International Journal of Modern Science and Technology
http://www.ijmst.co/
Vol. 2, No. 5, 2017. Page 188-192.
ISSN: 2456-0235.

Research Article

Determination of Mass Moment of Inertia with Experimental Validation Using Principle of Conservation of Energy

K. K. Dinesh, S. Jayakumar, K. M. Abubacker, V. Suresh babu

Department of Mechanical and Industrial Engineering, Caledonian College of Engineering, Muscat, Sultanate of Oman.

*Corresponding author's e-mail: dineshkk36@yahoo.co.in

Abstract

The moment of inertia of a body is a measure of how hard it is to get it rotating about some axis. Many physical objects require determination of mass moment of inertia for identifying and analyzing their practical implications during dynamic working conditions that are faced in real time applications. The moment of inertia of an object around a given axis determines the relationship between the applied mechanical torque and the angular acceleration produced. Mass moment of inertia measures the extent to which an object resists rotational acceleration about an axis. Present work aims in determining the mass moment of inertia by using principle of conservation of energy. Hitherto, there are well determined fundamental equations for geometrically simple objects that are easily interpreted mathematically. However, it is noticed that there is difficulty to mathematically express it for complex shaped bodies. Current work addresses a comparative study between a solid and circular ring with a unique mass of 2.9 kg with the thickness of 10mm. A typical dimension of 120 mm diameter was considered for solid disc and similarly, 130 mm outer diameter and 102 mm inner diameter for the circular ring. An experimental setup was initially modeled with Pro/E CREO software and later a test rig was fabricated to cater the objective. A series of experiments were conducted for the determination of Mass Moment of Inertia for both solid and circular ring specimen. A comparative study was conducted and the outcomes were tabulated and analyzed. It was noticed that the mass moment of inertia determined through the proposed experimental procedure on the specimens were within the error limit of 5%. It is further concluded that the procedure devised can be well utilized for determination of irregular objects that are used in real time applications.

Keywords: Mass moment of Inertia; Acceleration; Solid and Circular ring; Conservation of energy.

Introduction

Mass moment of inertia of a body about an axis is the resistance to a change in rotational motion around a specific axis. The magnitude of mass moment of inertia depends upon the distribution of mass and the axis about which mass moment of inertia has to be evaluated.

$$I = mr2$$

Where m is the mass element and r is the distance from that element to the axis [1-3]. Since there are many other axes to choose, and since all but a few yield easy moment of inertia calculations, axis of symmetry chosen as axis of rotation. When an object at rest is set into rotation about some axis, the rotational inertia keeps the object rotating at some angular speed characterized by a physical quantity called the moment of inertia, I, of the object. The kinetic energy of a rotating object is:

$$KE = ½\, I\, \omega^2 \qquad (1)$$

In the experiment, the object is being set rotating by attaching it to a hanging mass and allowing the mass to fall. Before dropping the mass, the energy is only potential energy that of the hanging mass. At the instant the mass hits the floor the energy is all kinetic energy, both in the rotating body and in the falling object. Ignoring friction and the rotational kinetic energy of the pulley, the Law of Conservation of Energy can be applied:

$$mgh = ½\, mv_f^2 + ½\, I\, \omega_f^2. \qquad (2)$$

The velocity (v) of the hanging mass is related to the angular velocity (ω) of the rotating object by v = ω r where r is the radius of the platform shaft of the object. The final velocity is related to the average velocity, and the average

Received: 03.04.2017; Received after Revision: 18.04.2017; Accepted: 20.04.2017; Published: 28.05.2017

velocity can be found from the distance covered between the sensors (h) and the time (t) of travel: vavg = (v_f + v_i) / 2 , and vavg = h/t, so that (with v_i = 0, i.e., starting from rest) v_f = 2h / t. Now substitute v_f and ω in Eq. (2) and solve for I in terms of the measurable quantities m, g, h, t and r:

$$I = mr\,2\,[\,(gt^2 / 2h) - 1\,] \qquad (3)$$

The equation 1 can now be used to find the moment of inertia experimentally for any object. The results are now compared for a solid disk and circular ring with those derived from theory.

$$I_{disk} = ½\, M_D R^2 \qquad (4)$$

$$I_{ring} = ½\, M_R\, (R_{IN}^2 + R_{OUT}^2) \qquad (5)$$

Where MD is the mass of the disk and MR is the mass of the ring, R is the radius of the disk, RIN is the inner radius of the ring, and ROUT is the outer radius of the ring.

Figure 1. 3-D assembled view of mass moment of inertia test rig

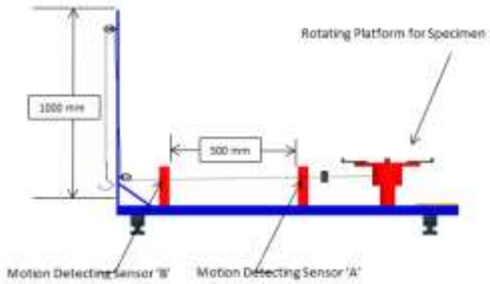

Figure 2. 2-D assembled view of mass moment of inertia test rig

Experimental methodology

To begin with the experiments the test rig is initially to be interfaced. The procedure for interfacing the software will be as per the flow diagram indicated in Figure 3.

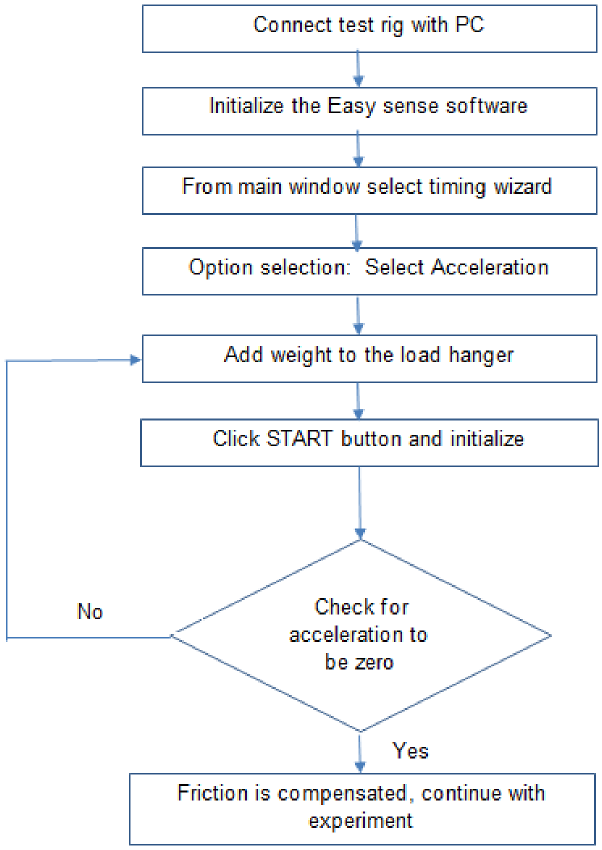

Figure 3. Flow diagram for software interface

Interface / Software Initial Setup

The easy sense software can be started and interfaced with the test rig. The acceleration option is selected under the timing wizard from the main window.

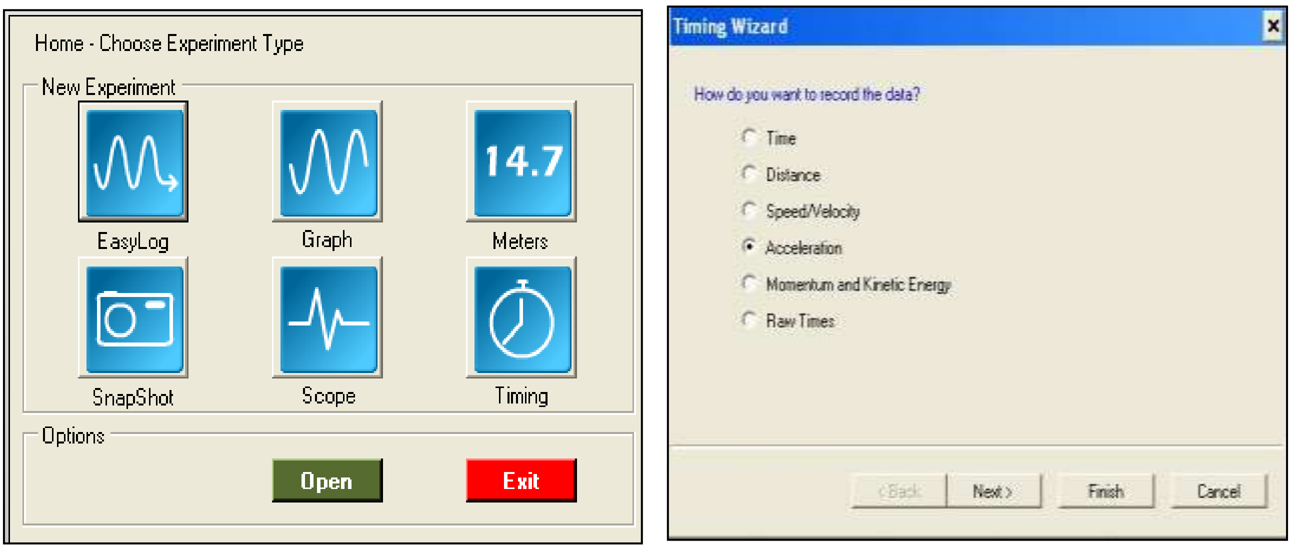

Figure 4. Software interface-1

After placing the weight on the load hanger in an incremental manner, the load at which the platform initiates its rotation is noted down and now the start button on the software is activated. If the acceleration displayed is zero, indicates that the frictional force is compensated

Figure 5. Software interface-II

Now the same method can be adopted to measure the time taken for the obstacle to travel between the two sensors by selecting the option Time under timing wizard between the two sensors A & B.

Compensating for friction

The frictional force is not counted while deriving equation 3, which could be compensated by following the below mentioned steps.
Bring the obstacle to its starting position by means of rotating the platform without any specimen. Now add some small masses to the load hanger and release the platform, if the platform doesn't rotate then add some more masses to initiate the rotation.
The system needs to move at a constant velocity (no acceleration) which can be confirmed with the help sensors integrated with easy sense software that shows the acceleration between two sensors equal to zero. If there is acceleration, remove some masses. When the system moves at constant velocity, which means the friction present in the system is compensated. Record the hanging mass required for compensating friction.

Procedure

(i) Start the experiment without any specimen on the rotating platform of the apparatus. For the selected mass 'm' suspended in the load hanger, determine the time taken (t) by the obstacle to travel between the two sensors. Stop the rotation of the platform once the obstacle crosses the second sensor to prevent possible tangling of the hanging mass (m) on the floor. Substitute time 't' value in Eq. (3) which has been derived based on the conservation law of energy [4-10] and the calculated value is termed as moment of inertia of the platform (I_{plat}).

(ii) Now place the ring in the rotating platform and find the moment of inertia as explained in Step 1 using Eq. (3). The moment of inertia obtained is the moment of inertia of the platform PLUS the moment of inertia of the ring ie., $I_{together}$ .Subtract the value of I_{plat} from $I_{together}$ to find the moment of inertia of the ring, I_{ring}.
3) Remove the ring and place the disk in the rotating platform and find the moment of inertia as explained in Step 1 using Eq. (3).). The moment of inertia obtained is the moment of inertia of the platform PLUS the moment of inertia of the disk ie., $I_{together}$ Subtract the value of I_{plat} from the value of $I_{together}$ to find the moment of inertia of the disk, I_{disk}.

Figure 6. Moment of inertia apparatus set-up with computer interface

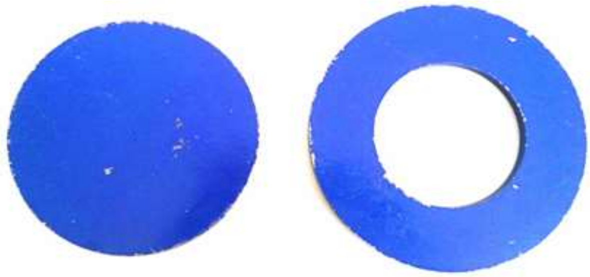

Figure 7. Moment of inertia specimens

Results and discussions

Mass of the Disc (M_D) : 2.9 kg
Mass of the Ring (M_R) : 2.9 kg
Radius of the Disc (R) : 102 mm
Outer radius of the Ring (R_{OUT}) : 120 mm
Inner Radius of the Ring (R_{IN}) : 65 mm
Radius of the platform shaft (r) : 30 mm
Distance between two sensors (h) : 500 mm
Frictional Mass (Apparatus) : 130 gms
Moment of inertia of the platform (I_{plat}) : 0.01591 kg m^2

Results are presented in the form of a table 1.

Table 1. Data collected for solid and hollow ring.

Mass moment of Inertia of Solid Disc				
Suspended Mass 'm' (kg)	Time Taken 't' (s)	Practical M.I ($I_{together} - I_{plat}$) (kg m^2)	Theoretical M.I (I_{disk}) (kg m^2)	% Error
0.02	13.4	0.0157	0.01508	4%

Mass moment of Inertia of Circular Ring				
Suspended Mass 'm' (kg)	Time Taken 't' (s)	Practical M.I ($I_{together} - I_{plat}$) (kg m^2)	Theoretical M.I (I_{ring}) (kg m^2)	% Error
0.02	15.5	0.0264	0.02700	2%

By trial and error it was found that the suspended mass required to overcome the frictional force is 130gms which is not counted in calculating Moment of Inertia using equation no (3). While performing the experiment it was observed that the time taken in case of circular ring specimen is found to be comparatively higher than that of the solid disc. The Moment of Inertia for circular ring is found to be more as expected because of the distribution of mass at a higher radial distance from the center in case of circular ring compared with that of a circular disc of identical mass. The variations between the theoretical and experimental values are found to be within 5% which is within the allowable limits for any experimental validation requirements.

Conclusions

The present work has determined the mass moment of inertia of solid and circular ring at preliminary level. The experimental test rig designed was noticed to be working as per the requirement and the readings were tabulated through the interfacing software. With the experimental procedure, the moment of inertia of known bodies could be obtained with a maximum relative error less than 5%, as seen in Table 1. The maximum relative error produced in the moment of inertia calculations is found to be very sensitive to the discrepancy generated from the time measurements. The specimen and circular platform's mass centers should have the same vertical centroid axis in order to get more accurate results. The initial position of the obstacle should be placed very near to sensor A that would ensure its initial velocity to be zero, as emphasized in the derivation earlier. The test rig could be extended for determining mass moment of inertia of irregular bodies with modification in the arrangement.

Conflict of interest

Authors declare there are no conflicts of interest.

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