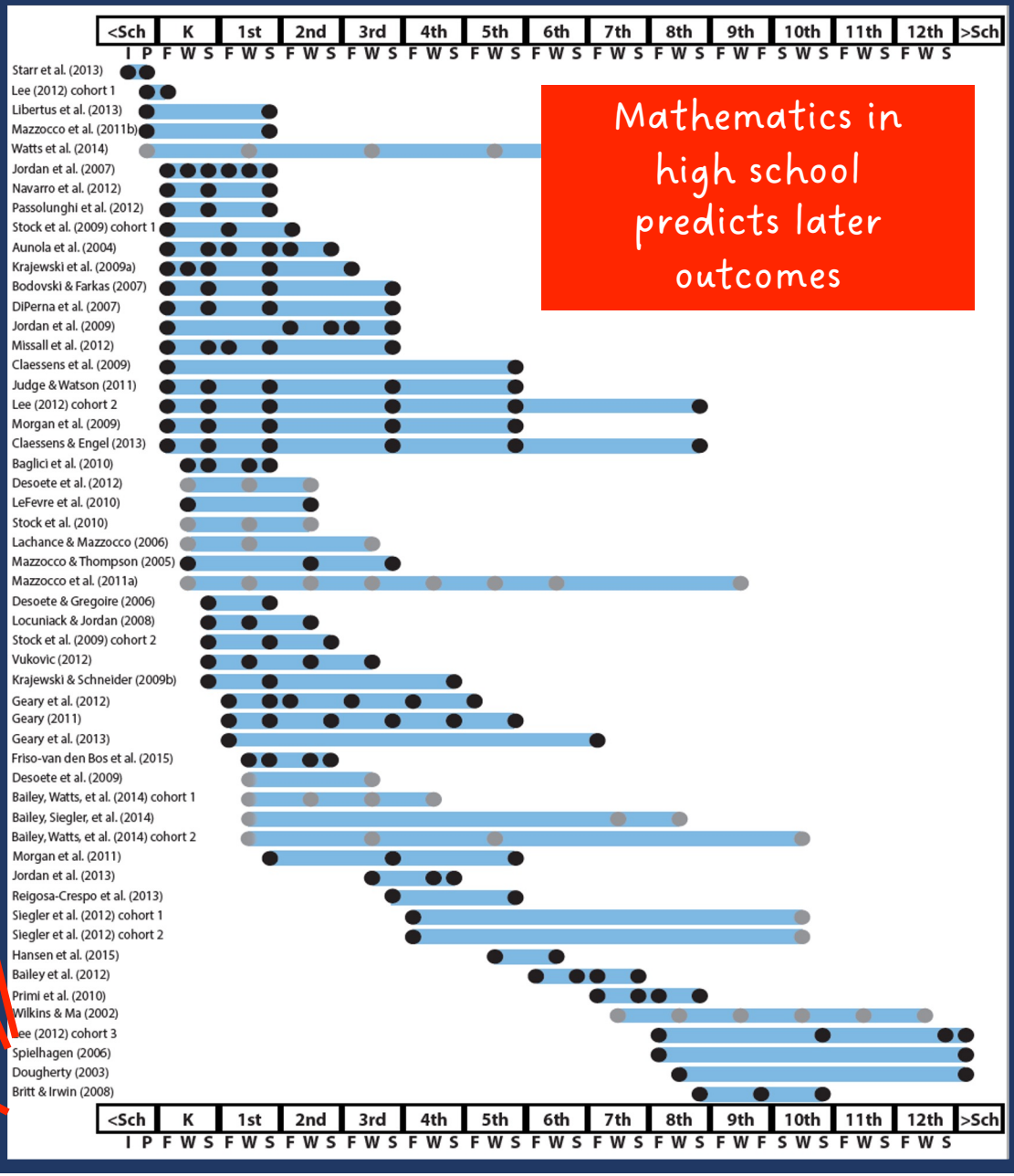
Broad math in grade 8 predicted completion of 4-year college degree

Students who took algebra in grades 8 took more advanced math courses and enrolled in 4-year colleges more often than students who took algebra in grade 9

Numeracy measured in adolescence impacted hourly earnings 7 to 15 years later



Mathematics in high school predicts later outcomes

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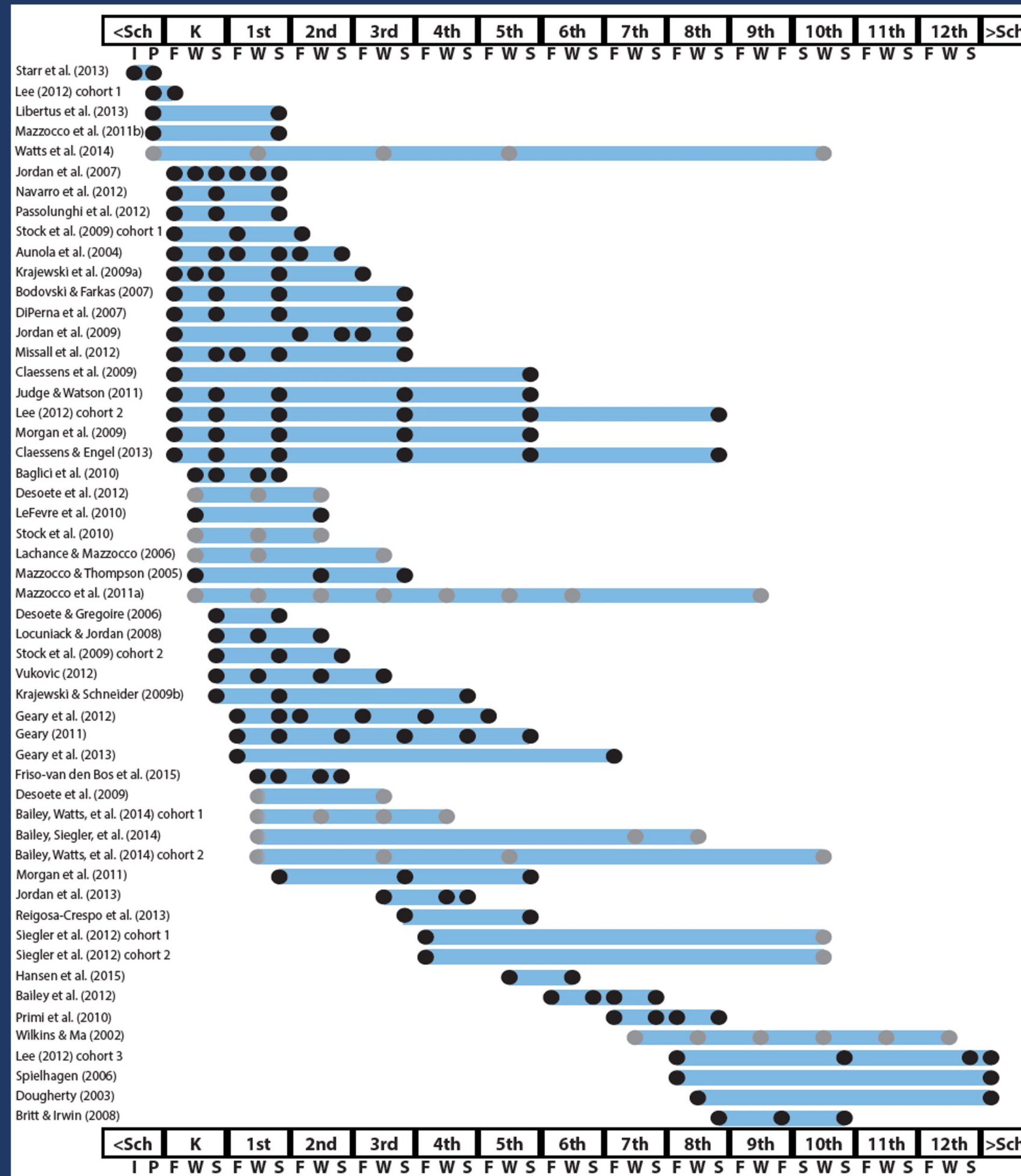
Mathematics in preschool predicts later mathematics

Mathematics in kindergarten predicts later mathematics

Mathematics in elementary school predicts later mathematics

Mathematics in middle school predicts later mathematics

Mathematics in high school predicts later outcomes



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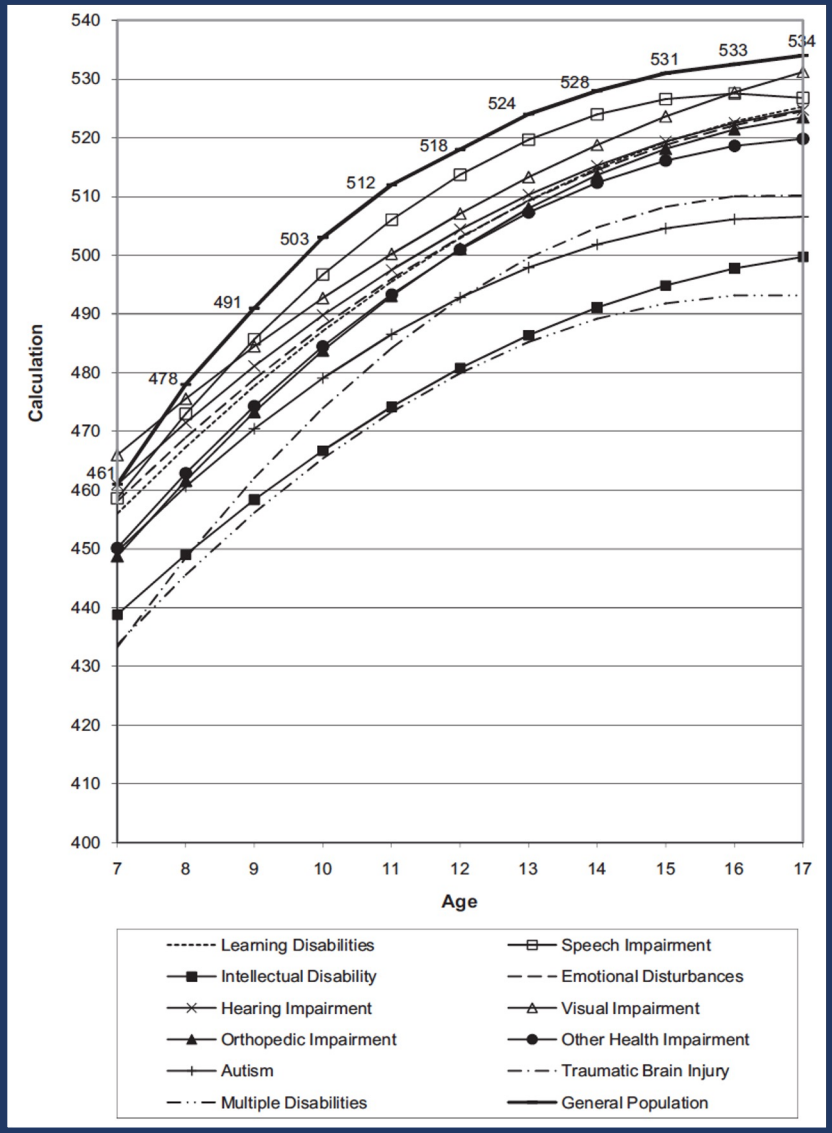
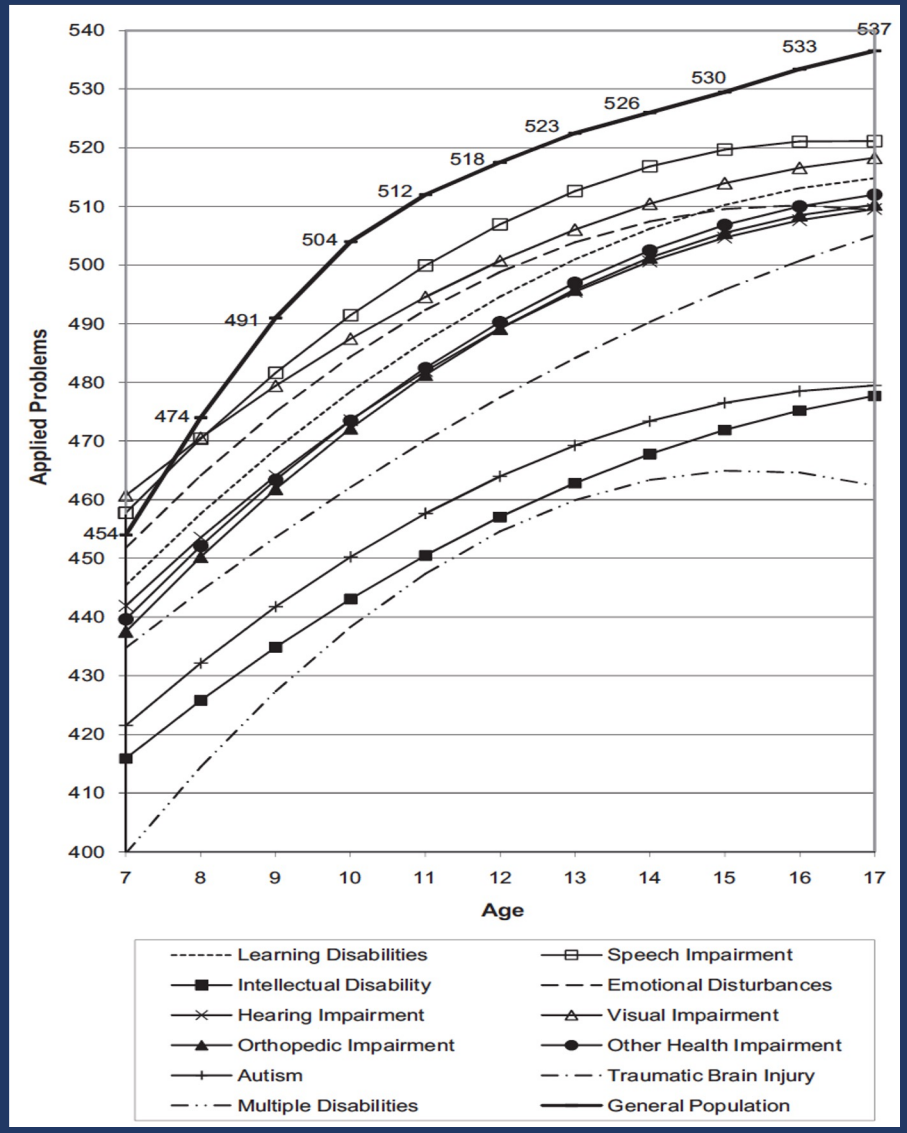


Computation



Problem Solving

Wei, Lenz, & Blackorby (2013)

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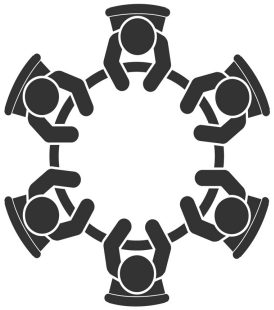

Provide examples of how you see earlier math skills relating to later math skills.

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Mathematical Progressions

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continuum of mathematics learning

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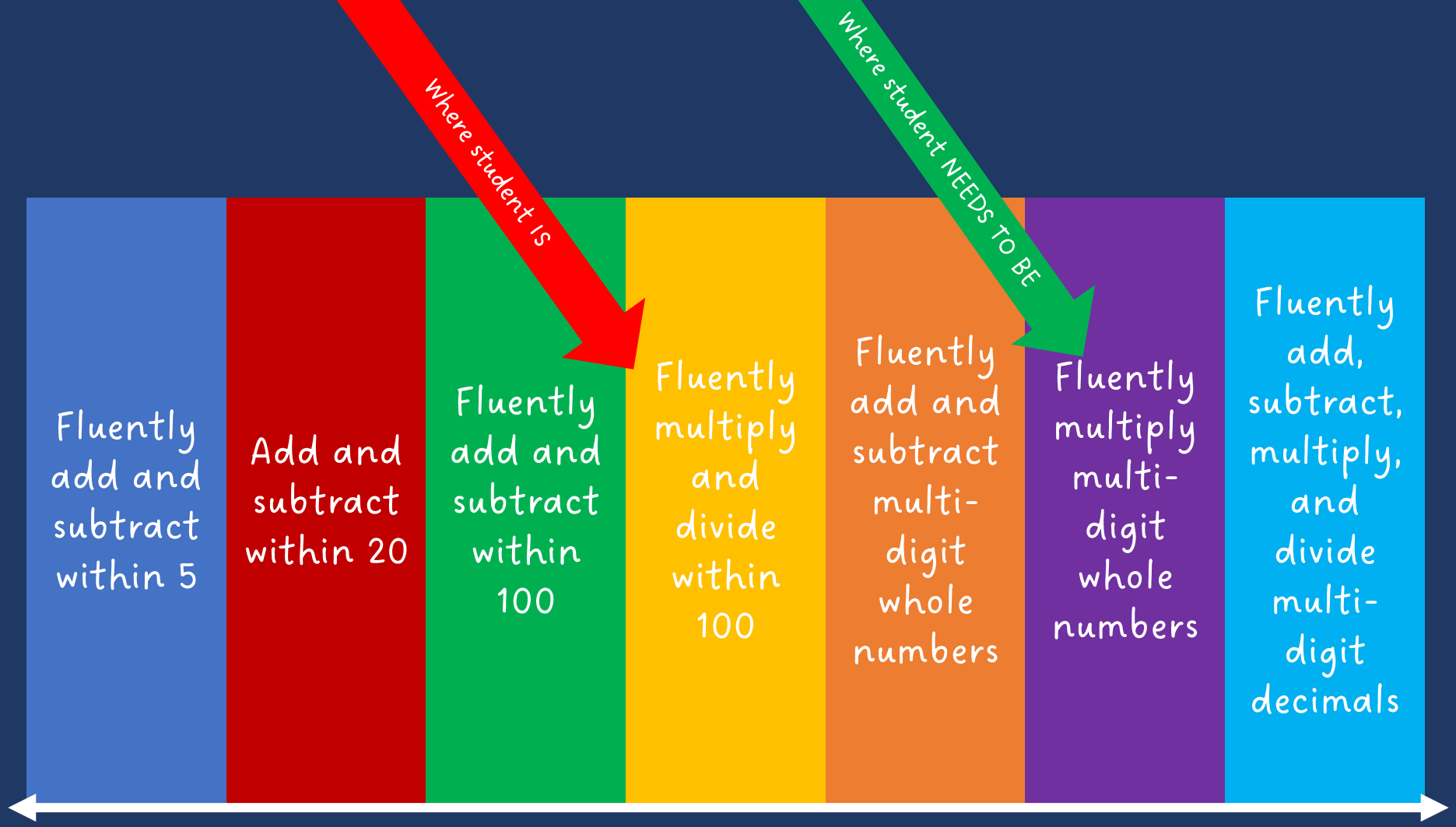

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Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones

Compose and decompose numbers from 11 to 19 into ten ones and some further ones

Understand that the two digits of a two-digit number represent amounts of tens and ones

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right

Use place value understanding to round whole numbers to the nearest 10 or 100

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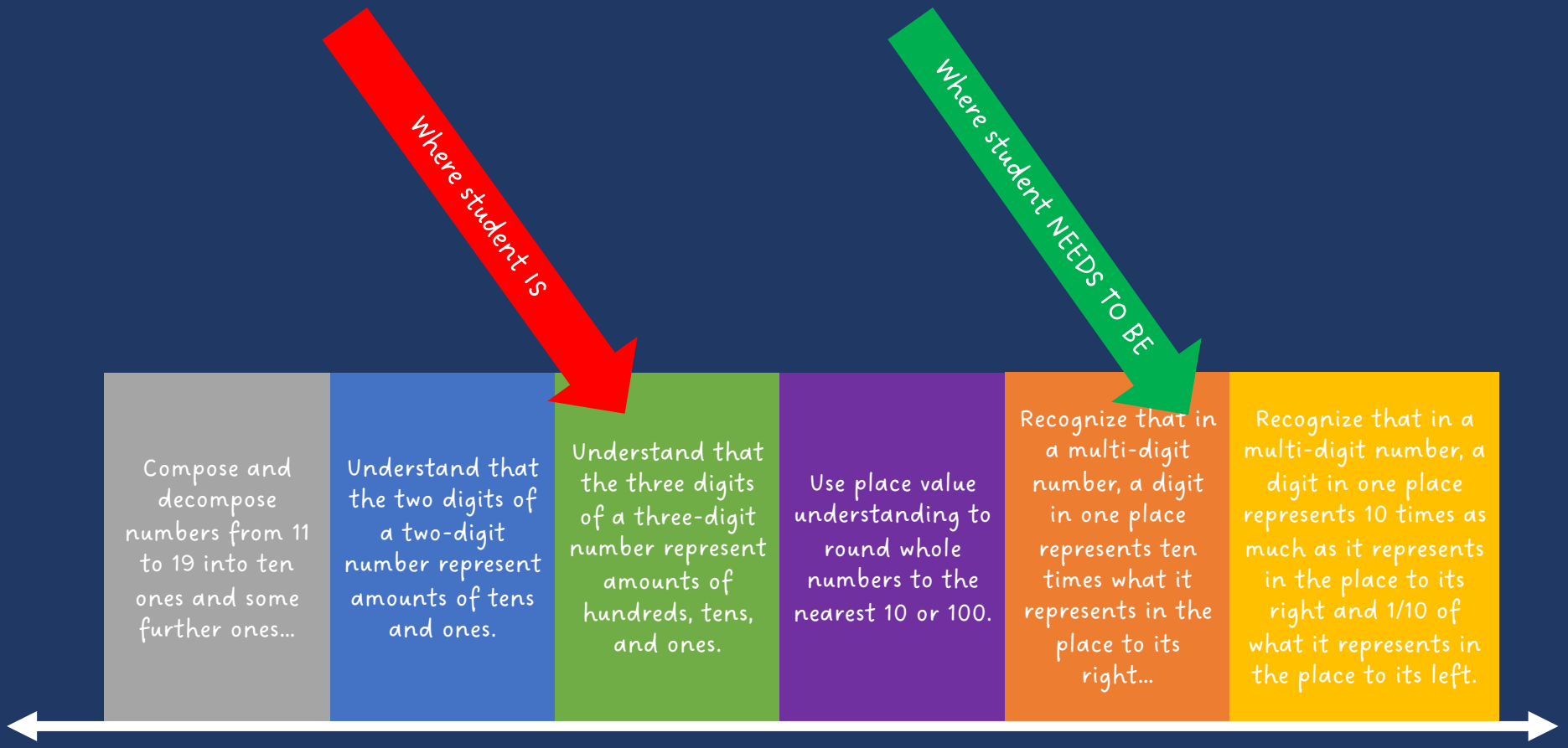

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Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20

Solve real-world and math problems involving the four operations with rational numbers.

Use addition and subtraction within 100 to solve one- and two-step word problems

Solve real-world and math problems leading to two linear equations in two variables

Use multiplication and division within 100 to solve word problems

Solve addition and subtraction word problems, and add and subtract within 10

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators

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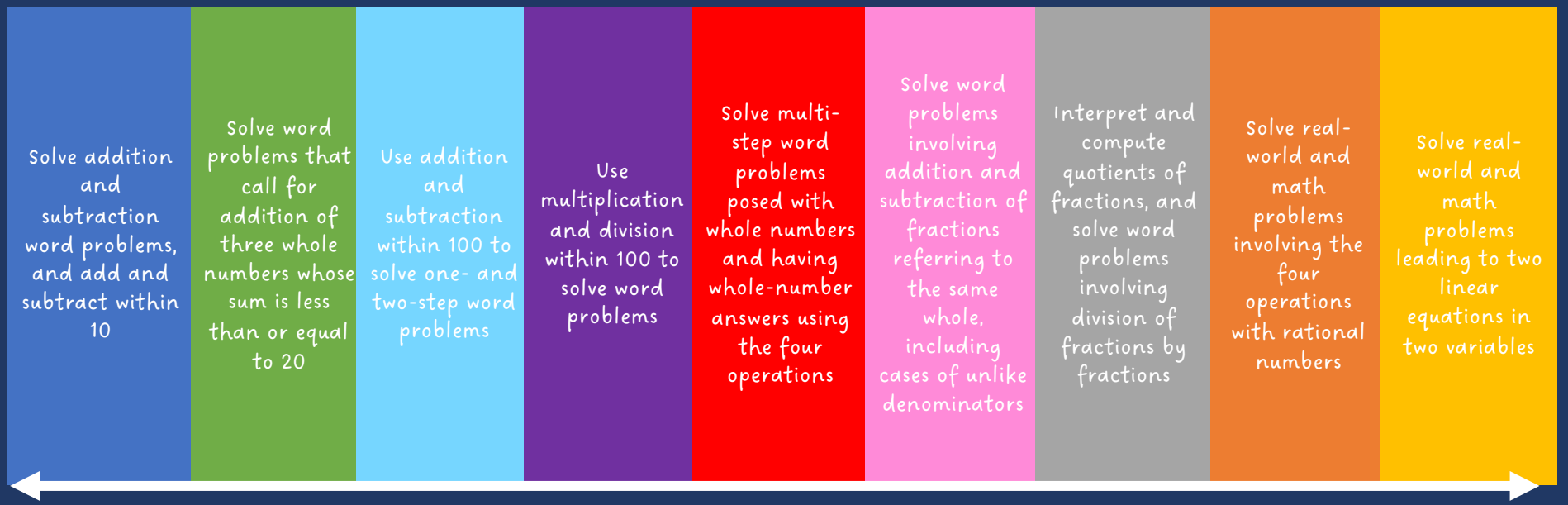

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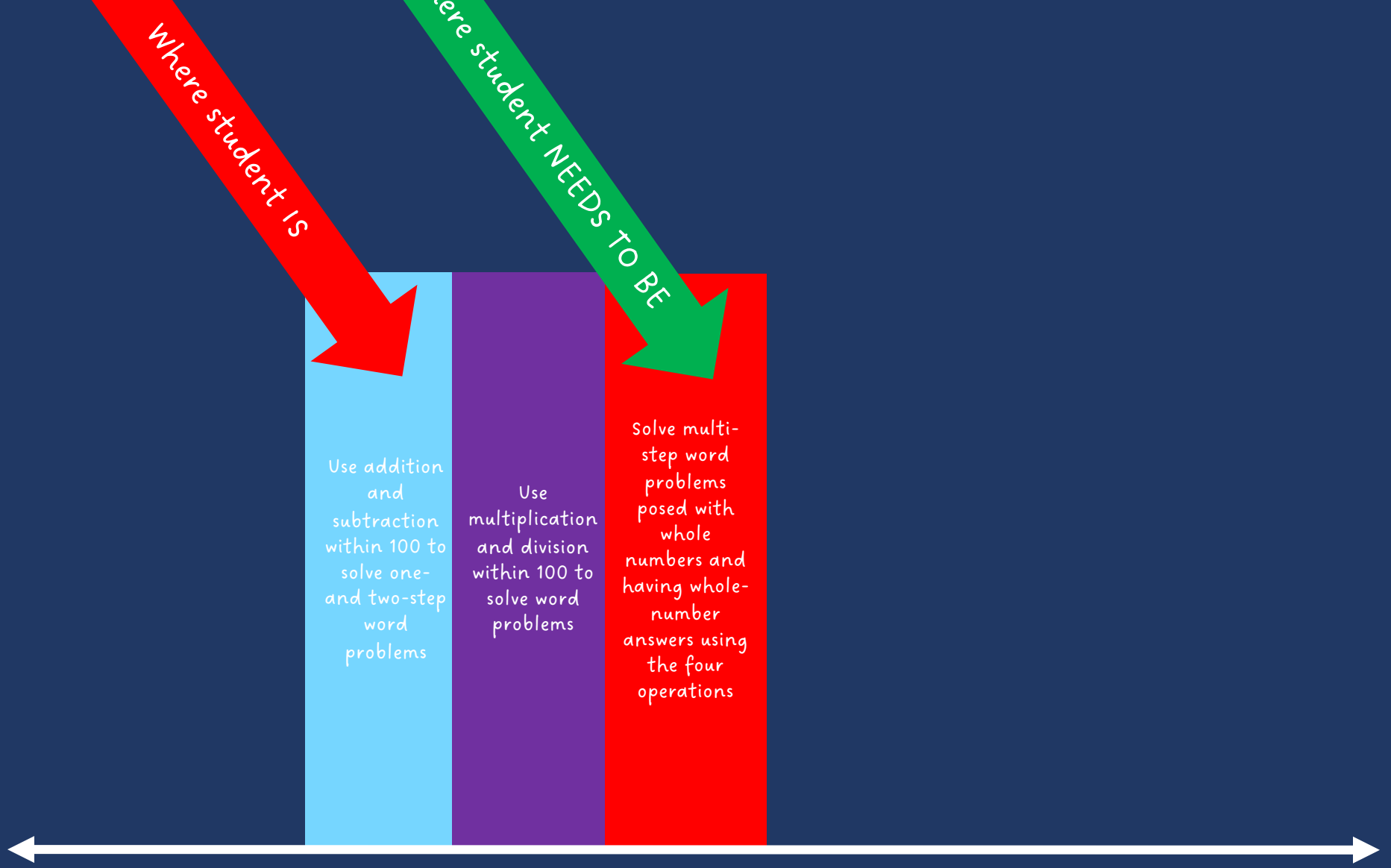

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An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.
** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

https://achievethecore.org/category/774/mathematics-focus-by-grade-level

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Table A.3. Grades 6–8 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*

Grade 6 · Grade 7 · Grade 8 · Outside grades 6–8 · Not identified at any level, pre-K–8

Curriculum Focal Points and Connections	Expectations of the Content Standards
Grade 6 Curriculum Focal Points	
***Number and Operations:* Developing an understanding of and fluency with multiplication and division of fractions and decimals** Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.	***Number and Operations, Grades 6–8***
	Work flexibly with fractions, decimals, and percents to solve problems
	Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line
	Develop meaning for percents greater than 100 and less than 1
	Understand and use ratios and proportions to represent quantitative relationships
	Develop an understanding of large numbers [identified in Grades 4 and 5 Curriculum Focal Points] and recognize and appropriately use exponential, scientific, and calculator notation
	Use factors, multiples, prime factorization, and relatively prime numbers to solve problems
	Develop meaning for integers and represent and compare quantities with them
	Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
	Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals
	Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems
	Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods
***Number and Operations:* Connecting ratio and rate to multiplication and division** Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.	
***Algebra:* Writing, interpreting, and using mathematical expressions and equations** Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that	

https://www.nctm.org/curriculumfocalpoints/

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What is the critical math content for your students?

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Instructional Platform

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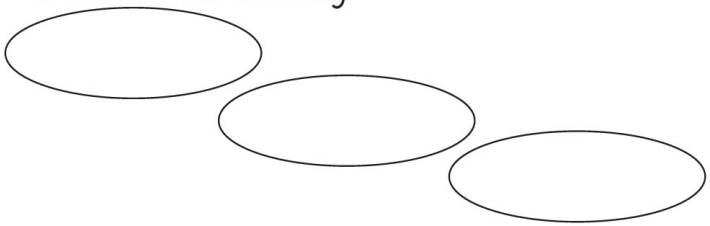
Instructional Platform

Instructional Delivery



Instructional Strategies

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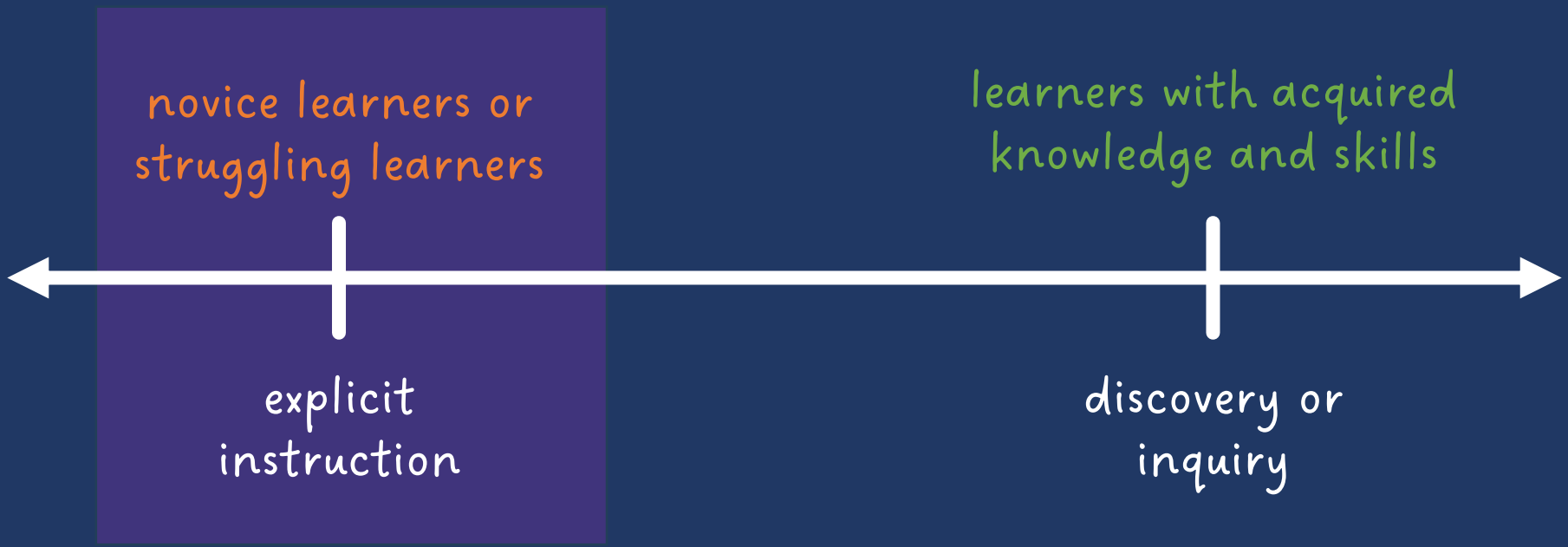
What's the continuum of mathematics support?



Anita Archer (2019) facebook/com/watch/?v=320845308601739

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Instructional Platform

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evidence-based practice

A practice that has shown **consistent and positive** results

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evidence-based practice

evidence-based intervention

An intervention (i.e., packaged program) that has shown **consistent and positive** results

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A method or strategy that has shown **consistent and positive** results

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Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

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Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit
- Vocabulary
- Representations

INSTRUCTIONAL STRATEGIES

- Fluency
- Word Problems

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Be Explicit

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Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit

INSTRUCTIONAL STRATEGIES

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Explicit Instruction

Research and Information

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

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Over a half century of research supports explicit (i.e., direct, systematic) instruction.
(Stockard et al., 2018)

When compared to discovery approaches, explicit instruction demonstrates higher outcomes.
(Alfieri et al., 2011; Kroesbergen et al., 2004; Poncy et al., 2010)

Numerous meta-analyses and large-scale studies have identified explicit instruction as essential for the teaching and learning of mathematics.
(Chodura et al., 2015; Ennis & Losinski, 2019; Jitendra et al., 2018; Kong et al., 2021; Morgan et al., 2015; Nelson & McMaster, 2019; Powell et al., 2021).

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Explicit Instruction

MODELING	PRACTICE

SUPPORTS

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MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

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Modeling is a dialogue between the teacher and students.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

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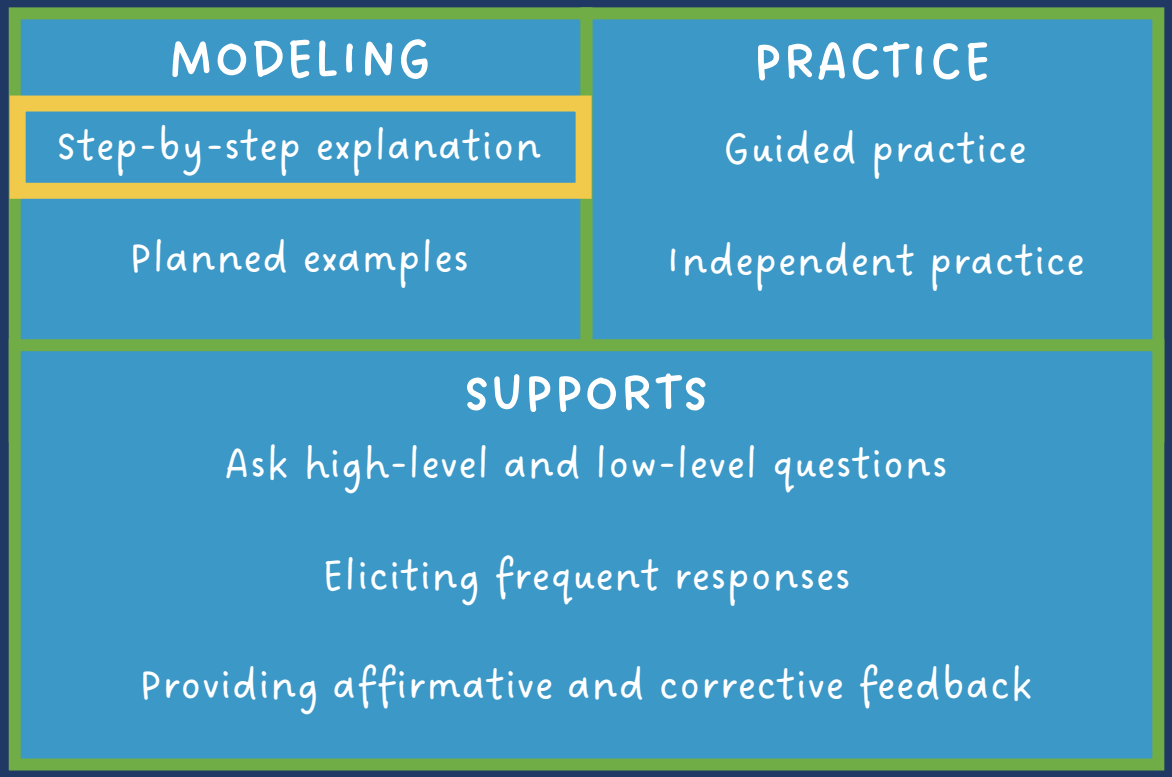
Modeling includes a step-by-step explanation of how to do a math problem.

A teacher may do 1 modeled problem or several.



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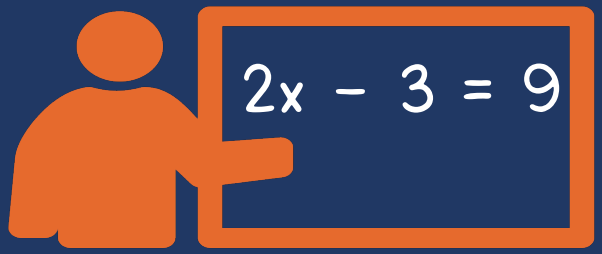

"Today, we are learning about solving equations. This is important because sometimes you have an unknown and you want to figure out the unknown."

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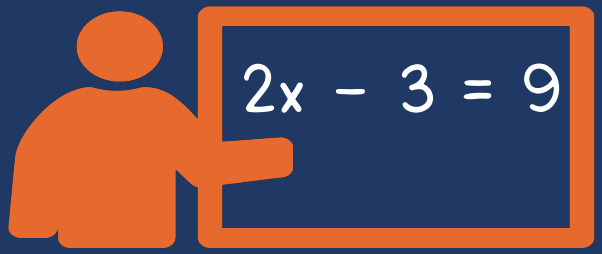

"Let's solve this problem. What's the problem?"

"2x minus 3 equals 9."

"To solve 2x minus 3 equals 9, first we want to isolate the term with the variable. What's the term with the variable?"

"2x."

"2x is our term with the variable. What's the coefficient?"

"2."

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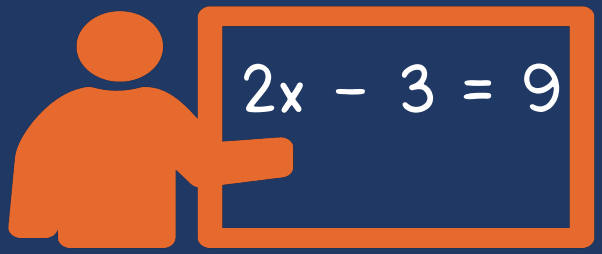

"And what's the variable?"

"x."

"The first step is to isolate the term with the variable. What do we want to isolate?"

"The term with the variable - 2x."

"To isolate the term with the variable, we can move the constant to the other side of the equal sign. To do that, we can use inverse operations. If we have minus 3, what's the inverse operation?"

"Plus 3."

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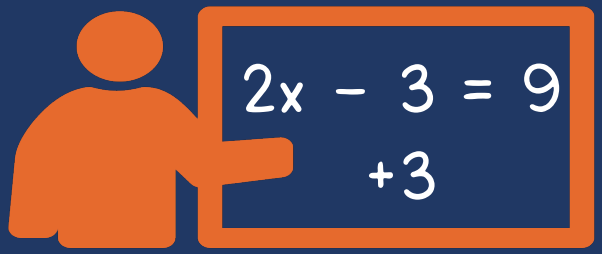

"So, let's write plus 3 below the minus 3. What do we write?"

"Plus 3."

"But if we add 3 to this side of the equal sign, what do we do to the other side of the equal sign?"

"Also add 3."

"Yes. Anytime you do something to one side of the equal sign you have to do the same thing to the other side of the equal sign. Describe that to your partner."

"Whenever you do something..."

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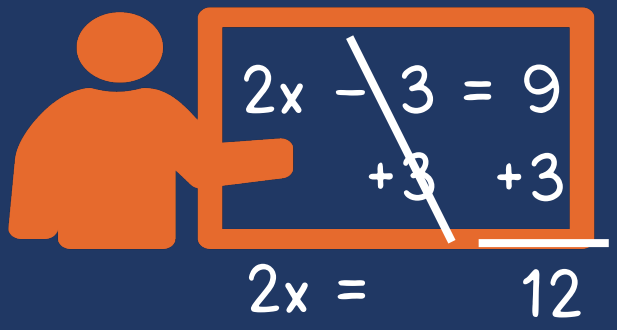

"So what should we write on that side of the equal sign?"

"Plus 3."

"So, minus 3 and plus 3 creates a zero pair. That would equal 0. What should we do?"

"Cross out the zero pair."

"Then we add 9 plus 3. What's 9 plus 3??"

"12."

"Let's write 12 here. So, now we have 2x equals 12. What is the equation now?"

"2x equals 12."

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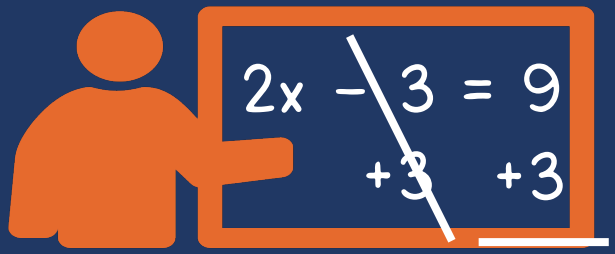

$$\frac{2x}{2} = \frac{12}{2}$$

$$x = 6$$

"Now we have 2x equals 12. We can isolate the variable of x from the coefficient of 2 by dividing by 2 on both sides of the equal sign What will we do?"

"Divide by 2 on both sides of the equal sign to balance the equation."

"So, 2x divided by 2 equals x. What's 12 divided by 2?"

"6."

"Let's write x equals 6. What?"

"x equals 6."

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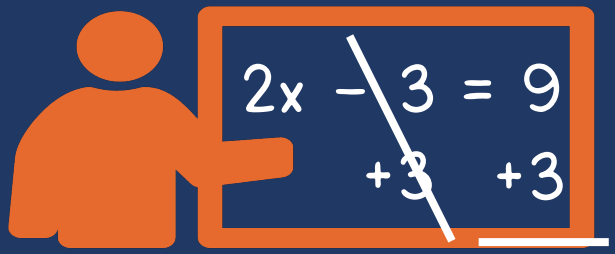

$$\frac{2x}{2} = \frac{12}{2}$$

$$x = 6$$

"So, x equals 6. Let's see if that makes sense. How would you check our work?"

"2 times 6 equals 12. Then, 12 minus 3 equals 9!"

"Excellent. Who can share how we solved this problem?"

"First, we isolated the term with the variable. So, we added 3 to both sides of the equation. Then, we isolated the variable by dividing by 2 on both sides of the equal sign."

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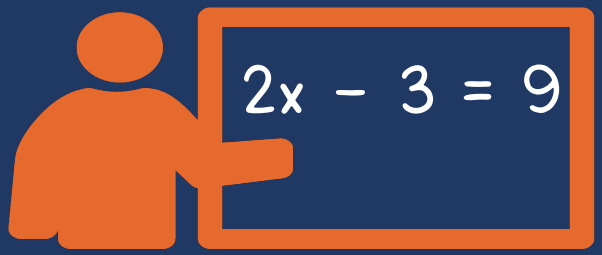

What did you observe?

How would you improve this example?

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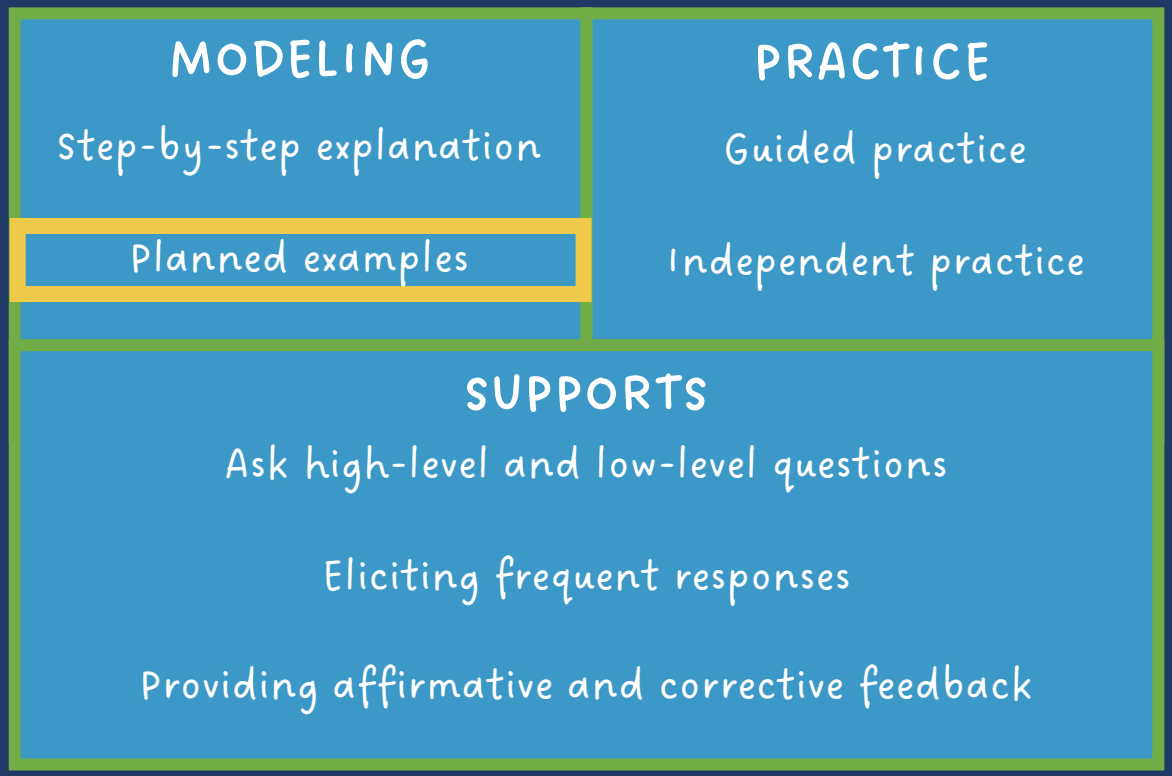
Modeling needs to include planned examples.

These examples should be sequenced so easier skills lead to more difficult skills.



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MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

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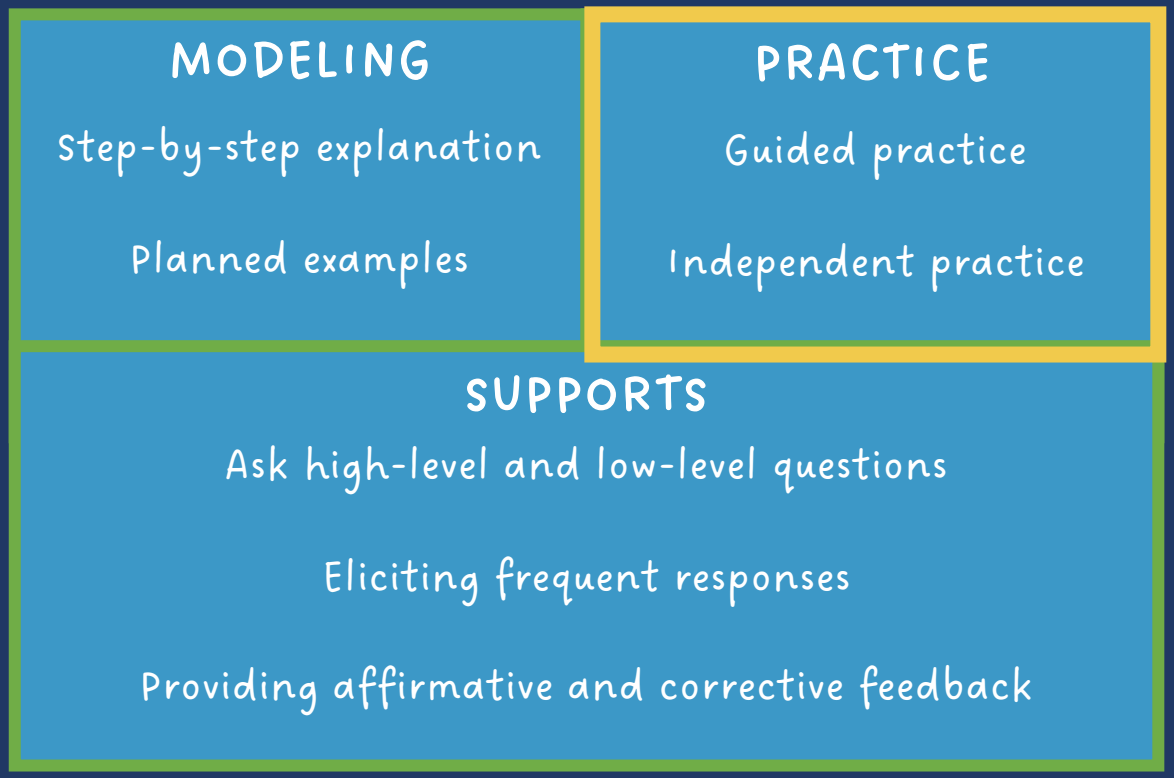

Practice continues as a dialogue between the teacher and students.

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MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Guided practice is practice in which the teacher and students practice problems together.

"Let's work on a problem together."

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MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Independent practice is practice in which the students practice independently with teacher support.

"Now, you'll practice a problem on your own. Use your attack strategy!"

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MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

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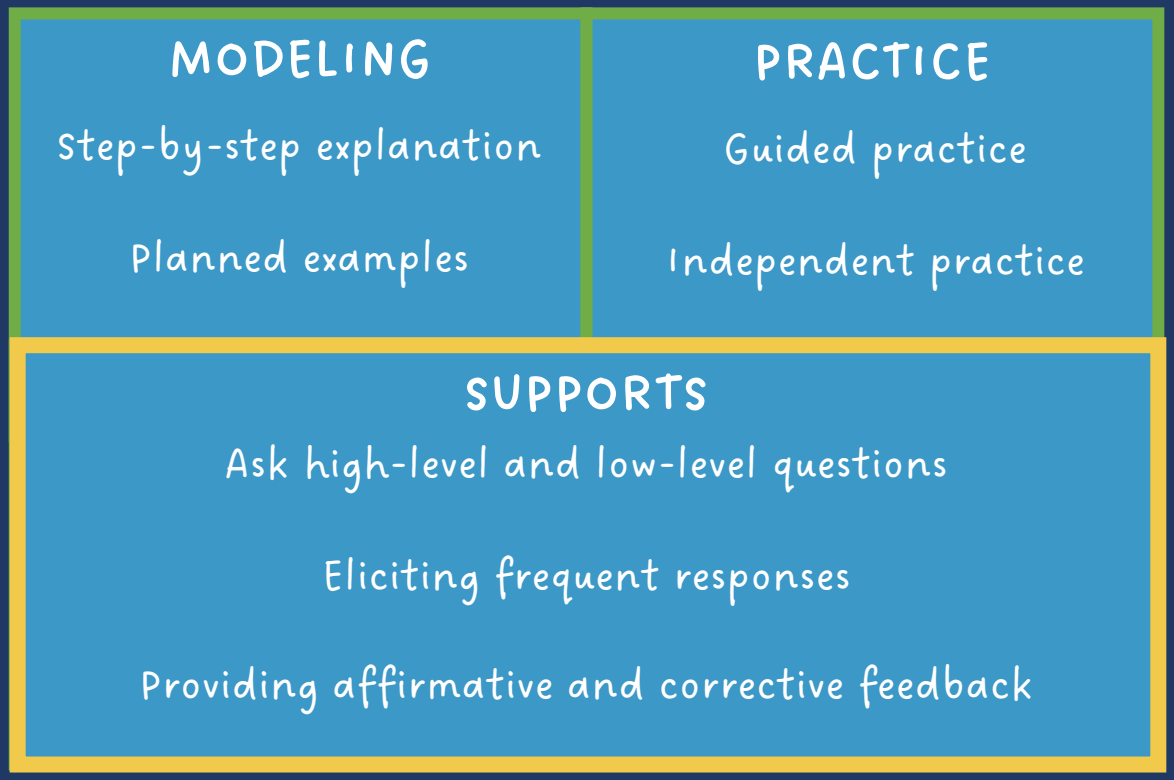

These **Supports** should be used in both **Modeling** and **Practice.**

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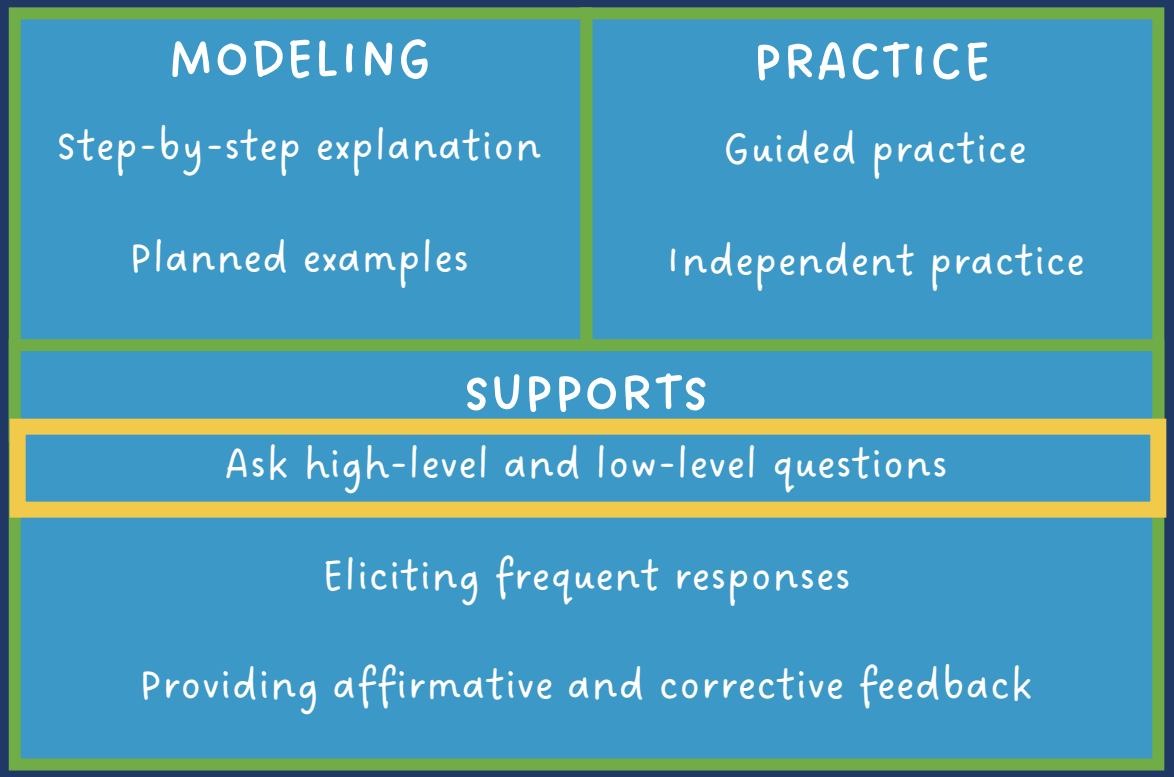

During **Modeling** and **Practice**, it is essential to engage students and check for understanding.

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MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"What is 7 times 9?"

"63."

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MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Why do you use zero pairs?"

"A positive 1 and a negative 1 equal 0 Zero pairs helps us balance on either side of the equal sign."

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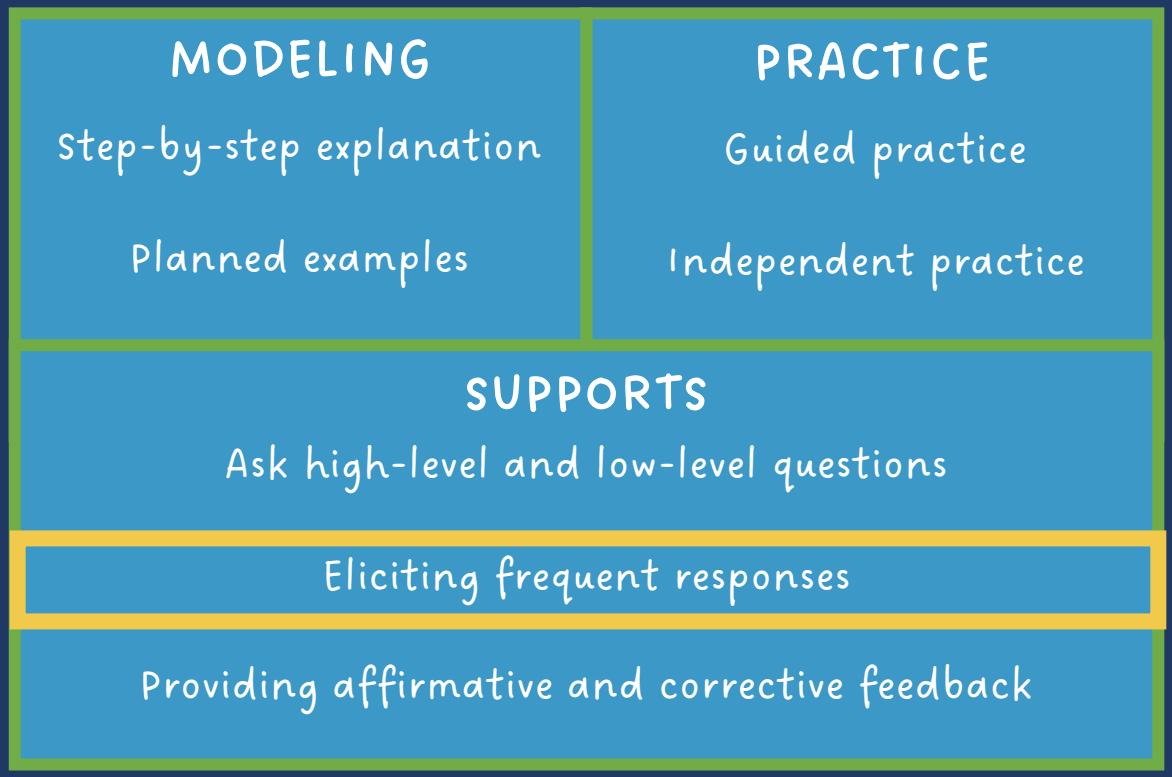

During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

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MODELING

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Planned examples

PRACTICE

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Independent practice

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Ask high-level and low-level questions

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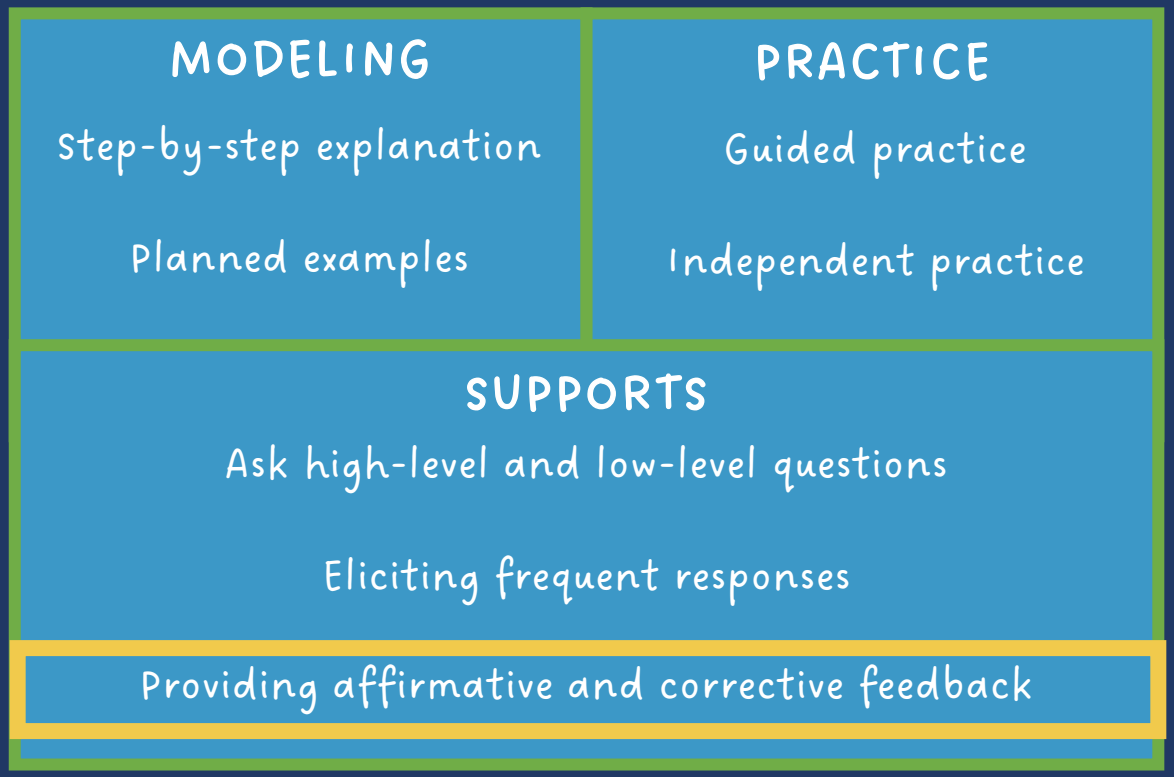

During **Modeling** and **Practice**, students should receive immediate feedback on their responses.

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MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Nice work using your word problem attack strategy."

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MODELING

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"Let's look at that again. Tell me how you used the formula for volume."

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Explicit Instruction

Research and Information

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

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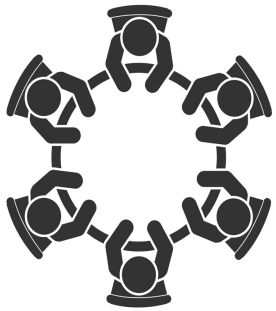
Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.



What are your strengths with explicit instruction?

What are the opportunities for growth?

What are your immediate next steps?

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Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

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Focus on Vocabulary

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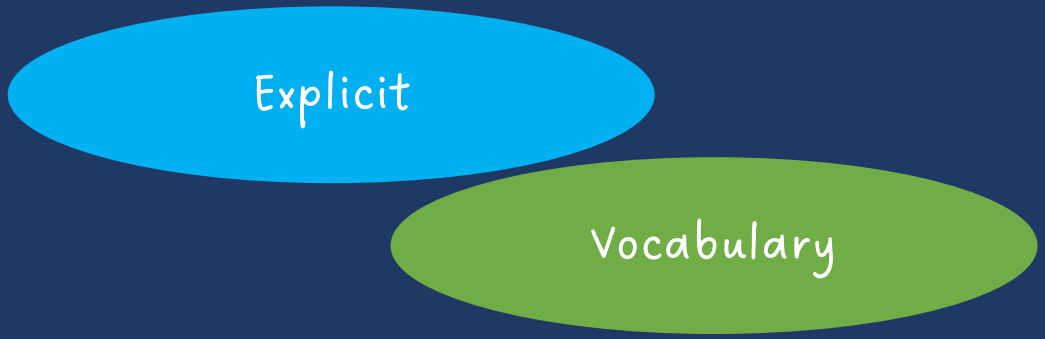
Instructional Platform

INSTRUCTIONAL DELIVERY



INSTRUCTIONAL STRATEGIES

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Math Vocabulary

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...

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Significant correlation (r = .49) between mathematics vocabulary and mathematics performance. Mathematics vocabulary appears most important for word-problem performance (r = .58).

(Lin et al., 2021)

Early mathematics vocabulary related to mathematics and literacy.

(Hornburg et al., 2018; Purpura et al., 2017)

Students who experience difficulty with mathematics demonstrate lower mathematics vocabulary performance.

(Hughes et al., 2020; Powell & Nelson, 2017; Powell et al., 2017; Unal et al., 2021)

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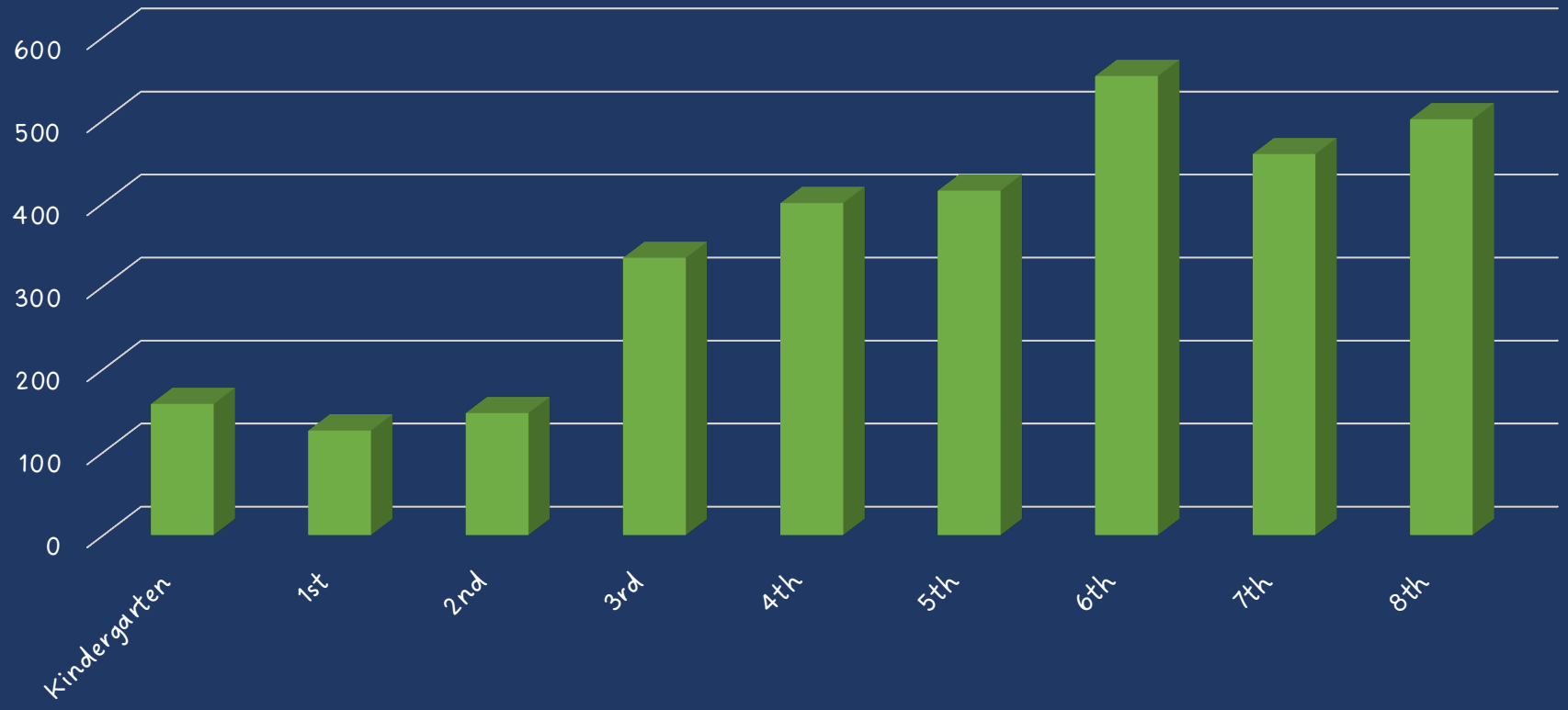

Powell, Bos, & Lin (2019)

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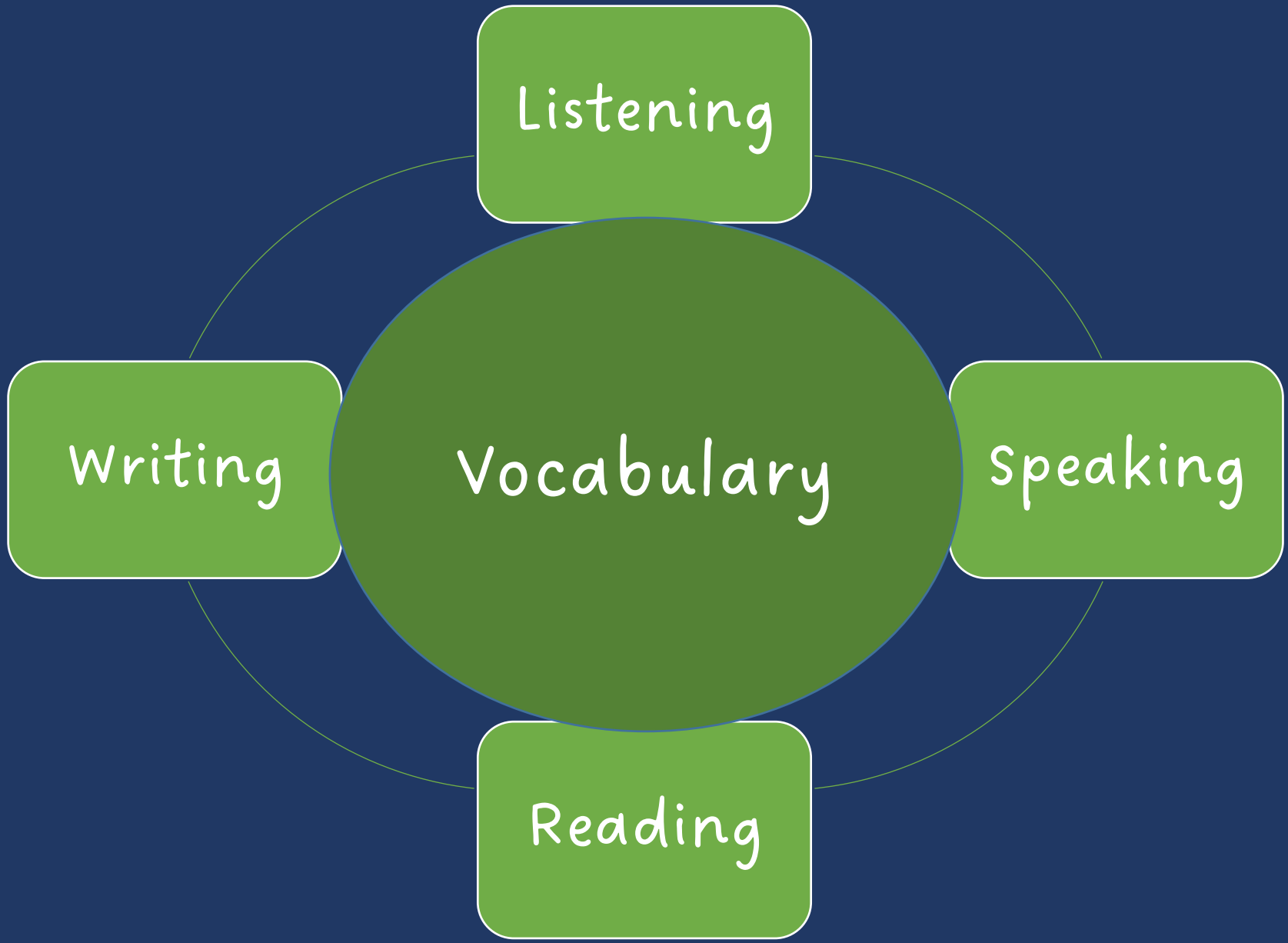

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1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

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1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)

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1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)

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1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)

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   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

divide vs. Continental Divide

variable vs. variably cloudy

Rubenstein & Thompson (2002)

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6. Some math terms are homophones

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)

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6. Some math terms are homophones

7. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)

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6. Some math terms are homophones

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)

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6. Some math terms are homophones

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

skip count vs. multiples

one-fourth vs. one quarter

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4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homophones

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

rhombus vs. diamond

10. Some math concepts are verbalized in more than one way

vertex vs. corner

11. Informal terms may be used for formal math terms

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Use formal math language

Use terms precisely

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$$\begin{array}{r} 167 \\ +\ 294 \\ \hline \end{array}$$

Why this is important...

- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.

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top number and bottom number

numerator and denominator

Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.

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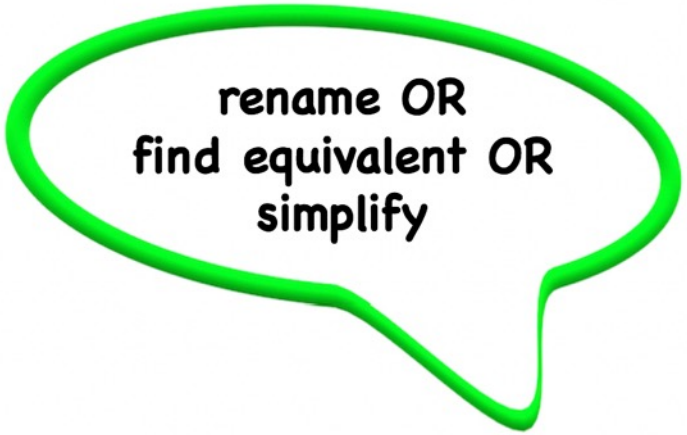

Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

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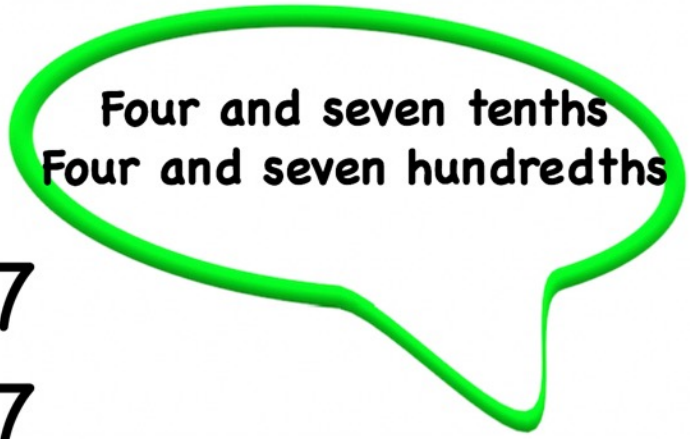

4.7

4.07

Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.

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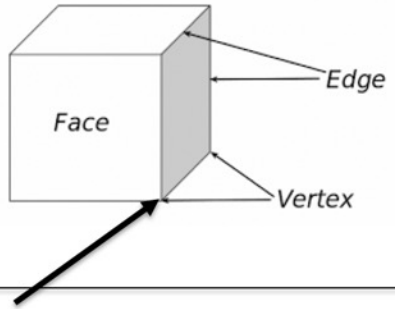
point

vertex



Why this is important...

- This is the endpoint where two or more line segments or rays meet.

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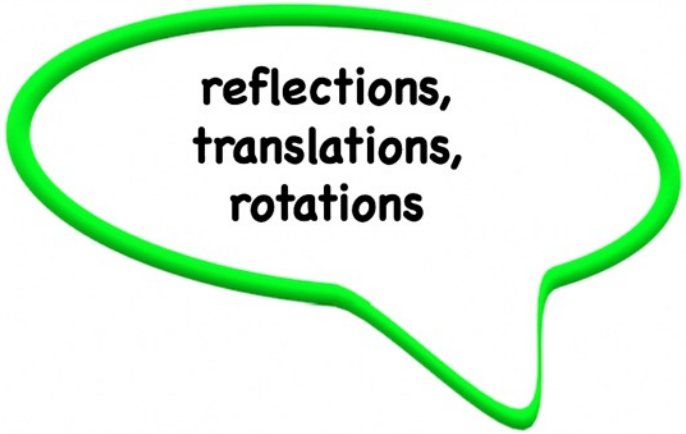

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.

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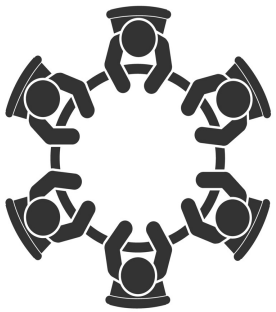
Math Vocabulary

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...



Identify examples of "Instead of ____, say ____."

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Use formal math language

Use terms precisely

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Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

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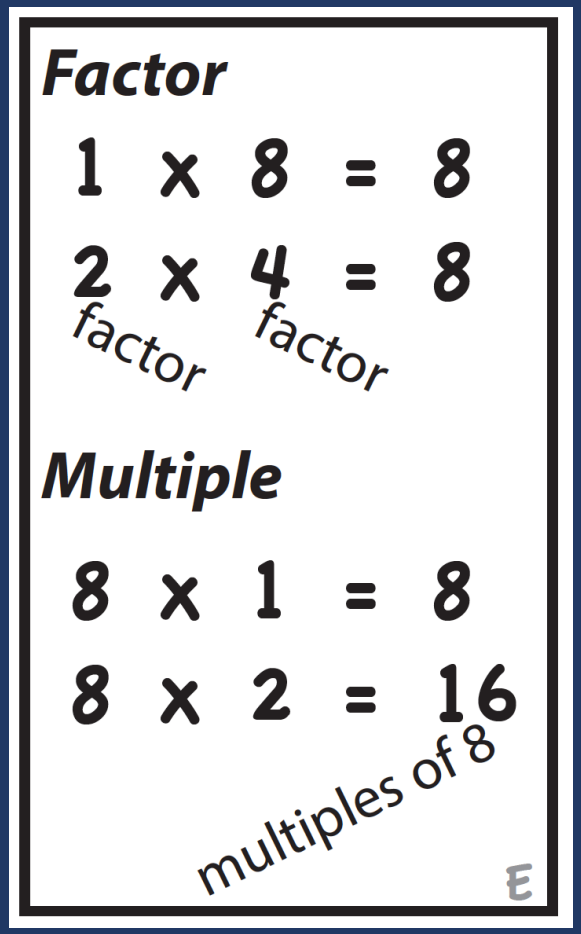

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Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

4:3

Proper fraction

$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

D

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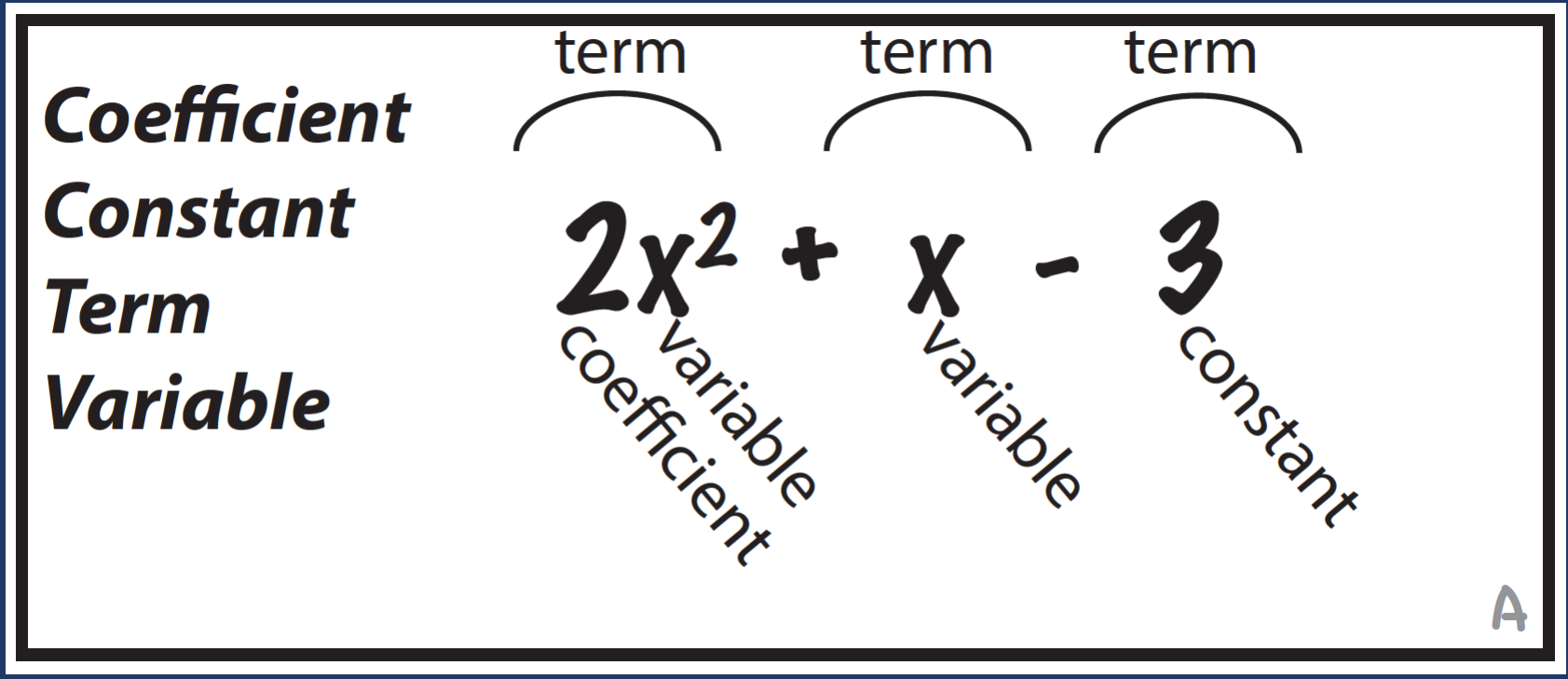

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Equation	$9x - 4 = 7x$
Expression	$9x - 4$
Formula	$a^2 + b^2 = c^2$
Function	$f(x)$
Inequality	$9x - 4 > 6x$

C

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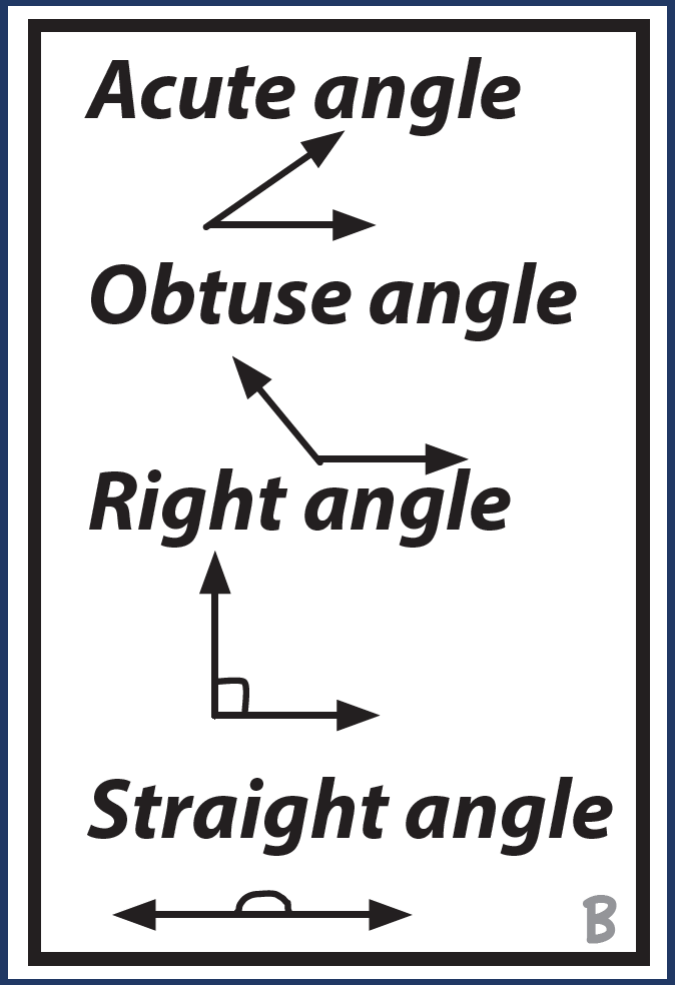

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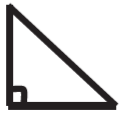
Acute triangle

Obtuse triangle

Right triangle



Equilateral triangle

Isosceles triangle

Scalene triangle

C

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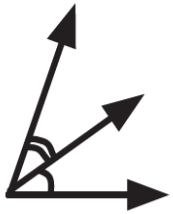
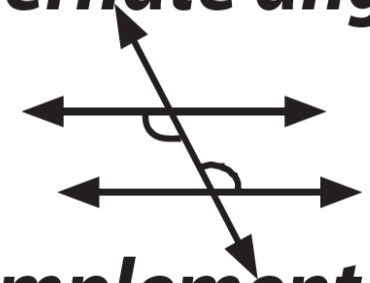
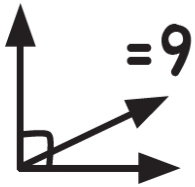
Adjacent angles



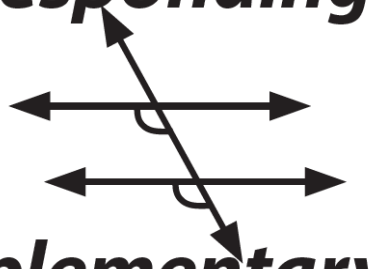
Alternate angles



Complementary angles



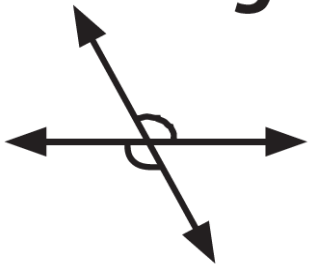
Corresponding angles



Supplementary angles

=180°

Vertical angles



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Congruent figures

Similar figures

E

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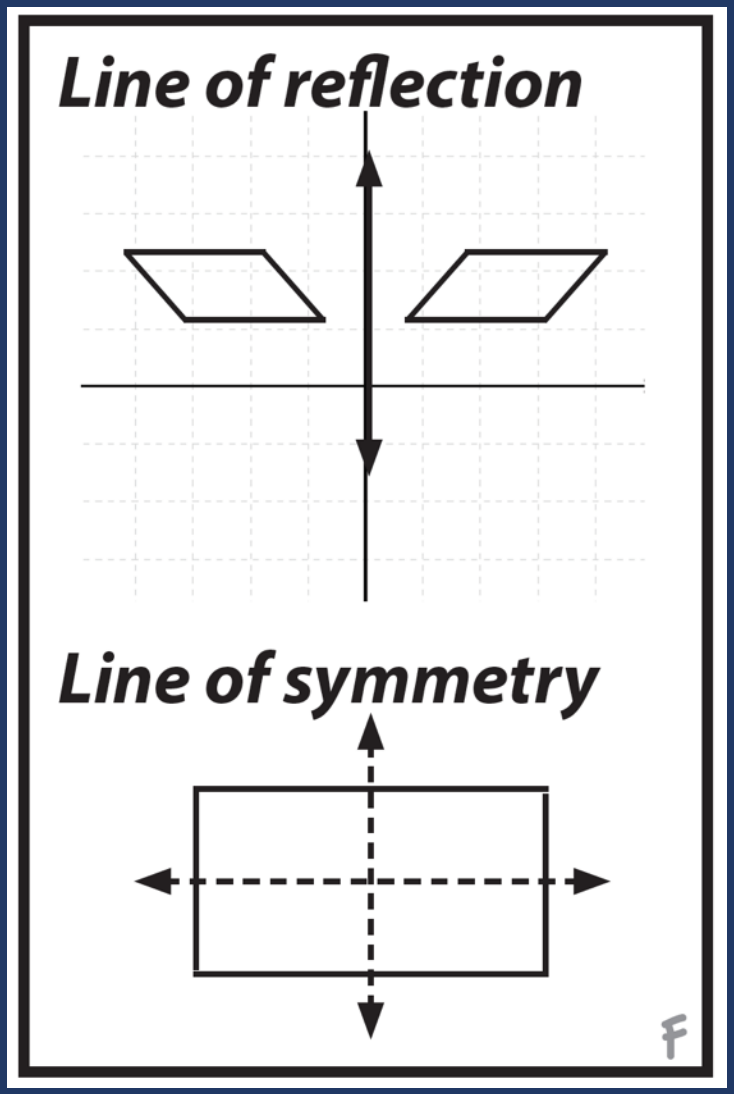

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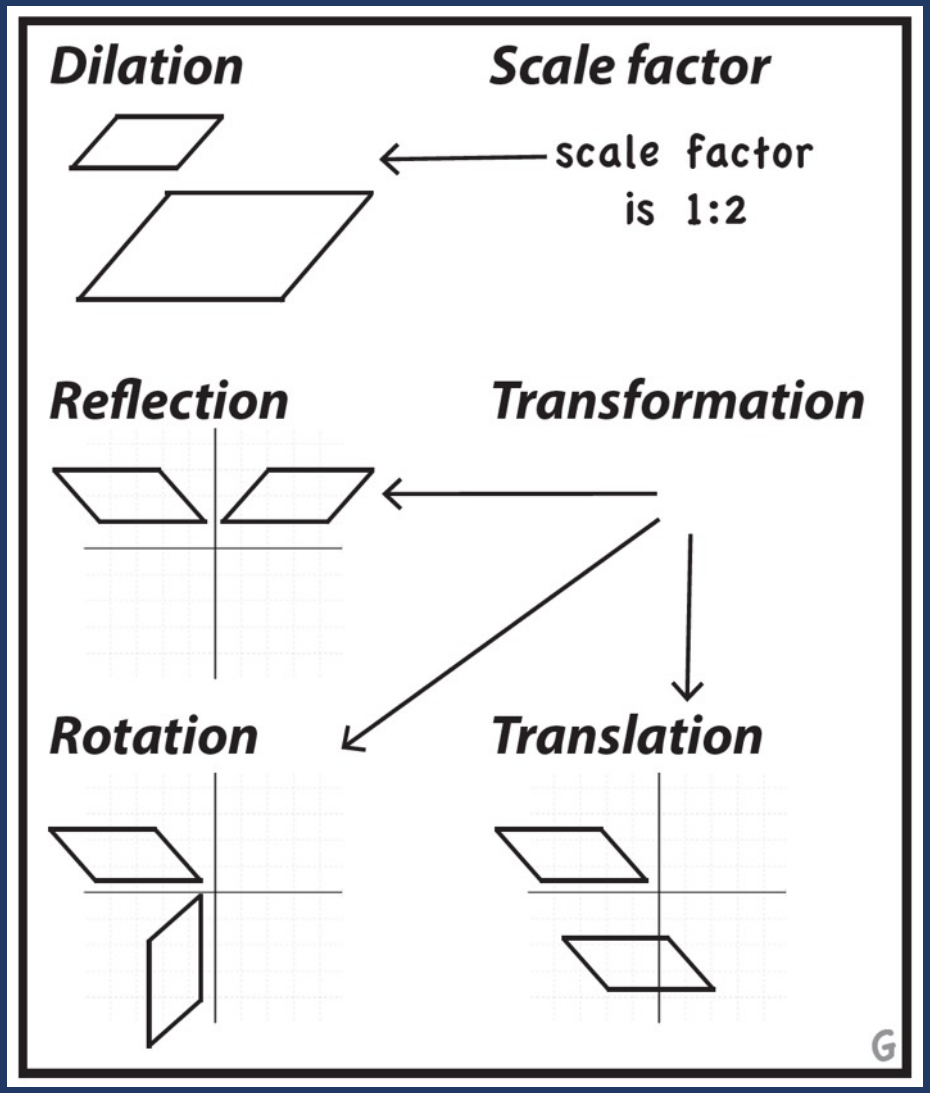

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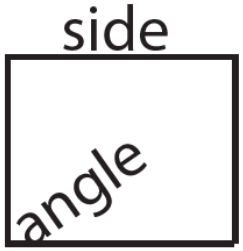
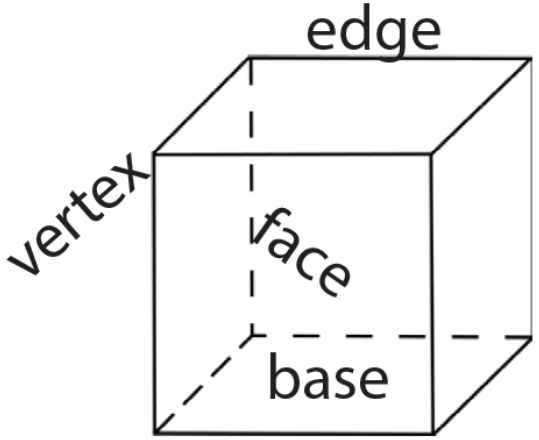
Angle
Base
Edge
Face
Side
Vertex





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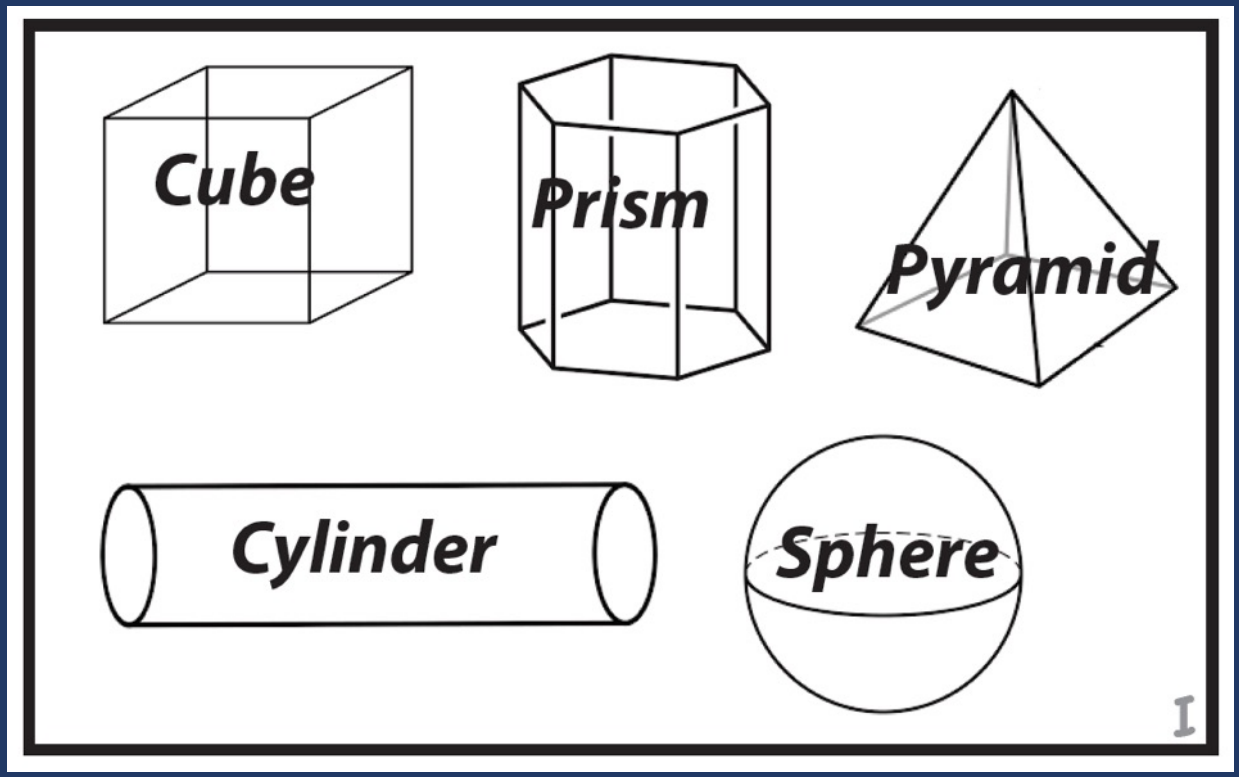

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Coordinate plane

Ordered pair

Quadrants

x

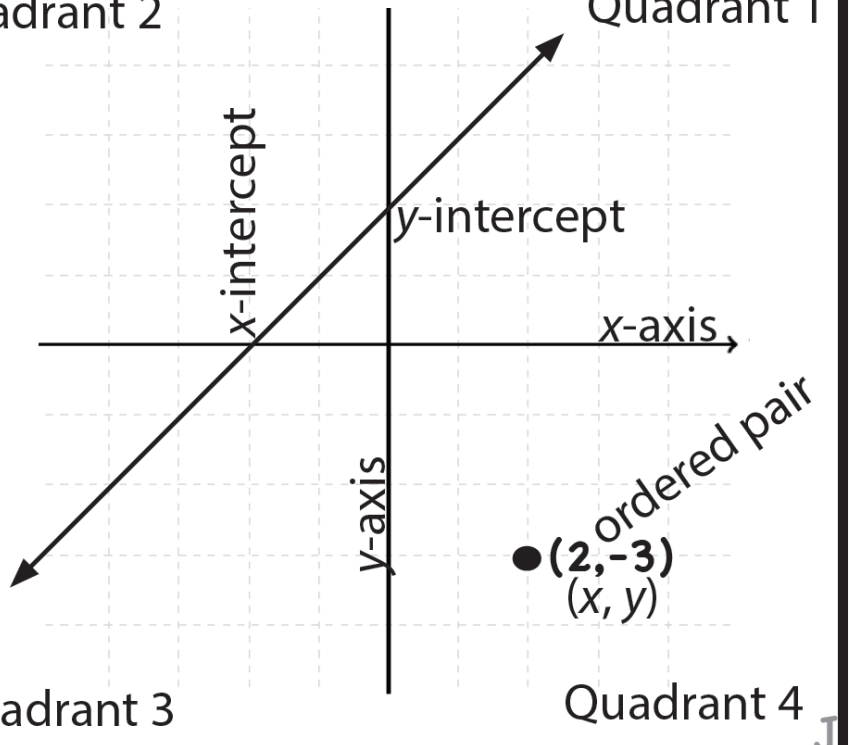
x-axis

x-intercept

y

y-axis

y-intercept



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Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

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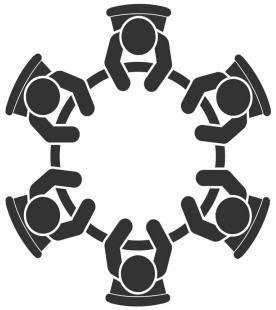

Discuss terms you want your students to use with precision.

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Use formal math language

Use terms precisely

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Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

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Use explicit instruction.
(Powell & Driver, 2015; Stevens et al., 2022)

Use semantic maps.
(Stevens et al., 2022)

Use visuals.
(Powell & Driver, 2015)

Use flashcards with spaced practice.
(Petersen-Brown et al., 2019)

Use read-alouds.
(Purpura et al., 2017)

Use explicit instruction.
Use multiple representations.
Create opportunities for discussion and feedback.
Monitor student progress.
Coordinate vocabulary instruction across settings.
Create additional practice opportunities.
(Nelson et al., 2020)

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Math Vocabulary

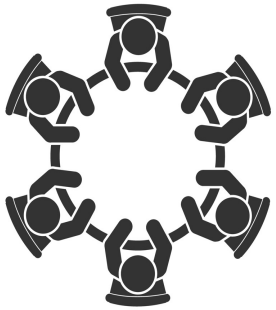
Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?



Discuss your strategy for focusing on mathematical vocabulary in your teaching.

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Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

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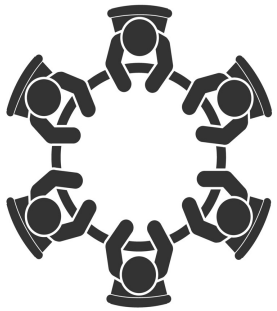
Students require modeling and practice on how to use the language of mathematics.



What are your strengths with mathematical language?

What are the opportunities for growth?

What are your immediate next steps?

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Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

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Use Representations

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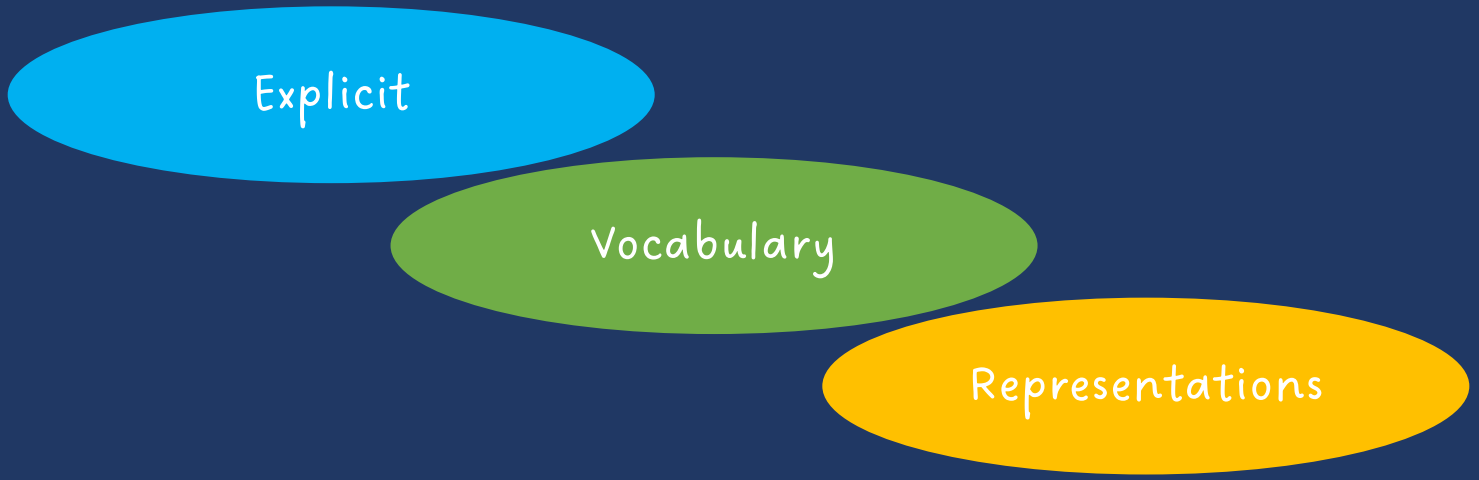
Instructional Platform

INSTRUCTIONAL DELIVERY



INSTRUCTIONAL STRATEGIES

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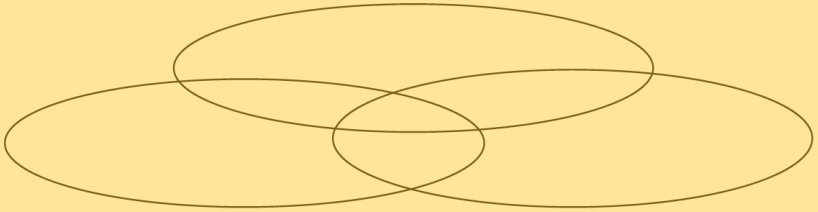
Representations

Research and Information



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Hands-on manipulatives contribute to increases in mathematics performance.
(Bouck & Park, 2018; Carbonneau et al., 2013; Namkung & Bricko, 2021; Sherman & Bisanz, 2009; Strickland & Maccini, 2012)

Virtual manipulatives contribute to increases in mathematics performance.
(Bouck et al., 2020; Satsangi et al., 2016)

Other visuals (e.g., graphic organizers) contribute to increases in mathematics performance.
(Jitendra et al., 2009; Sharp & Dennis, 2017; van Garderen, 2007; Xin, 2008)

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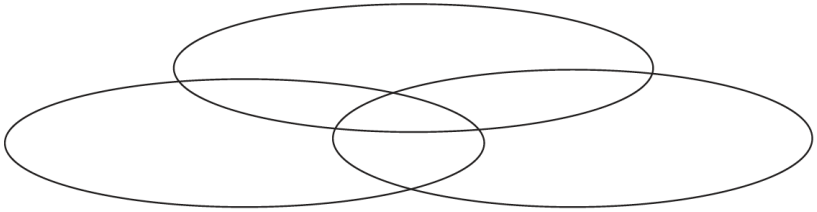
Representations

Research and Information



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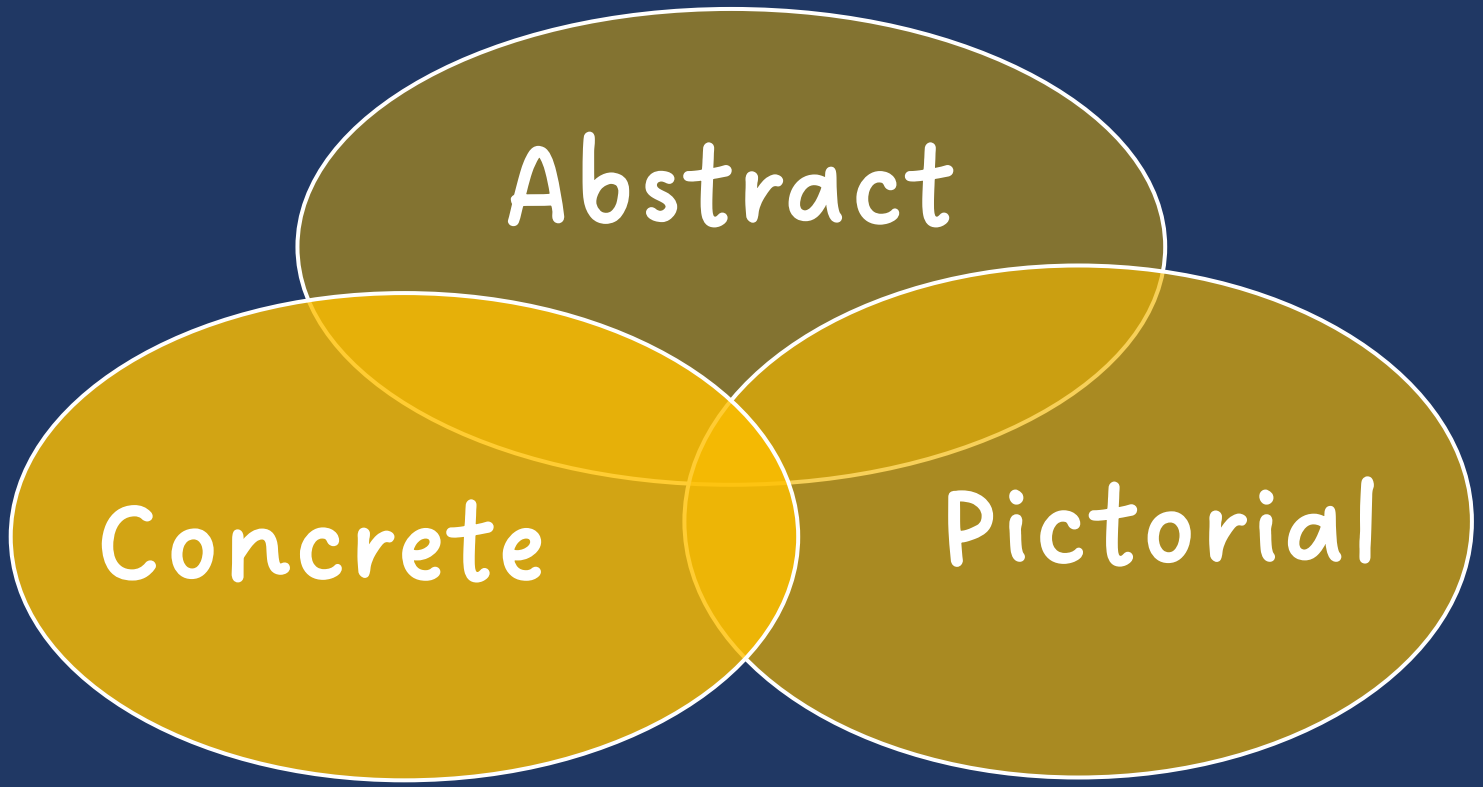

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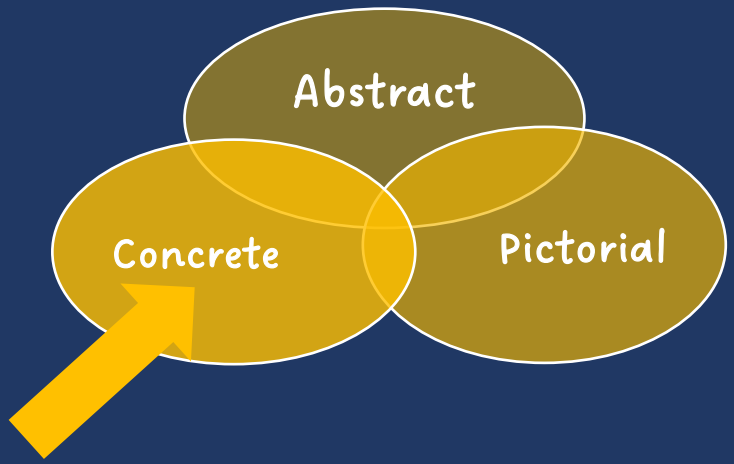

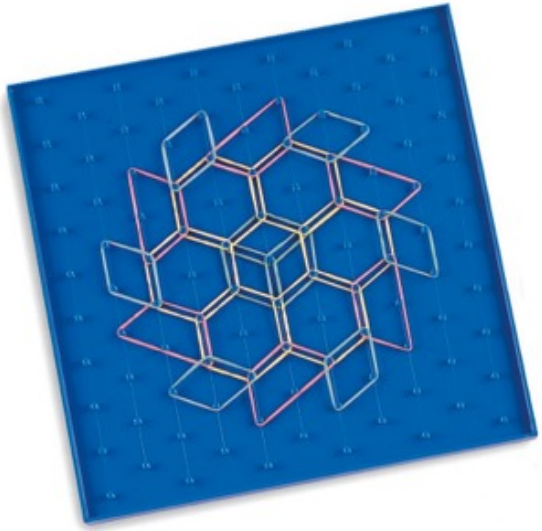

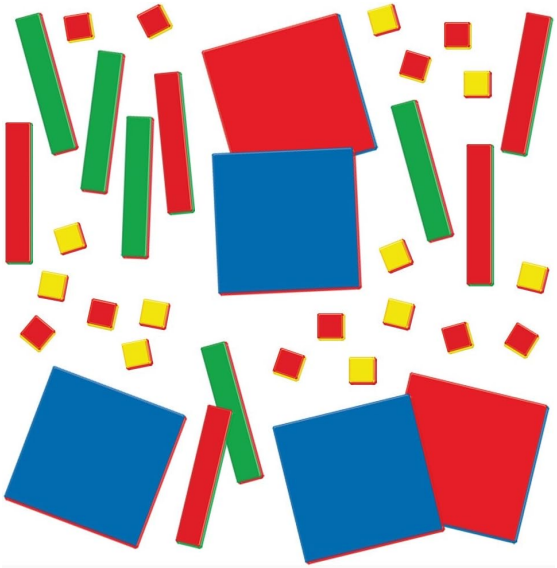

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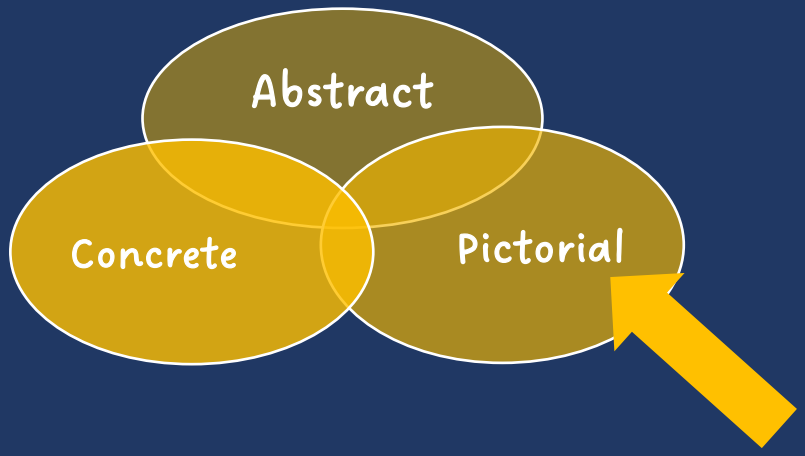

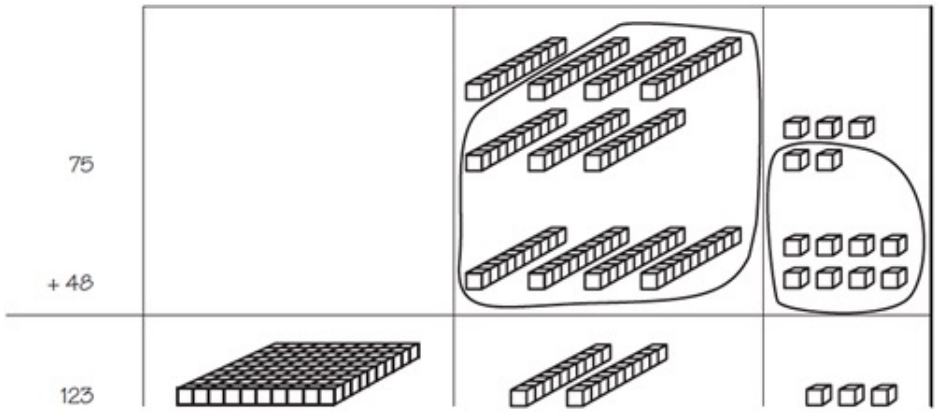

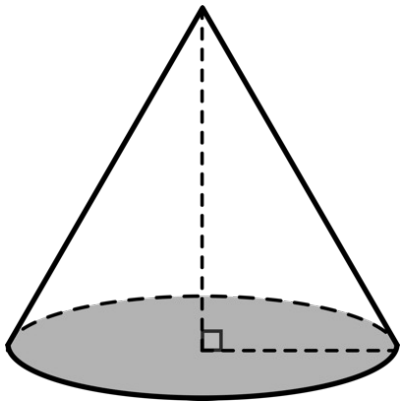

Find the rule and complete the table.

In	Out
9	5
19	15
	29
62	
95	91
96	92

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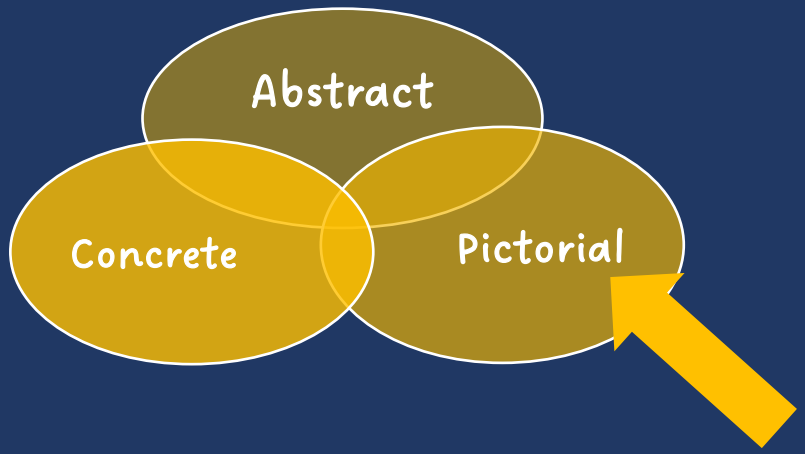

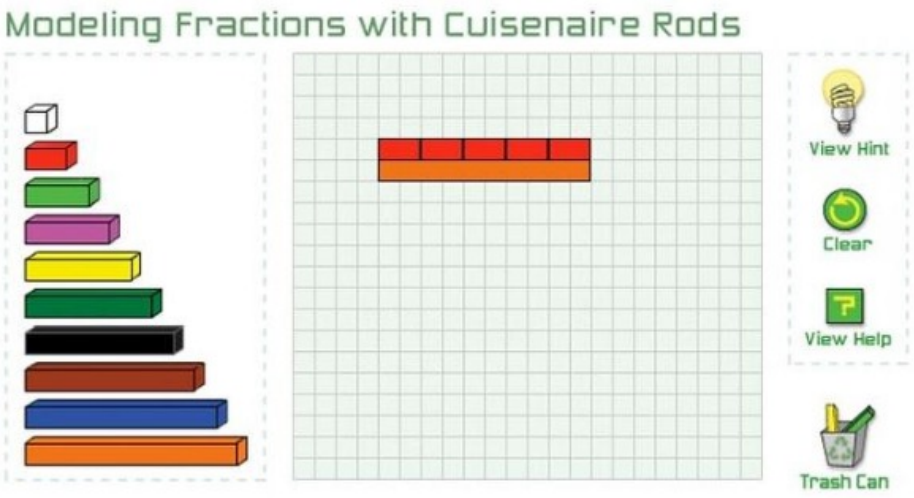

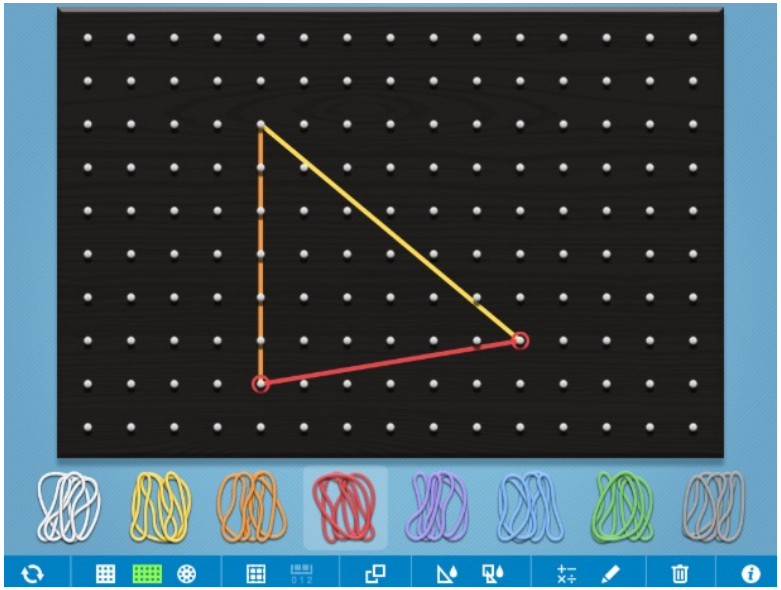

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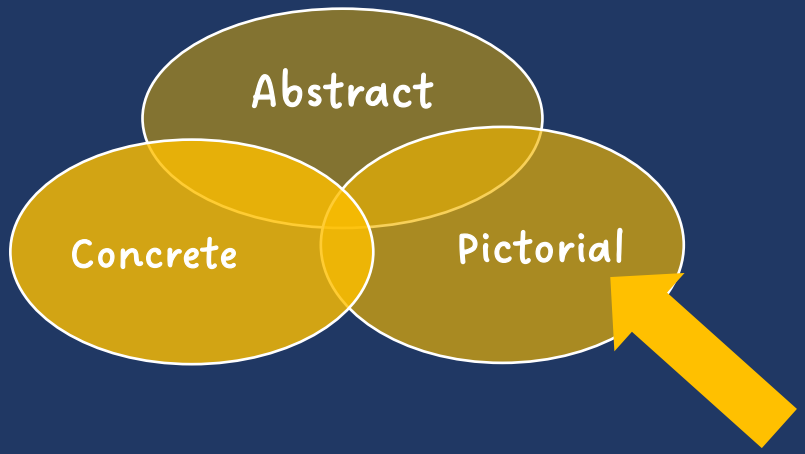

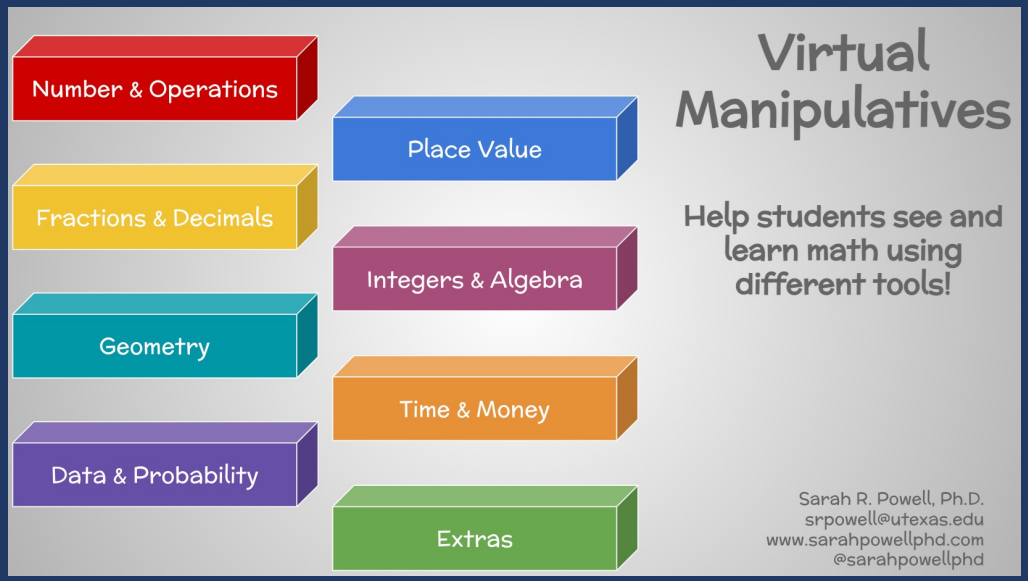

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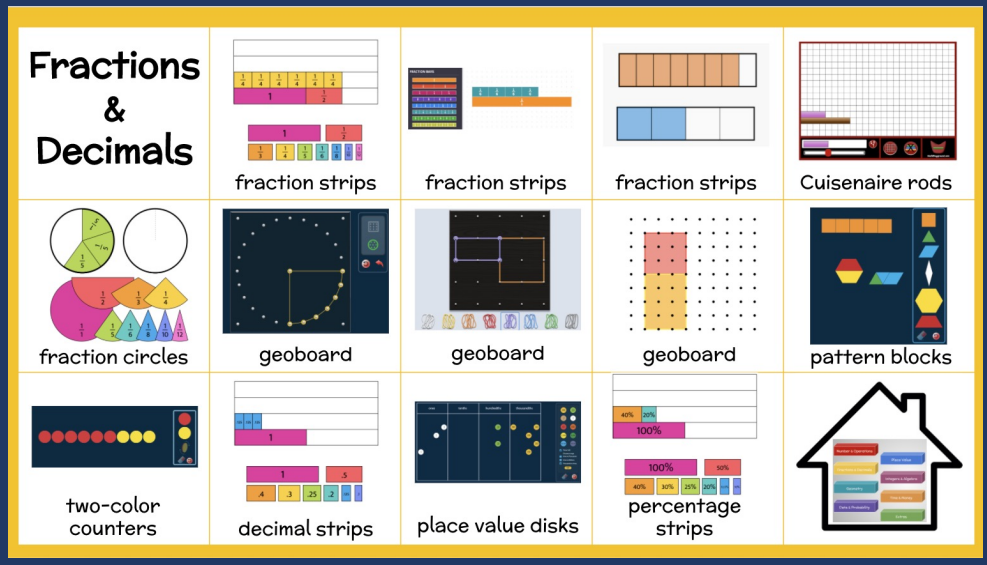

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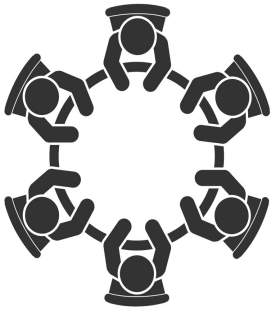

Explore 3 virtual manipulatives.

Share with a partner.

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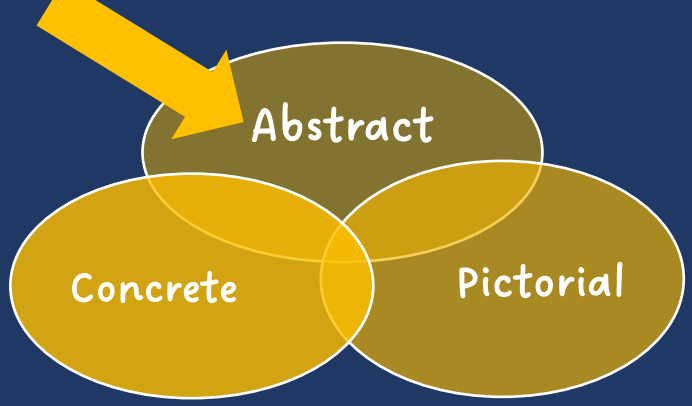

$2 + 8 = 10$

34 = 3 tens and 4 ones

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

$$\begin{array}{r} 4{,}179 \\ +\ \ 569 \\ \hline \end{array}$$

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Representations

 What are your strengths?

 What are your opportunities for growth?

 What are your immediate next steps?

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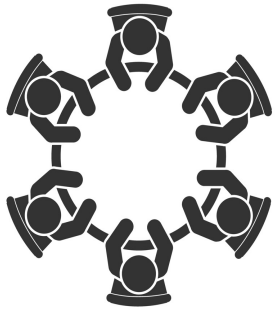
Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.



What are your strengths with multiple representations?

What are the opportunities for growth?

What materials do you need?

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Build Fluency

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Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit

Vocabulary

Representations

INSTRUCTIONAL STRATEGIES

Fluency

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Fluency

Research and Information

Types of Fluency

Type	Memorization?	
	Yes	No

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Mathematics fluency, particularly fluency with facts and computation, is related to overall mathematics performance.

(Bailey et al., 2012; Cirino et al., 2019; Koponen et al., 2007, 2017; Vukovic et al., 2014

Fact fluency practice improves mathematics fact performance.

(Burns et al., 2010; Codding et al., 2011; McCallum et al., 2004; Nelson et al., 2013; Poncy et al., 2010; Schutte et al., 2015; Stocker & Kubina, 2017)

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Fluency is doing mathematics easily and accurately.

Fluency in mathematics makes mathematics easier.

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both conceptual learning and procedural learning.

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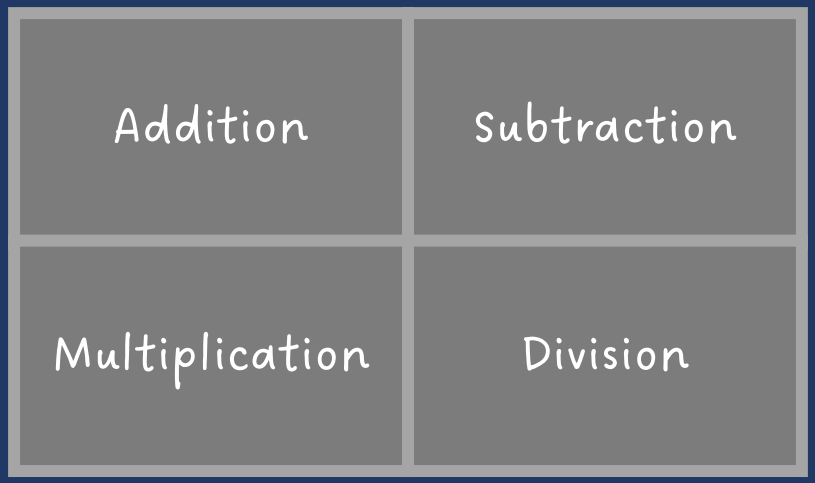

Benchmark fractions

Knowing multiples

Equivalent fractions & decimals

Equivalent decimals & percentages

Knowing formulas

Reading place values

Knowing vocabulary

Reading word problems

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Memorization or automaticity

Ease and accuracy

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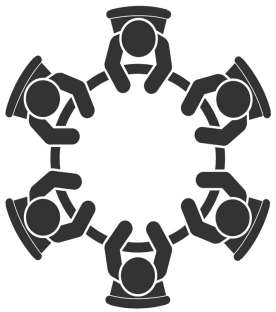
Fluency

Research and Information



Types of Fluency

Type	Memorization?	
	Yes	No

List different types of fluency. Discuss whether they require memorization.

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Fluency with Facts

Addition

Subtraction

Multiplication

Division

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Addition

100 addition facts

Single-digit addends sum to a single- or double-digit number

```
   5     (addend)
+  4     (addend)
------
   9     (sum)
```

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100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{rl} 16 & \text{(minuend)} \\ -\ \ 8 & \text{(subtrahend)} \\ \hline 8 & \text{(difference)} \end{array}$$

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Multiplication

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{rl} 2 & \text{(factor)} \\ \times\ 3 & \text{(factor)} \\ \hline 6 & \text{(product)} \end{array}$$

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Division

90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

8 ÷ 4 = 2

(dividend) (divisor) (quotient)

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Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ 8 \\ \hline \end{array}$$

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Fluency

Strategies for Building Fluency

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

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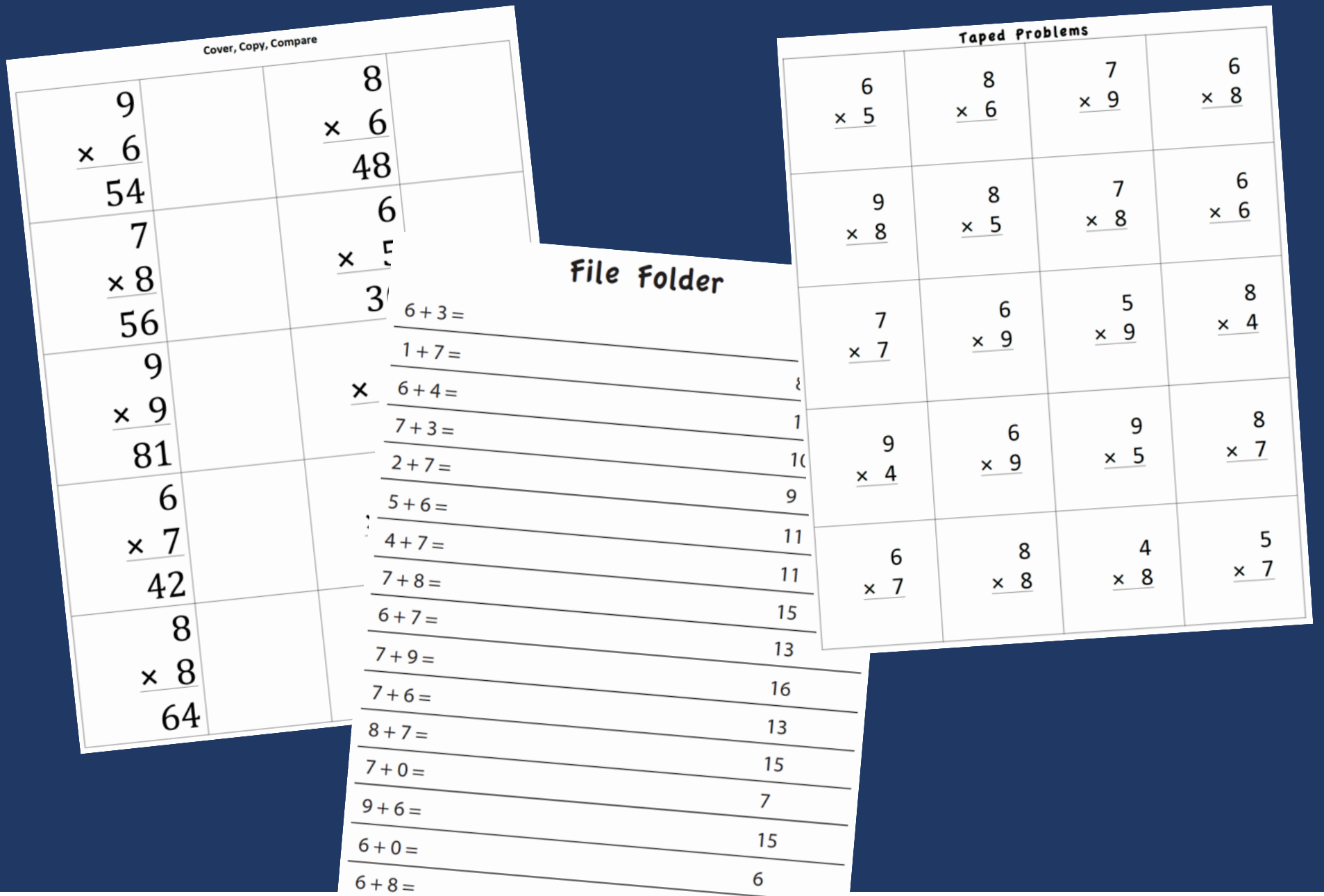

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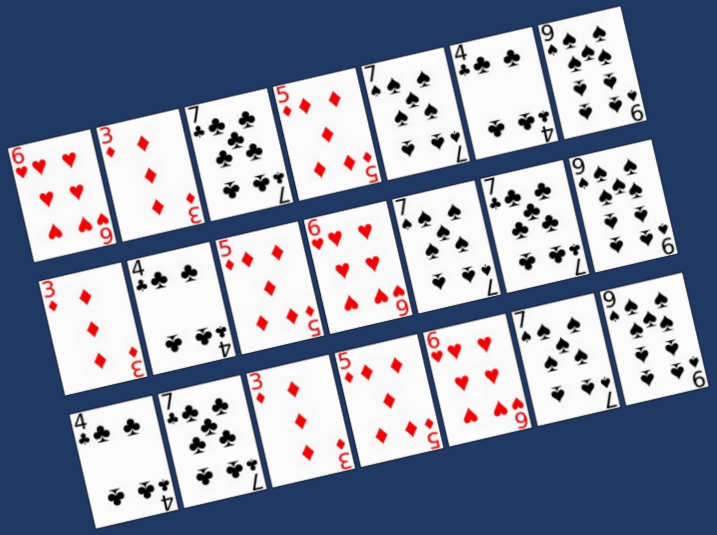

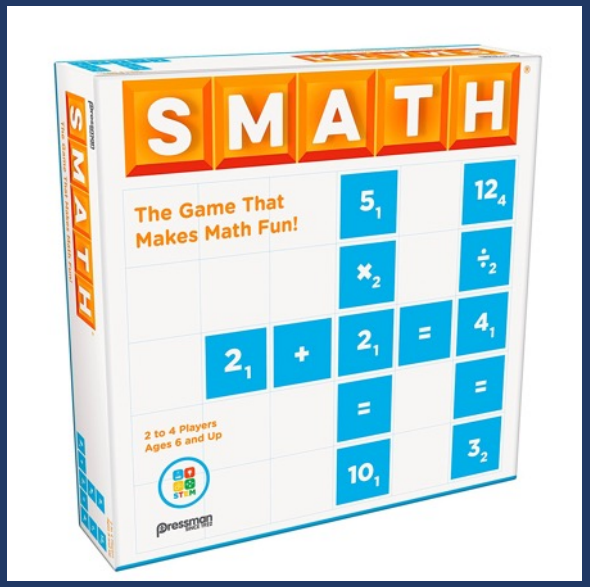

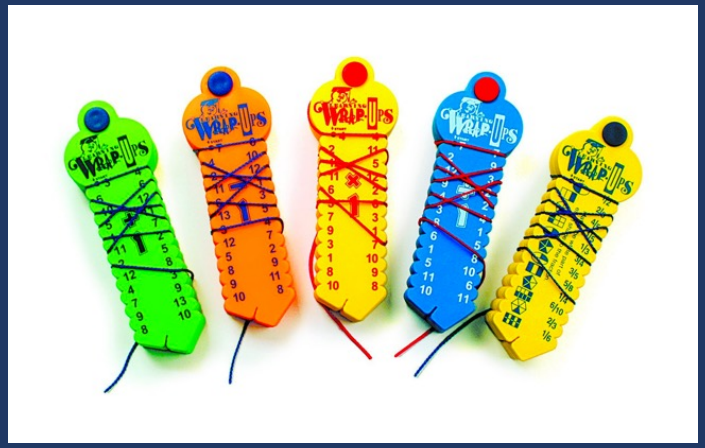

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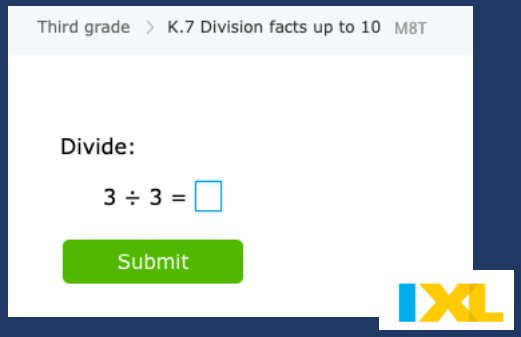

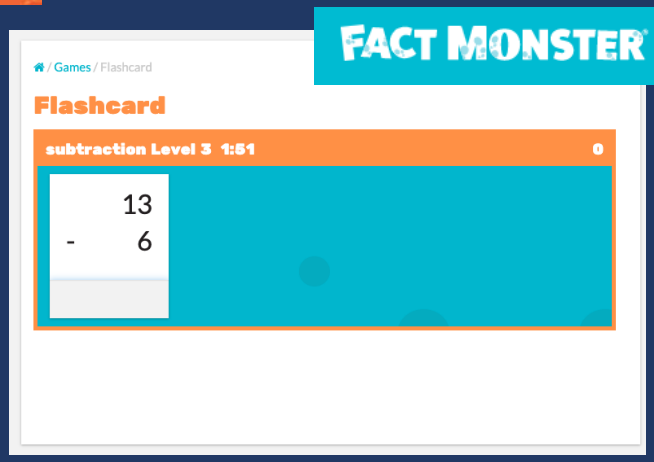

DAILY and BRIEF

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Addition	Subtraction
Multiplication	Division

What are five ways you help students build fact fluency?

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Fluency with Computation

Addition

Subtraction

Multiplication

Division

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Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ +\ 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ -\ 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div\ 15 \\ \hline \end{array}$$

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Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ +\ 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div\ 0.14 \\ \hline \end{array}$$

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Addition	Subtraction
Multiplication	Division

Build fluency with integer computation

$$\begin{array}{r} 1.4 \\ +\ -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times\ -12 \\ \hline \end{array}$$

$-14 - (-7) =$

$-135 \div 2 =$

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Partial Sums

A.

$$\begin{array}{r} 74 \\ +\ 18 \\ \hline 80 \\ +12 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ +\ 365 \\ \hline 1{,}000 \\ 80 \\ +\ 10 \\ \hline 1{,}090 \end{array}$$

$$\begin{array}{r} 227 \\ +\ 185 \\ \hline \end{array}$$

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Opposite Change

A.

$$\begin{array}{r} 74 \xrightarrow{-4} 70 \\ +\ 18 \xrightarrow{+4} +22 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \xrightarrow{+5} 730 \\ +\ 365 \xrightarrow{-5} +360 \\ \hline 1{,}090 \end{array}$$

$$\begin{array}{r} 227 \\ +\ 185 \\ \hline \end{array}$$

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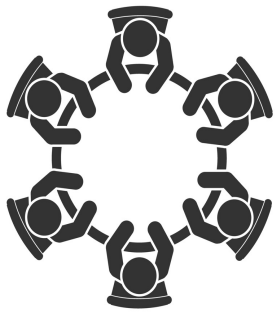
Partial Differences

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline +50 \\ -\ 5 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline +300 \\ -90 \\ -1 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

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Same Change

A.

$$\begin{array}{r} 62 \xrightarrow{+3} 65 \\ -\ 17 \xrightarrow{+3} -20 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \xrightarrow{+4} 309 \\ -\ 96 \xrightarrow{+4} -100 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

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Add Up

A.

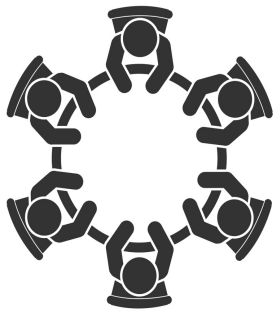
$$\begin{array}{r} 62 \\ -\ 17 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 20 \quad 3 \\ 60 \quad 40 \\ 62 \quad +\ 2 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ 100 \quad 4 \\ 300 \quad 200 \\ 305 \quad +\ 5 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

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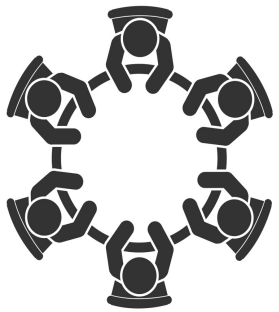
Partial Products

A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$



$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

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Area (Array)

A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$

	20	4	
	800	160	40
	60	12	3

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$

	100	30	2	
	5000	1500	100	50
	300	90	6	3

$$\begin{array}{r} 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$

$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

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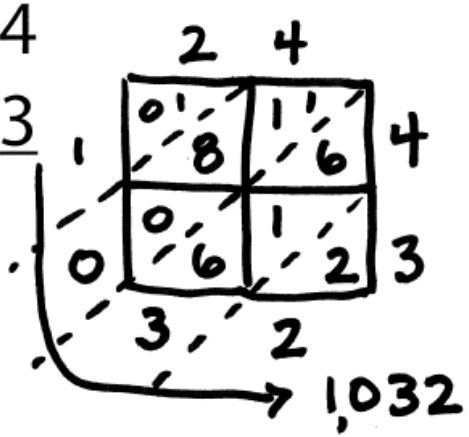
Lattice

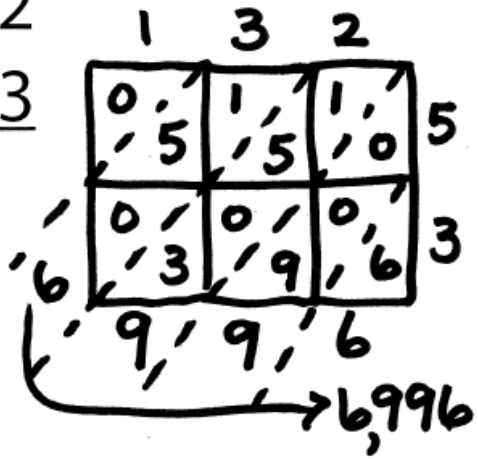
A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$



B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

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Partial Quotients

A.

$$
\begin{array}{r|l}
12\overline{)158} & \\
-120 & 10 \\
\hline
38 & \\
-36 & +3 \\
\hline
2 & 13\ \text{R}2
\end{array}
$$

B.

$$
\begin{array}{r|l}
34\overline{)970} & \\
-680 & 20 \\
\hline
290 & \\
-170 & 5 \\
\hline
120 & \\
102 & +3 \\
\hline
18 & 28\ \text{R}18
\end{array}
$$

$$
\begin{array}{r}
804 \\
\div \quad 12 \\
\hline
\end{array}
$$

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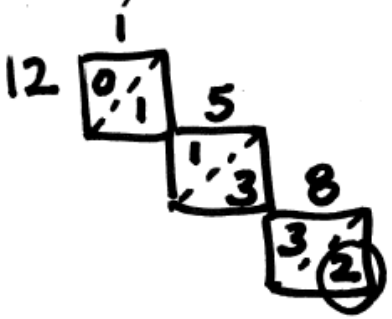
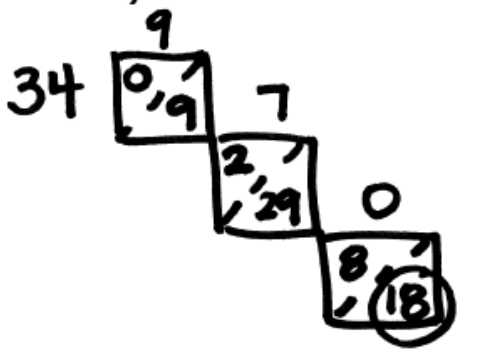
Lattice

A. $12\overline{)158}$ = 13 R2



B. $34\overline{)970}$ = 28 R 18



804 ÷ 12

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Fluency

Strategies for Building Fluency

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

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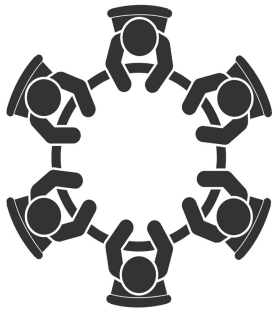
Teachers should use fluency building activities to build counting fluency and fluency with the operations.



What are your strengths with building fluency?

What are the opportunities for growth?

What are your immediate next steps?

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Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

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Solve Word Problems

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Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit
- Vocabulary
- Representations

INSTRUCTIONAL STRATEGIES

- Fluency
- Word Problems

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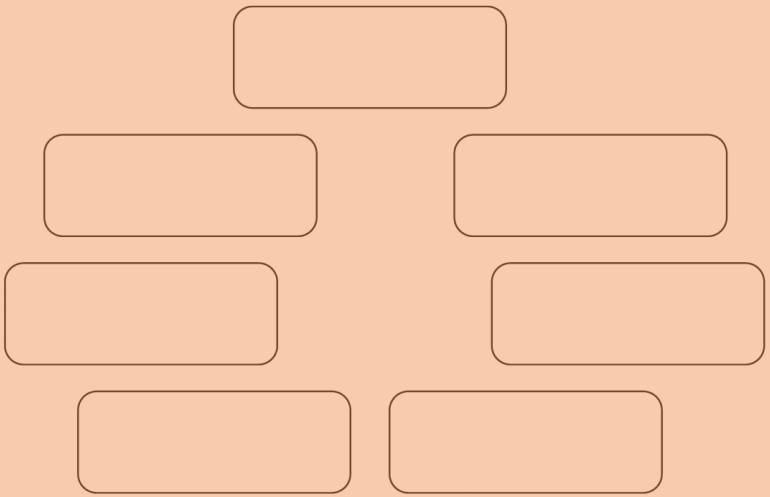
Word Problems

Research and Information



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Key words tied to operations is an ineffective word-problem strategy.
(Karp et al., 2019; Powell et al., 2022)

Using a meta-cognitive strategy improves word-problem performance.
(Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

A focus on schemas improves word-problem performance.
(Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin & Xhang, 2009; Zheng et al., 2013)

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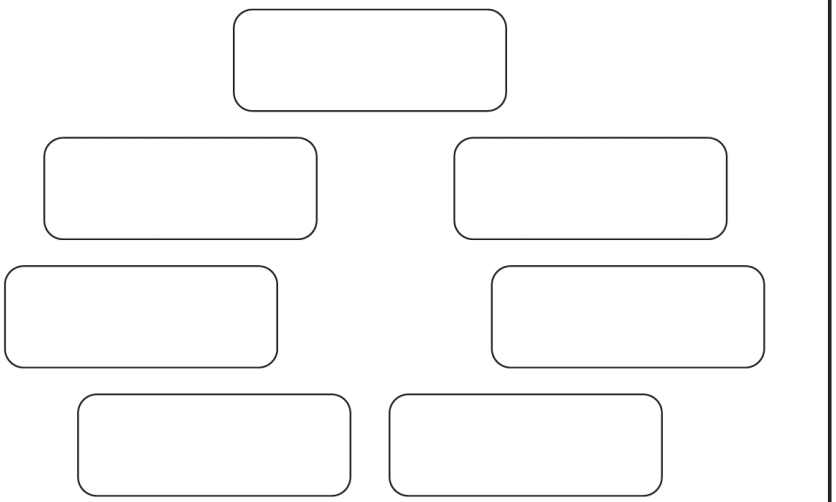
Word Problems

Research and Information



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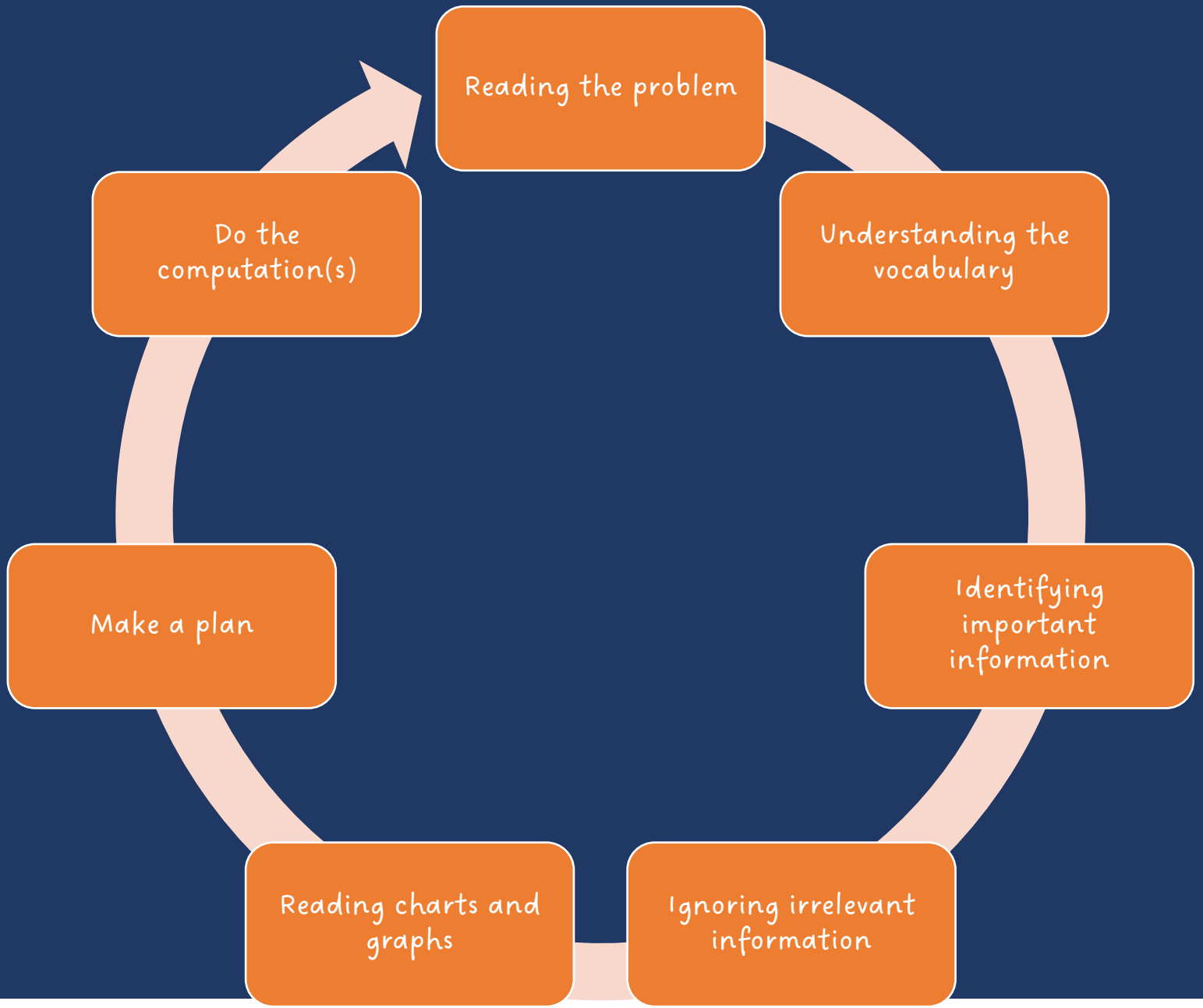

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1. Keywords tied to operations

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The bakery had 75 fewer cookies than brownies. If the bakery had 200 brownies, how many brownies did the bakery have?

The bakery had 75 fewer cookies than brownies. If the bakery had 200 cookies, how many cookies did the bakery have?

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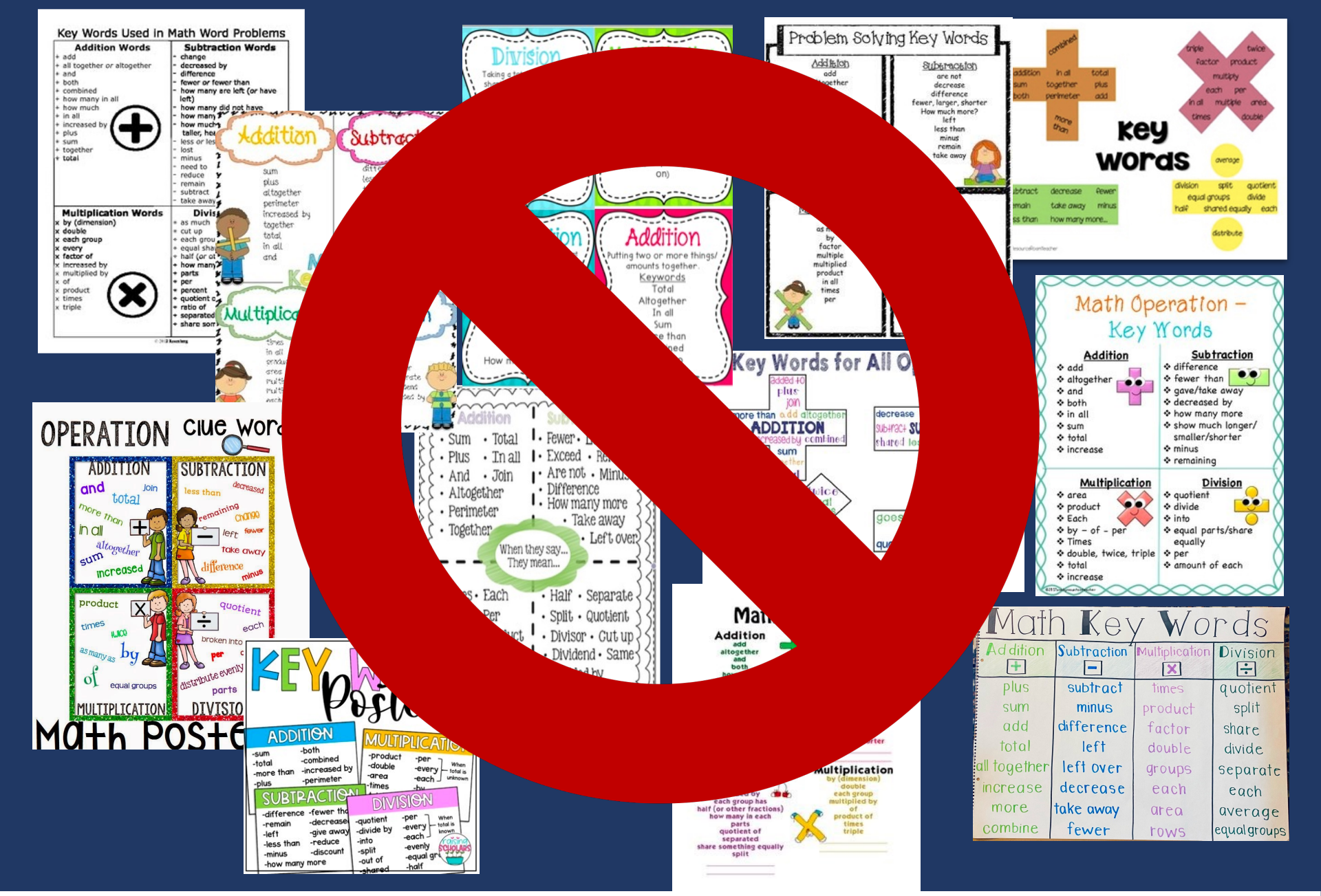

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Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords[a]		Multiple keywords[a]		Keyword(s) led to correct solution[a]	
	n	%	*n*	%	*n*	%	*n*	%	*n*	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

[a]When a problem featured a keyword.

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Description of Multi-Step Word Problems (n = 84)

	Occurrence of schema[a]		Any keyword		Keyword(s) led to correct solution[b]	
Schema	*n*	%	*n*	%	*n*	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.

[b]When a problem featured a keyword.

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Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")

But, *do not* tie a keyword to a specific operation!

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2. Presenting problems by operation

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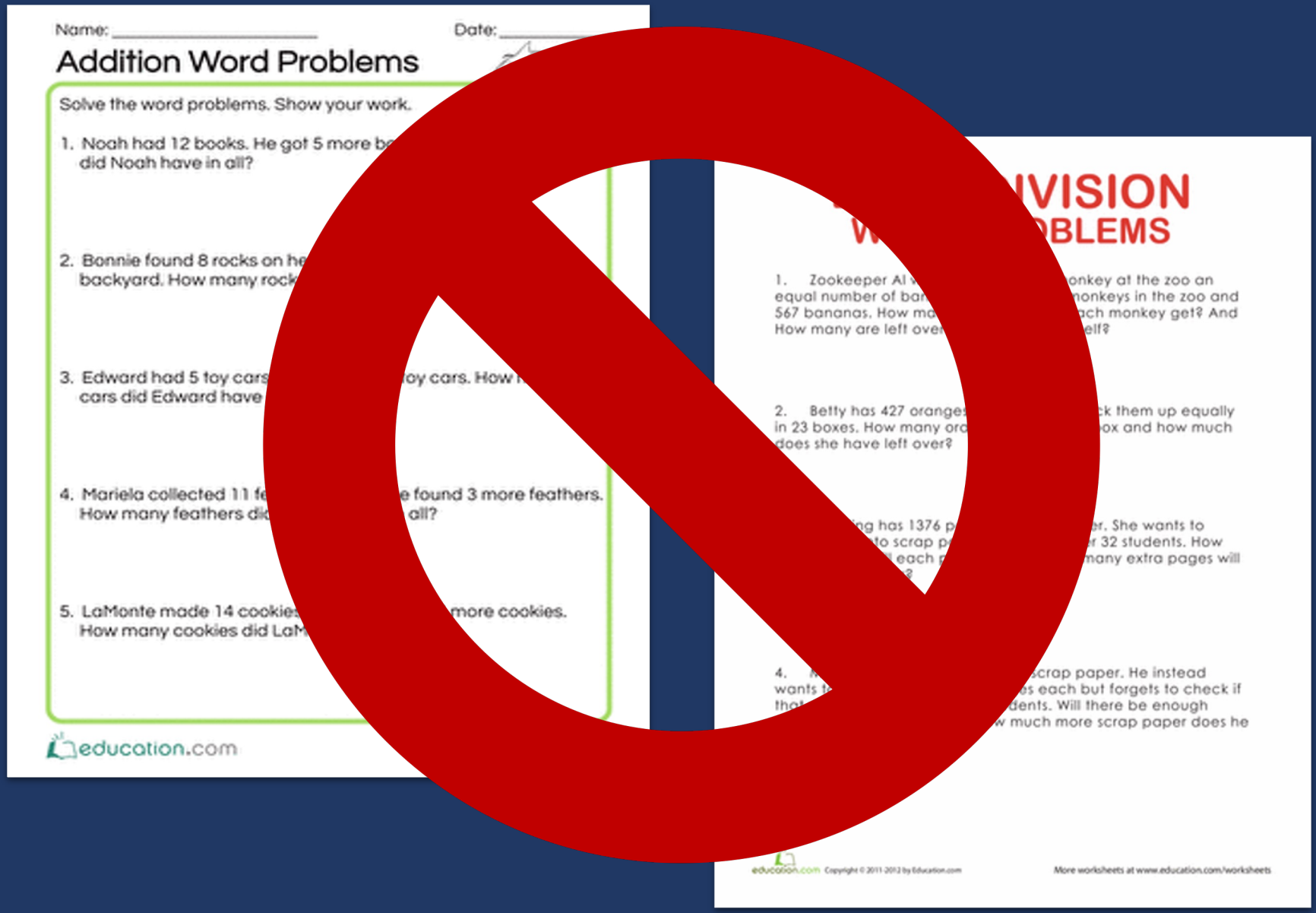

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Word Problems

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

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Teach an attack strategy

Teach about schemas

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RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RICE

Read and record the problem.
Illustrate your thinking.
Compute.
Explain your thinking.

RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

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SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

STAR

Stop and read the problem carefully.
Think about your plan and the strategy you will use.
Act. Follow your plan and solve
the problem.
Review your answer.

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SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

UPS✓

UNDERSTAND
Read and explain.

PLAN
How will you solve the problem?

SOLVE
Set up and do the math!

✓CHECK
Does your answer make sense?

Created by: Sarah Powell (srpowell@austin.utexas.edu)

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Share your favorite attack strategy.

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Teach an attack strategy

Teach about schemas

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Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

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Total

Part-part-whole
Combine

Parts put together into a **total**

Daniela saw 3 canoes and 8 kayaks. How many boats did Daniela see?

Total

Daniela saw 11 boats. If 3 of the boats were canoes, how many were kayaks?

Part

Daniela saw 11 boats. 8 of the boats were kayaks, how many were canoes?

Part

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Difference

Compare

Greater and **lesser** amounts compared for a **difference**

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have?

Difference

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Greater amount

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does Tracy have?

Lesser amount

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Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

End amount

Change amount

Start amount

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Change

Separate

An amount that increases or **decreases**

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Change amount

Samantha baked some cookies. She ate 3 of the cookies and has 17 cookies left. How many cookies did Samantha bake?

Start amount

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Equal Groups

Array
Vary

Groups multiplied by **number in each group** for a **product**

Toni has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Toni have altogether?

Product

Toni has 24 crayons. They want to place them equally into 2 boxes. How many crayons will Toni place in each box?

Number in each group

Toni has 24 crayons. They put them into boxes with 12 crayons each. How many boxes did Toni use?

Groups

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Comparison

Set multiplied by a number of times for a product

Brooke ran 6 minutes. Shaleeni ran 4 times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of times

Product

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Ratios/Proportions

Description of relationships among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

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Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

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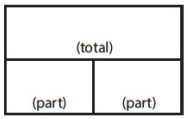
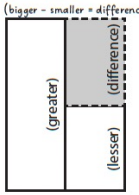
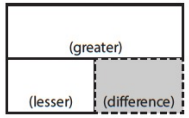
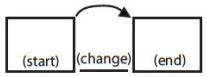
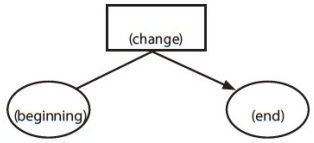
Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total)  	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B - s = D (bigger - smaller = difference)  G - L = D (greater - less = difference)  	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/- C = E (start +/- change = end)   	End (increase) unknown: Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now?	Change (increase) unknown: Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting?	Start (increase) unknown: Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with?	Multiple changes: Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?
		End (decrease) unknown: Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	Change (decrease) unknown: Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	Start (decrease) unknown: Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	

Powell & Fuchs (2018).
Material collected from: Griffin & Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch & Greeno, 1985; Van de Walle, Karp, & Bay-Williams, 2013.

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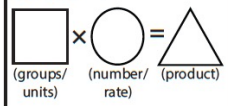
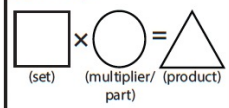
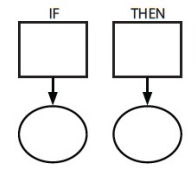
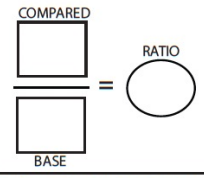
Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units	GR x N = P  	Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set	S x T = P  	Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Ratios/ Proportions (Percentages; Unit Rate) Relationships among quantities Ratio	  	Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?		With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
		Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	Compared unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With unit rate: Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.

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Teach an attack strategy

Teach about schemas

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Word Problems

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

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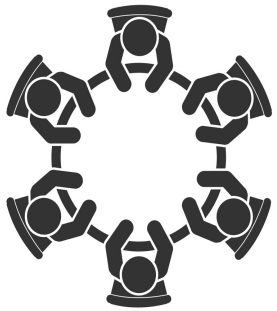

What are your strengths with word-problem solving?

What are the opportunities for growth?

What's one thing you can start doing next week?

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Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit
- Vocabulary
- Representations

INSTRUCTIONAL STRATEGIES

- Fluency
- Word Problems

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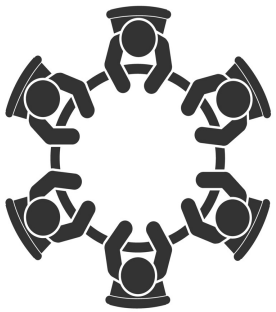
Checklist

Evidence-Based Practice	Description; Look-Fors



Describe evidence-based practices important for your teachers. What are the look-fors?

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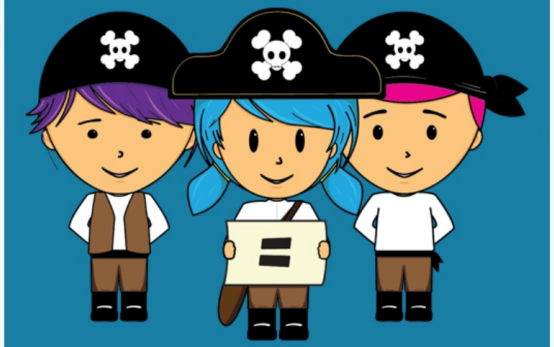
Pirate Math Equation Quest

About | Research | Individual | Small Group | STAAR | Videos

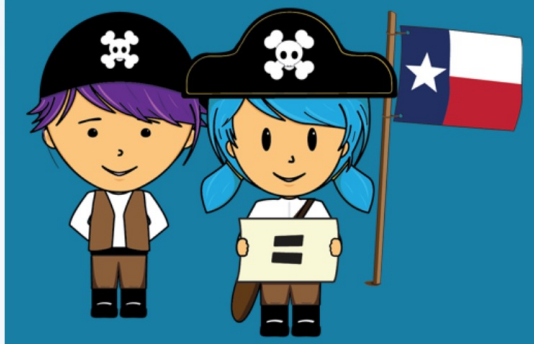
Welcome to Pirate Math Equation Quest!

Individual Word-Problem Intervention

Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



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https://intensiveintervention.org/intensive-intervention-math-course

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INTENSIVE INTERVENTION
at American Institutes for Research

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Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a high-leverage practice in special education, and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center, developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.

MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES

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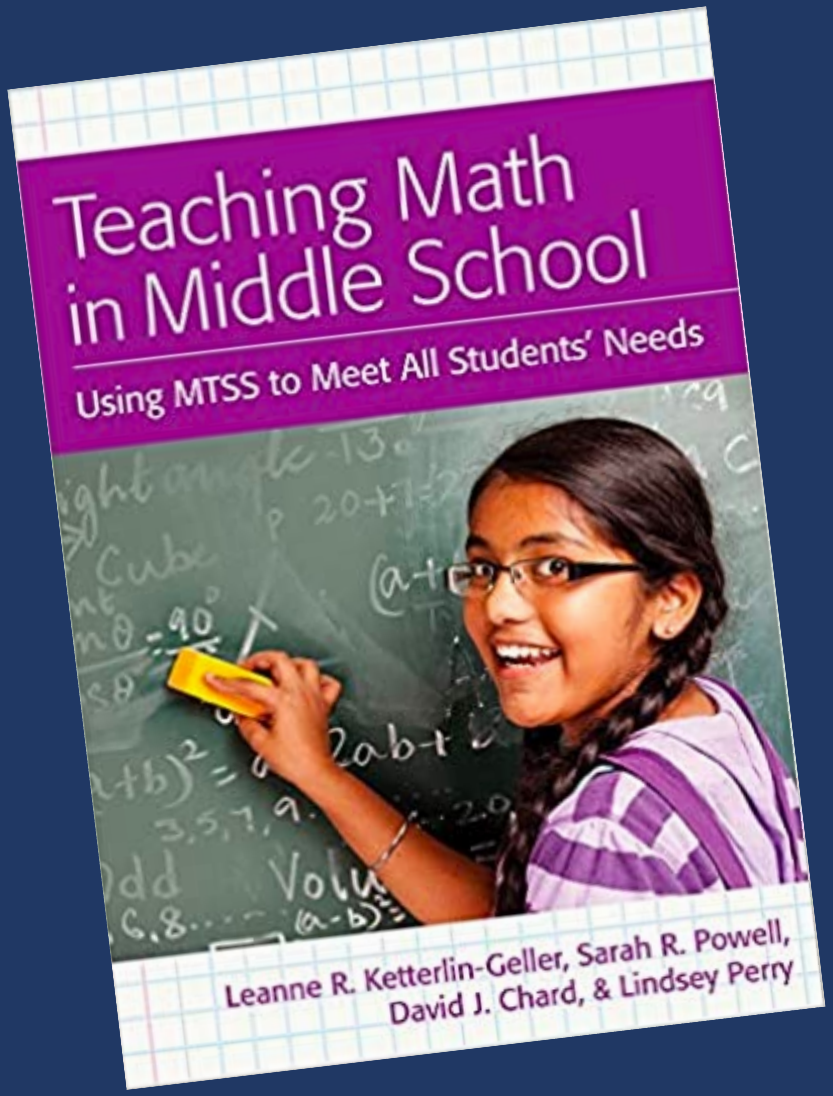
https://www.amazon.com/Teaching-Math-Middle-School-Students/dp/1598572741



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