Solutionbank FP2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

a Show that $r = \frac{1}{2}(r(r+1) - r(r-1))$.

b Hence show that $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$ using the method of differences.

Solution:

a $\frac{1}{2}(r(r+1) - r(r-1))$

Consider RHS.

$= \frac{1}{2}(r^2 + r - r^2 + r)$

Expand and simplify.

$= \frac{1}{2}(2r)$

$= r$

$=$ LHS

b $\sum_{r=1}^{n} r = \frac{1}{2}\sum_{r=1}^{n} r(r+1) - \frac{1}{2}\sum_{r=1}^{n} r(r-1)$

Use above.

$$\begin{array}{lll}
r = 1 & \cancel{\tfrac{1}{2} \times 1 \times 2} & -\tfrac{1}{2} \times 1 \times 0 \\
r = 2 & \cancel{\tfrac{1}{2} \times 2 \times 3} & \cancel{-\tfrac{1}{2} \times 2 \times 1} \\
r = 3 & \tfrac{1}{2} \times 3 \times 4 & \cancel{-\tfrac{1}{2} \times 3 \times 2} \\
\dots & \dots & \\
r = n-1 & \cancel{\tfrac{1}{2}(n-1)(n)} & -\tfrac{1}{2}(n-1)(n-2) \\
r = n & \tfrac{1}{2}n(n+1) & \cancel{-\tfrac{1}{2}n(n-1)}
\end{array}$$

Use method of differences.

When you add, all terms cancel except $\frac{1}{2}n(n+1)$.

Hence $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

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Exercise A, Question 2

Question:

Given $\frac{1}{r(r+1)(r+2)} \equiv \frac{1}{2r(r+1)} - \frac{1}{2(r+1)(r+2)}$

find $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$ using the method of differences.

Solution:

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \sum_{r=1}^{n} \frac{1}{2r(r+1)} - \sum_{r=1}^{n} \frac{1}{2(r+1)(r+2)}$$

Use the information given and equate the summations.

Put $r = 1$ $\quad \frac{1}{2\times1\times2} - \cancel{\frac{1}{2\times2\times3}}$

Use method of differences.

$r = 2$ $\quad \cancel{\frac{1}{2\times2\times3}} - \cancel{\frac{1}{2\times3\times4}}$

All terms cancel except first and last.

$r = 3$ $\quad \cancel{\frac{1}{2\times3\times4}} - \frac{1}{2\times4\times5}$

$\vdots$

$r = n$ $\quad \cancel{\frac{1}{2n(n+1)}} - \frac{1}{2(n+1)(n+2)}$

Add $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$

First and last from above.

$$= \frac{(n+1)(n+2)-2}{4(n+1)(n+2)}$$

Simplify.

$$= \frac{n^2+3n+2-2}{4(n+1)(n+2)}$$

$$= \frac{n(n+3)}{4(n+1)(n+2)}$$

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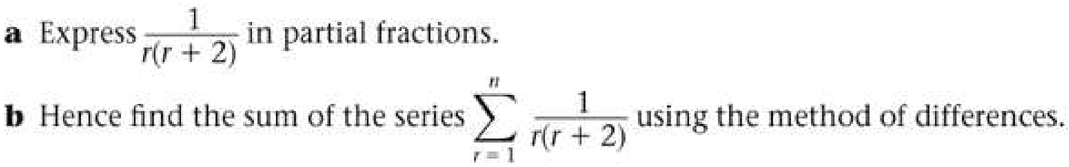
Exercise A, Question 3

Question:



a Express $\frac{1}{r(r+2)}$ in partial fractions.

b Hence find the sum of the series $\sum_{r=1}^{n} \frac{1}{r(r+2)}$ using the method of differences.

Solution:

a $$\frac{1}{r(r+2)} \equiv \frac{A}{r} + \frac{B}{r+2}$$

Set $\frac{1}{r(r+2)}$ identical to $\frac{A}{r} + \frac{B}{r+2}$.

$$\equiv \frac{A(r+2)+Br}{r(r+2)}$$

Add the two fractions.

$$1 \equiv A(r+2)+Br$$

$$\begin{aligned}
\text{Put } r &= 0\\
1 &= 2A\\
A &= \tfrac{1}{2}\\
\cancel{\tfrac{1}{2}} &\cancel{= A}
\end{aligned}$$

$$\begin{aligned}
\text{Put } r &= 1\\
1 &= \tfrac{1}{2}(3) + B\\
B &= -\tfrac{1}{2}
\end{aligned}$$

$$\therefore \frac{1}{r(r+2)} = \frac{1}{2r} - \frac{1}{2(r+2)}$$

b $$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \sum_{r=1}^{n} \frac{1}{2r} - \sum_{r=1}^{n} \frac{1}{2(r+2)}$$

Use method of differences.

$$\begin{array}{lccc}
r = 1 & \frac{1}{2\times 1} & - & \cancel{\frac{1}{2\times 3}}\\
r = 2 & \frac{1}{2\times 2} & - & \cancel{\frac{1}{2\times 4}}\\
r = 3 & \cancel{\frac{1}{2\times 3}} & - & \cancel{\frac{1}{2\times 5}}\\
\vdots & & & \\
r = n-1 & \cancel{\frac{1}{2(n-1)}} & - & \frac{1}{2(n+1)}\\
r = n & \cancel{\frac{1}{2n}} & - & \frac{1}{2(n+2)}
\end{array}$$

All terms cancel except $\frac{1}{2}$, $\frac{1}{4}$ $\frac{1}{2(n+1)}$ and $\frac{1}{2(n+2)}$

Add

$$\begin{aligned}\sum_{r=1}^{n}\frac{1}{r(r+2)} &= \frac{1}{2}+\frac{1}{4}-\frac{1}{2(n+1)}-\frac{1}{2(n+2)}\\ &= \frac{2(n+1)(n+2)+(n+1)(n+2)-2(n+2)-2(n+1)}{4(n+1)(n+2)}\\ &= \frac{2n^2+6n+4+n^2+3n+2-2n-4-2n-2}{4(n+1)(n+2)}\\ &= \frac{3n^2+5n}{4(n+1)(n+2)}\\ &= \frac{n(3n+5)}{4(n+1)(n+2)}\end{aligned}$$

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Exercise A, Question 4

Question:

a Express $\frac{1}{(r+2)(r+3)}$ in partial fractions.

b Hence find the sum of the series $\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)}$ using the method of differences.

Solution:

a

$$\frac{1}{(r+2)(r+3)} \equiv \frac{A}{r+2} + \frac{B}{r+3}$$

Set $\frac{1}{(r+2)(r+3)}$ identical to $\frac{A}{r+2} + \frac{B}{r+3}$.

$$\equiv \frac{A(r+3) + B(r+2)}{(r+2)(r+3)}$$

Add the two fractions.

$$1 \equiv A(r+3) + B(r+2)$$

Compare numerators as they are equivalent.

$$r = -3 \Rightarrow B = -1$$
$$r = -2 \Rightarrow A = 1$$

Solve for A and B.

$$\therefore \frac{1}{(r+2)(r+3)} = \frac{1}{r+2} - \frac{1}{r+3}$$

b

$$\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)} \equiv \sum_{r=1}^{n} \frac{1}{(r+2)} - \sum_{r=1}^{n} \frac{1}{(r+3)}$$

Use the method of differences.

$r = 1$ $\quad \frac{1}{3} - \cancel{\frac{1}{4}}$

$r = 2$ $\quad \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}$

$r = 3$ $\quad \cancel{\frac{1}{5}} - \cancel{\frac{1}{6}}$

$\vdots$

$r = n$ $\quad \cancel{\frac{1}{n+2}} - \frac{1}{n+3}$

All cancel except first and last.

$$\text{Add } \sum_{r=1}^{n} \frac{1}{(r+2)(r+3)} = \frac{1}{3} - \frac{1}{n+3}$$

$$= \frac{n+3-3}{3(n+3)}$$

$$= \frac{n}{3(n+3)}$$

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Exercise A, Question 5

Question:

a Express $\frac{5r + 4}{r(r + 1)(r + 2)}$ in partial fractions.

b Hence or otherwise, show that $\sum_{r=1}^{n} \frac{5r + 4}{r(r + 1)(r + 2)} = \frac{7n^2 + 11n}{2(n + 1)(n + 2)}$

Solution:

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Exercise A, Question 6

Question:

Given that $\frac{r}{(r+1)!} \equiv \frac{1}{r!} - \frac{1}{(r+1)!}$

find $\sum_{r=1}^{n} \frac{r}{(r+1)!}$

Solution:

$$\sum_{r=1}^{n} \frac{r}{(r+1)!} \equiv \sum_{r=1}^{n} \frac{1}{r!} - \sum_{r=1}^{n} \frac{1}{(r+1)!}$$

Use given.

$$\begin{array}{lccc}
r = 1 & \frac{1}{1!} & - & \frac{\cancel{1}}{\cancel{2}!} \\
r = 2 & \frac{\cancel{1}}{\cancel{2}!} & - & \frac{\cancel{1}}{\cancel{3}!} \\
r = 3 & \frac{\cancel{1}}{\cancel{3}!} & - & \frac{\cancel{1}}{\cancel{4}!} \\
\vdots & & & \\
r = n & \frac{\cancel{1}}{\cancel{n}!} & - & \frac{1}{(n+1)!}
\end{array}$$

Use method of differences.

All cancel except first and last term.

$\therefore$ Add $\sum_{r=1}^{n} \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!}$

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Exercise A, Question 7

Question:

Given that $\frac{2r+1}{r^2(r+1)^2} \equiv \frac{1}{r^2} - \frac{1}{(r+1)^2}$

find $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$.

Solution:

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = \sum_{r=1}^{n} \frac{1}{r^2} - \sum_{r=1}^{n} \frac{1}{(r+1)^2}$$

Use given.

Use method of differences.

$$\begin{array}{lccc}
r=1 & \frac{1}{1} & - & \cancel{\frac{1}{2^2}} \\
r=2 & \cancel{\frac{1}{2^2}} & - & \cancel{\frac{1}{3^2}} \\
r=3 & \cancel{\frac{1}{3^2}} & - & \cancel{\frac{1}{4^2}} \\
\vdots & & & \\
r=n & \cancel{\frac{1}{n^2}} & - & \frac{1}{(n+1)^2}
\end{array}$$

All terms cancel except first and last.

$$\begin{aligned}
\text{So adding } \sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} &= 1 - \frac{1}{(n+1)^2} \\
&= \frac{(n+1)^2 - 1}{(n+1)^2} \\
&= \frac{n^2+2n}{(n+1)^2} \\
&= \frac{n(n+2)}{(n+1)^2}
\end{aligned}$$

Simplify.