HOW DO WE KNOW?

THE LENGTH OF ONE MET DE BY ANDREW ROBINSON

Defining the length of a metre meant a revolution in accurate scientific measurement. From metal bars to spectrums of light to today's laser-based system, discover how pioneers rewrote the rules

HE EARLIEST UNITS for measuring length were based on the human body – most obviously the arm and the foot – and on seeds and grains. An Ancient Greek relief sculpture dated to about 450BC, kept at the Ashmolean Museum in Oxford, shows a male figure with two arms

stretched out and a foot-shaped depression above one of the arms. The sculpture may have been set up in a public place as a set of standard measurements. If its damaged arm is completed by symmetry, then the full arm span, known as the fathom, measures 2.08 metres (m); the forearm, known as the cubit, 0.52m; and the foot 0.297m.

In the Roman Empire, the *milliare* gave its name

to the mile, a length of 1,760 yards (1,609m). The milliare equalled *mille passuum*, a 'thousand paces' of a length taken by a Roman legionary on a long march. This makes a 'pace' about 1.6m – clearly an impossible average length for walking. The Roman 'pace' must therefore refer to the full cycle of left-right-left or right-left-right, giving a reasonable single pace of about 0.8m.

In Medieval England, the barleycorn defined the inch, the foot and the yard. During the reign of Edward II, in the 14th Century, the inch was defined as 'three grains of barley, dry and round, placed end to end, lengthwise'. For smaller lengths, the barleycorn was split into four equal parts, to create the line. So there were 12 lines in an inch, 12 inches (and 36 barleycorns) in a foot, and three feet in a yard.

In France, before the 1789 Revolution paved the way for the introduction of the metric system, a staggering 250,000 measures were in use under the guise of some 800 names, including the *aune* for length. Since the time of Charlemagne, eight French kings – including Louis XIV, the 'Sun King' – had attempted to lay down the law regarding standard weights and measures. All failed, because uniformity did not please the feudal aristocracy, who habitually manipulated the diversity of customary units to the disadvantage of their peasantry. Inconsistency particularly

bedevilled the cubit, a unit approximately equal to the distance from a man's

> This Ancient Greek relief from 450BC shows an early attempt at standardising measurement

How do we know?

The Length Bar Interferometer at the National Physical Laboratory uses laser light to define a given length up to a metre

> IN A NUTSHELL

From early forms of defining length with seeds and parts of the body, to defining a length based on accurate measurements of the size and shape of the Earth, it's taken hundreds of years for a standard metre to be used throughout the world. elbow joint to the farthest fingertip of his extended hand, or about half a metre. In ancient Egypt, the short cubit equalled six palms, while the royal cubit (used in the construction of the Pyramids) equalled seven palms. The eight different cubits known in the early civilisations vary in length from Roman 0.444m to Palestinian 0.641m.

Clearly a measurement of length defined by a universally agreed standard was needed, especially for international trade. The Greek mathematician and geographer Eratosthenes, who became director of the library at Alexandria in 235BC, was the first to attempt such a definition by determining the Earth's circumference using astronomy and geometry.

Eratosthenes employed a well in far-off Syene (modern Aswan) and a vertical obelisk in the grounds of the Alexandria library. On the day when the overhead Sun shone directly into the well, casting no shadow on its walls – that is, the summer solstice – Eratosthenes, further north in Alexandria, measured the angle created between the obelisk and its shadow. Since Syene and Alexandria lay almost on the same meridian of longitude, this angle measured the difference in latitude between Syene and Alexandria.

If the Earth were a perfect sphere, the angle at the Earth's centre subtended by the well and the obelisk - by definition the difference in their latitudes – had to equal the angle of the shadow, from simple geometry. (This assumed, reasonably, that the Sun was so far away that all its rays were parallel at the Earth.) The angle was 7.2°. Given the distance by camel from Alexandria to Syene, known to be about 5,000 stadia, the circumference of the Earth (subtending 360°) could be calculated. Multiply 5,000 by the ratio 360:7.2 and you get 250,000 stadia. The length of the stadium is unfortunately disputed, but one

modern conversion of 250,000 stadia would give 39,690km (24,662 miles) – very close to the current value of Earth's equatorial circumference, 40,075km (24,901 miles).

TRUSTED TRIANGULATION

As the Earth's geography became clearer through the great sailing voyages of the 16th Century, so did the accuracy of surveying techniques on land. Triangulation was first reported in a book published in Antwerp in 1533. In the 17th Century came telescopes with cross-hairs for sighting the stations being triangulated. These were introduced in the 1670s in France, the earliest country to attempt an exact survey of itself, under the direction of Jean-Dominique Cassini, the first of four generations of Cassini surveyors of France. The survey shifted the western coastline of existing maps about one-anda-half degrees of longitude east in

THE KEY DISCOVERY

NEWTON USED THE

trigonometrical survey of the Paris meridian in his calculation of the force of gravity. His theory led him to predict, in his Principia Mathematica of 1687, that the Earth could not be a perfect sphere. Centrifugal force, caused by axial spinning, was balanced by gravitational force. But since the equator moved faster than the poles, the equator must bulge very slightly, while the poles must be slightly flattened, resulting in an oblate spheroid like a flattened tomato. Gravitational attraction at the equator must be slightly less than at the poles, since gravity weakens with distance from the centre of the Earth.

To prove this, Newton first re-analysed the French survey data to show that a degree of latitude appeared to lengthen slightly as one moved north – an increase to be expected from a bulging equator and flattened poles. Second, he noted that a pendulum clock carried to the equator should beat slightly slow, since gravity was weaker there - which had indeed been observed when a French savant took such a clock to the Caribbean in 1672. Third, he pointed out that astronomers had observed Jupiter to be flattened at the poles. Finally, he showed how the gravitational pull of the Sun and the Moon on a bulging equator could account for the swivel in the Earth's axis that had been known to be the cause of the precession of the equinoxes since the time of the ancient astronomers.

Isaac Newton's theory of gravity predicted the true shape of the Earth, and therefore the varying length of a degree of latitude. It would prove a crucial step in defining the metre

Newton found that the Earth was in fact an oblate spheroid relation to the Paris meridian, and the southern coastline about half a degree of latitude to the north. Brest moved 177km (110 miles), Marseilles 64km (40 miles). When in 1682 Louis XIV paid a visit to the Paris Observatory and saw the new map, he exclaimed to Cassini: "Your journey has cost me a major portion of my realm!"

The French survey data also adjusted the shape of the Earth: from the perfect sphere imagined by Eratosthenes to a prolate spheroid: that is, a sphere slightly flattened at the equator and slightly bulging at the poles. But Isaac Newton, working across the English Channel, disagreed with Cassini (see 'The Key Discovery'). Newton calculated an oblate Earth, bulging at the equator and flattened at the poles. Not until the 1730s-40s, after Newton's death, was the question resolved in Newton's favour. Two gruelling expeditions led by French savants measured a degree of latitude in Lapland, near the pole, and in Peru, at the equator. The results were conclusive enough, said the French writer Voltaire - a devotee of Newton - "to flatten both the poles and the Cassinis".

THE FRENCH CONNECTION

The very first proposal to relate all length measurements to a decimal system based on the Earth's dimensions came from a French churchman, the Abbé Gabriel Mouton, writing in 1670. He suggested that the primary length unit should equal 1 minute of arc of a great circle (ie the circumference) of the Earth, a length not far short of 2,000m. But instead, a standard length based on the length of a pendulum was considered.

By the late 18th Century, scientists were aware that the rate of swing of a pendulum depended not at all on the weight of the bob, only on the length of the pendulum. Its period could therefore be used to define length. In fact, the so-called seconds pendulum, swinging once every second, has a length of 0.994m at sea level and latitude 45°, halfway between the equator and the poles, under conditions of standard gravity.

However, the pendulum measurement of the metre was rejected, partly because the period depended on gravity, which was known to vary with altitude and latitude, and partly because the units of time were themselves



CAST OF CHARACTERS From an Ancient Greek to a French emperor, it's taken an eclectic mix to standardise measurement



Isaac Newton

(1642–1727), the son of an English yeoman farmer who needs no introduction as a mathematician and physicist, also studied alchemy and religion. Although he never left the shores of England, he took a keen interest in calculating the shape of the Earth from both survey data and gravitational theory.





Jean Baptiste Joseph Delambre (1749-1822), a

French astronomer, he surveyed the northern part of the meridian arc from Dunkirk to Barcelona in collaboration with Méchain. After the death of Méchain, Delambre became director of the Paris Observatory and published three volumes defining the basis of the metric system in 1806–10.





Eratosthenes

(c. 276BC c. 195/194BC) was a Greek mathematician, astronomer and geographer, as well as a poet and music theorist, who became director of the library at Alexandria in 235BC. He is principally known for his accurate calculation of the circumference of the Earth and his map of the world based on parallels and meridians.

Pierre François André Méchain (1744-1804),

a French astronomer and hydrographer, he surveyed the southern part of the meridian arc from Dunkirk to Barcelona during the 1790s in collaboration with Jean Baptiste Joseph Delambre. Director of the Paris Observatory from 1799, Méchain died in Spain while checking his survey, which was completed by François Arago in 1806-09.

Napoléon Bonaparte

(1769-1821) strongly encouraged the development of science in France. This began with his invasion of Egypt in 1798, accompanied by a party of scientific savants, and continued after he proclaimed himself emperor in 1804. Although he was a leading supporter of the new metric system, he personally refused to use it.

TIMELINE

Becoming ever more accurate, there have been many methods of defining length over the centuries



krypton, and later in terms of the speed of light.

under Revolutionary review. The unit of length finally selected by the French Academy of Sciences in 1791 was one ten-millionth of a quarter of a great circle, in other words one ten-millionth of the distance from the equator to the North Pole (assuming the Earth to be spherical). Since the circumference of the Earth was 40,075km (24,901 miles), a quarter was just under 10,019km. Dividing this

figure by 10 million gave just over 1m. In 1792, two French scientists, Jean Baptiste Joseph Delambre and Pierre François André Méchain, set out to measure the length of the Paris meridian from Dunkirk to Barcelona by triangulation; the latitudes of Dunkirk and Barcelona were figured by astronomical observations. This challenging task, conducted at a time of political turmoil in France and war between France and Spain, took more than seven years to complete, ending in the death of Méchain.

Its accuracy was formidable. Even so errors crept in because of the oblate Earth, instrument error and human error in making endless finicky observations. But the great scientific expedition did give legitimacy to the metre and prestige to the metric system, which was officially imposed on France by Napoléon Bonaparte's government in 1801. For all its initial unpopularity, metrication was inevitable in the long run. As Napoléon farsightedly congratulated Delambre in 1806: "Conquests will come and go, but this work will endure."

GOING GLOBAL

For a century and a half, until the introduction of the Système International (SI) in 1960, the surveybased metre was embodied in the length of a metal bar kept in a vault at the International Bureau of Weights and Measures in Sèvres near Paris, copies of which had been distributed to national standards institutions in other countries.

In 1889, a new prototype bar was made from a dense platinum-iridium alloy. It had an X-like cross-section intended to minimise sag and distortion when the bar was properly supported. On the polished facets at both ends, there were fine horizontal graticules, lines made for visual settings by micrometer, and thicker vertical lines to monitor the expansion of the metal in the temperature range



NEED TO KNOW

These terms will help you understand the story of the metre

1 MERIDIANS

└ Circles of constant longitude, passing through a given place and through the terrestrial poles. The French meridian passing through Paris was used to define the metre, while the British meridian passing through Greenwich still defines the world's time zones.

$2 \, \underset{\text{spheroid}}{\text{oblate/prolate}}$

An oblate spheroid is a sphere that is flattened at the poles, bulging at the equator, like a tomato. A prolate spheroid, by contrast, is elongated at the poles, flattened at the equator, like a lemon. Earth is an oblate spheroid, as first demonstrated by Isaac Newton.

TRIANGULATION

✓ Triangular surveying proceeds by building up a chain of interlinked triangles. The first triangle is constructed from a precisely measured horizontal baseline. Its two base angles are measured through sighting instruments aimed at a prominent landmark – the triangle's third vertex. These angles are then used to calculate the lengths of the triangle's other two sides by simple trigonometry, after allowing for any altitude difference between the vertex and the baseline.

0-20°C. The standard length was always measured at 0°.

The disadvantages are obvious, and during the first half of the 20th Century scientists made progressive attempts to find techniques to redefine the metre in terms of the wavelength of light – an invariable standard that could be measured in any laboratory with the right equipment. In 1960, the metre was redefined in terms of a spectrum line of krypton. Then in 1983, the current definition was adopted, based on the speed of laser light. The metre is now the length of the path travelled by light in vacuum during a time of 1/299,792,458 second.

A METRIC WORLD

After France, the next countries to metricate were its neighbours, which had come under direct French rule. Spain went metric in the 1850s, followed by Germany and Italy as part of their political unifications. In 1875. 17 nations and their empires signed the Convention Of The Metre. By 1900, well over half of European nations had gone metric. The first Asian nation to convert was Mongolia in 1918. Before this, Japan and China had both shown interest, but neither nation moved until the 1920s. In Japan, there was popular opposition and conversion was shelved until the 1950s; in China, metrication did not revive until 1959, 10 years after the Communist Revolution. Political upheaval promoted metrication, as it had in France, Italy, Germany and the Soviet

Union, which metricated in 1924 after the Russian Revolution.

Britain officially committed itself in 1965, then dragged its feet and abolished the Metrication Board in 1979. Since 1974, the metric system has been taught in British schools and metric packaging has been gradually introduced alongside imperial. But there is no plan to convert road signs. As for the United States, there is little political will to metricate. Even in science, the old measures are sometimes used alongside the metric system – as became embarrassingly obvious in 1999 when a NASA probe sent to Mars was lost because one of its design teams had used the traditional units while the other had used metric.

Since 1791, when the metre was measured as one ten-millionth of the quarter meridian of the Earth to within 0.06mm, scientists have made drastic strides in accuracy. When you're told the length of something in metres, you can be confident that we now know the length of a metre to within 0.0000002mm, or one-hundredth of the width of DNA's double helix.

Andrew Robinson is the author of *The Story Of Measurement* (Thames & Hudson)

Find out more

Watch Marcus du Sautoy explain the quest to measure the metre in *Time And Distance* http://youtu.be/x1DT8flqqZAc