Math 4315 - PDEs   Home Work 1

1. Solve the following PDEs by using the chain rule $u_s = u_x \cdot x_s + u_y \cdot y_s$

$$\begin{aligned}
(i) &\quad 2u_x - 3u_y = 1, \\
(ii) &\quad xu_x + (x+2y)u_y = x, \\
(iii) &\quad xu_x + (u+y)u_y = 1, \\
(iv) &\quad u_x + 2xu_y = 4\,xy, \quad u(x,0) = x^4, \\
(v) &\quad xu_x + 2yu_y = u + x^2, \quad u(x,x) = x^2 + 1.
\end{aligned}$$

2. Show that under the change of variables

$$r = R(x+y), \quad s = s(x,y),$$

the PDE

$$u_x - u_y = 0,$$

becomes

$$u_s = 0.$$

For the following boundary conditions, show that it is possible to choose $R(x+y)$ and $s(x,y)$ such that the boundary in the (r,s) plane is $s = 0$ and the two boundaries can be connected via $x = r$.

$$\begin{aligned}
(i) &\quad u(x,0) = f(x) \\
(ii) &\quad u(x,1) = f(x) \\
(iii) &\quad u(x,x) = f(x) \\
(iv) &\quad u\left(x,x^2\right) = f(x)
\end{aligned}$$

Due: Sept. 15, 2023.