Error Probability of OFDM with Carrier Frequency Offset in AWGN and Fading Channels

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***Abstract*— In this paper we investigate the error probability for OFDM with carrier frequency offset (CFO) in additive white Gaussian noise (AWGN) and Rayleigh fading channels. The statistical nature of the intercarrier interference (ICI) due to CFO and the validity of the usual Gaussian distributed ICI assumption are studied. In AWGN channels, our simulations indicate that the theoretical symbol error rate (SER) obtained by assuming Gaussian characteristics for the ICI is not accurate. The effect of adjacent channel ICI coefficients must be considered. However in a fading channel provided that the channel correlation effects on the useful OFDM signal and the ICI are considered, the theoretical BER results due to Gaussian ICI approach and simulations closely agree. The theoretical SER for MQAM signaling in fading channels is also derived in closed form.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a popular technology used to realize high data rates over wireless channels. Applications of OFDM include digital audio broadcasting, digital video broadcasting (DVB), wireless local (802.11.a/g/n, Hiperlan/2), metropolitan area networks (802.16) and ultra-wideband systems.

However compared to single-carrier techniques, OFDM is more sensitive to frequency synchronization errors due to carrier frequency offset (CFO) and Doppler shift [1-4]. Since OFDM subcarriers are closely packed compared to the system bandwidth (for example in DVB 2048 or 8196 subcarriers) the amount of tolerable frequency offset is a small fraction of the OFDM bandwidth. CFO, the difference between the carrier and receiver local oscillator frequency introduce rotation for the useful signal and intercarrier interference (ICI) which cause loss of orthogonality among the demodulated OFDM subcarriers [3]. The rotation common to all the subcarriers is known as the common phase error (CPE). Although the CPE could be removed using pilot tones, ICI due to CFO potentially introduce low data throughputs and significantly increase the system error performance.

The importance of the CFO problem in OFDM has resulted in many authors analyzing the impact of CFO and proposing suppression techniques [5]. Several papers have investigated the theoretical performance degradation due to the CFO [4], [6]–[8]. These include efforts to study the signal-to-interference-noise ratio (SINR) reduction [9], [10] and the symbol error rate (SER) [4], [8]. A very common assumption made in previous literature for theoretical SER calculation is that the ICI due to CFO has a Gaussian distribution. This greatly simplifies the SER analysis [11]. Since the total noise contribution due to the ICI and additive white Gaussian noise (AWGN) is treated as Gaussian, simple error formulas can be used to predict the error performance both for AWGN and fading channels. An exact error analysis is also reported in [7] using the characteristic function of an ICI distribution. But the work of [7] assume no fading effects. Again to perform the error calculation over a fading channel, two Gaussian ICI modeling approaches are reported in the past.

1). Channel faded ICI is assumed to be independent of the OFDM signal. The ICI is modeled as Gaussian [10], [12]. In the subsequent analysis we refer to this approach as the "channel independent" error calculation method.

2). The channel fading effect on both the ICI and the OFDM signal is simultaneously considered. The ICI is modeled as a Gaussian process [4], [8].

In this paper we investigate the error performance of OFDM in the presence of CFO over AWGN and fading channels. Although the subject has been touched by a *large* volume of previous publications in technical literature there seems to be *disparities* among them. Our contributions are summarized as:

1). The validity of assuming ICI as a Gaussian random variable was analyzed by varying the normalized CFO. In AWGN channels, it shown that a theoretical error performance based on a Gaussian ICI assumption yields highly pessimistic values compared to the same for a fading channel. The joint fading effects between the OFDM signal and ICI must be considered in fading conditions [4].

2). In AWGN channels, by considering significant adjacent channel ICI coefficients a better theoretical result can be derived. This approach provides a flexible means of adjusting the theoretical result closer to the simulations.

The following is an outline of the paper organization. In Section II we describe the OFDM system model with CFO. The ICI distribution is analyzed in Section III. The theoretical error performance due to CFO degradation is studied for AWGN and fading channels in Sections IV and V. Finally some conclusions are reported in Section VI.

II. OFDM System Model with CFO

An OFDM system with N subcarriers is considered. The binary data bits are serial-to-parallel converted and mapped

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onto a signal constellation. The OFDM transmissions consist of symbol blocks each with a period of T_s. In this time N subcarriers $X(k)$ for $k = 0, 1, \ldots, (N-1)$ get modulated by a signal alphabet $\mathcal{A}$ and used for information carrying. For 4-QAM signaling $\mathcal{A} \in \{\pm 1 \pm j\}$. The discrete time domain OFDM signal is given by

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) \exp\left(\frac{j2\pi nk}{N}\right) \quad (1)$$

A cyclic prefix $P \geq L$ is appended to $x(n)$s and the composite signal is transmitted to the channel after upconverting. L is the number of multipaths. It is assumed that the transmitted signals propagate through a multipath wireless channel modeled by

$$h(t, \tau) = \sum_{l=0}^{L-1} h_l[t] \delta\left(t - \tau_l[l]\right) \quad (2)$$

where $\delta(\cdot)$ is the dirac function and $h_l[t]$s are complex Gaussian random variables. Let $y(n)$ denote the received signal. $y(n)$ can be expressed as

$$y(n) = \sum_{l=0}^{L-1} h(l) x(n-l) + w(n) \quad (3)$$

In (3) $w(n)$ is the complex AWGN component, $w(n) \sim \mathcal{N}(0, \sigma^2_{\text{AWGN}})$. In the presence of CFO the received OFDM signal for the k-th subcarrier after DFT processing is

$$\begin{aligned} Y(k) &= \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{\frac{-j2\pi kn}{N}} \quad (4) \\ &= \frac{1}{N} \sum_{n=0}^{N-1} e^{\frac{j2\pi \epsilon n}{N}} \sum_{l=0}^{N-1} H(l) X(l) e^{\frac{2\pi(l-k)n}{N}} + W(k) \end{aligned}$$

Discrete time domain OFDM signal $y(n)$ is multiplied by the factor $e^{j2\pi\epsilon n/N}$ in (4) to represent the CFO effect and ϵ is the normalized CFO. The k-th subcarrier frequency domain channel coefficient is denoted by $H(k)$. Mathematically $H(k) = \sum_{l=0}^{L-1} h_l e^{\frac{j2\pi lk}{N}}$ and $E\{|H(k)|^2\} = \sigma^2_h$. Eq. (4) can be rearranged as

$$\begin{aligned} Y(k) &= S(0) H(k) X(k) \quad (5) \\ &+ \sum_{l=0,\, l\neq k}^{N-1} S(l-k) H(l) X(l) + W(k) \end{aligned}$$

where $S(\cdot)$ indicates the ICI coefficients. An expression for the ICI coefficients is given in [7].

$$S(k) = \frac{\sin \pi(k+\epsilon)}{N \sin \frac{\pi}{N}(k+\epsilon)} \exp\left[j\pi(1 - \frac{1}{N})(k+\epsilon)\right] \quad (6)$$

and

$$S(0) = \frac{\text{sinc}(\epsilon)}{\text{sinc}(\epsilon/N)} \exp\left(j\pi(1 - \frac{1}{N})\epsilon\right) \quad (7)$$

As evident from (5), the modulated symbol for the k-th subcarrier is attenuated and rotated by the CPE, $\arg\{S(0)\}$. The ICI is denoted by the second term in (5) and cause loss of orthogonality among the received subcarriers. The average

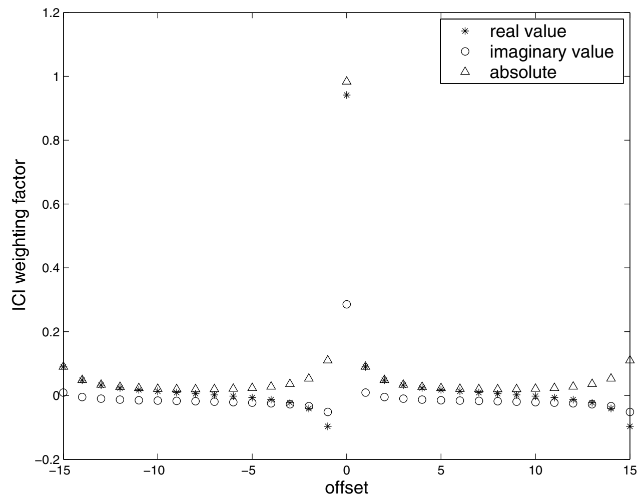

Fig. 1. Absolute and real/imaginary parts of the complex ICI coefficients. $N = 16$ and $\epsilon = 0.1$.

SINR for the k-th subcarrier, $\gamma(k)$ without CPE correction is given by [7]

$$\gamma(k) = \frac{|S(0)H(k)X(k)|^2}{E\left\{\left|\sum_{l=0,\, l\neq k}^{N-1} S(l-k)H(l)X(l)\right|^2\right\} + \sigma^2_{\text{AWGN}}} \quad (8)$$

where $E(\cdot)$ signifies the expectation operator.

III. SER Analysis in AWGN Channels

In this Section we analyze the SER for CFO affected OFDM in AWGN. The data subcarriers are assumed to be modulated by an M-QAM alphabet. The use of M-QAM is not a limitation and extensions for other modulation schemes are straightforward. In an AWGN channel note that $H(k) = 1$ for all k. Hence Eq. (6) is simplified to get

$$Y(k) = S(0)X(k) + \sum_{l=0,\, l\neq k}^{N-1} S(l-k)X(l) + W(k) \quad (9)$$

A. Statistical Characterization of the ICI

In this Section we look into the statistical behavior of the ICI. The usual assumption of an independent OFDM data stream is made, however coding in practice can introduce some correlation. Here our aim here is to obtain general comments rather than to discuss about a specific system.

By statistically categorizing the ICI, the error performance of the system can be efficiently studied. Surprisingly previous authors discussing the CFO effects have skipped the topic or have assumed Gaussian characteristics for the CFO using a simple central limit theorem argument. However note that ICI term depends on many system parameters such as N, the modulation alphabet and the ICI coefficients. The similarity of ICI to a Gaussian distribution can be easily analyzed from simulations by using measures such as the probability density function (pdf) and the Kurtosis excess. For a Gaussian random variable Kurtosis excess is zero. In Fig. 2 we plotted the

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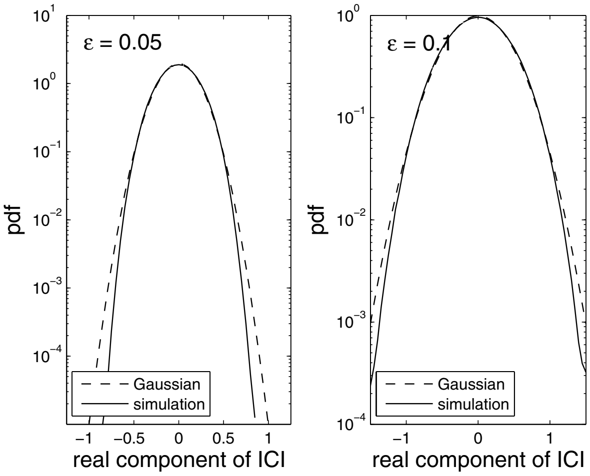

Fig. 2. Probability density function of the real part of ICI.

ICI pdf for $N = 64$ and normalized CFOs 0.05 and 0.1. Data symbols were drawn from the 16-QAM constellation. For comparison, theoretical Gaussian curves are also shown. The mean and variance were obtained from the simulations. As seen from Fig.2, ICI can deviate from Gaussian characteristics. Results not presented here showed similar observations with different N values.

B. Gaussian Approximated ICI Analysis

Past literature has analyzed the theoretical SER by modeling the ICI as a Gaussian interference source. This approach produces simple mathematical manipulations. However, the mismatch between the simulation and theoretical values can be substantial (for some ϵ range). Since the ICI is modeled as a Gaussian process, a conventional error analysis valid for AWGN channels can be applied [11]. The total noise power σ_T^2 is given by

$$\sigma_T^2 = (1 - |S(0)|^2)\sigma_x^2 + \sigma_{\text{AWGN}}^2 \tag{10}$$

and the SER is expressed by [11]

$$\begin{aligned} P_e &= 1 - \left(1 - 2\alpha_1\, Q\left(\sqrt{\alpha_2 \gamma}\right)\right)^2 \\ &= 4\alpha_1 Q\left(\sqrt{\alpha_2\gamma}\right)\left(1 - \alpha_1 Q\left(\sqrt{\alpha_2\gamma}\right)\right) \end{aligned} \tag{11}$$

where $\alpha_1 = (1 - 1/\sqrt{M}), \alpha_2 = 3/(M-1)$ and $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}\, dt$. The SINR $\gamma = \sigma_x^2/\sigma_T^2$.

Fig. 3 shows that the theoretical results due to (11) are pessimistic. The reason for this is that the Gaussian analysis does not consider the significant adjacent channel ICI correlation [6]. That is contributions from $S(1)$ and $S(-1)$. In order to elaborate this we rewrite (9) as,

$$Y(k) = S(0)X(k) + N(k) + \sum_{\substack{l=0 \\ l\neq k,\pm 1}}^{N-1} S(l-k)X(l) + W(k) \tag{12}$$

where $N(k) = S(-1)X(k-1) + S(1)X(k+1)$ is the ICI contribution for the k-th subcarrier from the immediate left and right neighboring subcarriers. Note that $N(k)$ depends on $X(k-1)$ and $X(k+1)$, i.e., the adjacent channel modulated subcarrier values. Let us treat the second summed ICI term as a Gaussian variable distributed as $\mathcal{N}(0, k_1\sigma_n^2)$. σ_n^2 can be analytically calculated and k_1 is a tuning parameter which can be used to fit the theoretical analysis with the simulations accurately. Using this two term ICI approximation, error rate of an OFDM system (any signal constellation mapped) can be easily calculated. However without loss of generality, we explain the derivations using 16-QAM mapped OFDM signals. In Fig. 3 simulated results are provided to corroborate this analysis. Let us assume that $X(k) = (1+j) = \sqrt{2}e^{j\frac{\pi}{4}}$. We can write (12) the real part of $\Re\{Y(k)\} = Y^{\mathcal{R}}(k)$ as (analysis for the imaginary part is similar)

$$\begin{aligned} Y^{\mathcal{R}}(k) = &\frac{\sqrt{2}\text{sinc}(\epsilon)}{\text{sinc}(\epsilon/N)}\cos\left(\frac{\pi}{4} + \pi(1-\frac{1}{N})\epsilon\right) \\ &+ N^{\mathcal{R}}(k) + I^{\mathcal{R}}(k) + \Re\{W(k)\} \end{aligned} \tag{13}$$

where $N^{\mathcal{R}}(k) = \Re\{N(k)\}$, $I_{\mathcal{R}}(k)$ is the inphase component of $\sum_{l=0, l\neq k,\, \pm 1}^{N-1} S(l-k)X(l)$. Note that $N^{\mathcal{R}}(k)$ is also a random variable since it is conditioned on $X(k-1)$ and $X(k+1)$. $I^{\mathcal{R}}(k)$ and $\Re\{W(k)\}$ are uncorrelated and the combined $\tilde{N}^{\mathcal{R}}(k) = I^{\mathcal{R}}(k) + \Re\{W(k)\}$ term is treated as a Gaussian random variable with zero mean and variance, $(k_1\sigma_n^2 + \sigma_{\text{AWGN}}^2)/2$. The exact SER analysis can be performed assuming all possible combinations for $N^{\mathcal{R}}(k)$ due to the pair $X(k-1), X(k+1)$ and writing individual error probability for all. For the sake of simplicity here a mean value of $E\{N^{\mathcal{R}}(k)\} = N^{\mathcal{R}}$ is used. In the theoretical results shown in Fig. 3 we have assumed all possible combinations of $N^{\mathcal{R}}(k)$. Hence the CPE and attenuation corrected k-th subcarrier can be rewritten using (13) as,

$$Y^{\mathcal{R}}(k) = X^{\mathcal{R}}(k) + N^{\mathcal{R}} + \tilde{N}^{\mathcal{R}}(k) \tag{14}$$

In (14), $x = N^{\mathcal{R}} + \tilde{N}^{\mathcal{R}}(k)$ is simply a Gaussian random variable with mean, $m = N^{\mathcal{R}}/\Re\{S(0)\}$ and variance $\sigma^2 = (k_1\sigma_n^2 + \sigma_{\text{AWGN}}^2)/2\Re\{S(0)\}^2$. Hence the symbol error probability $P_e^{\mathcal{R}}$ for this inphase component can be calculated as

$$\frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{-d} e^{\frac{-(x-m)^2}{2\sigma^2}}dx + \frac{1}{\sqrt{2\pi\sigma^2}}\int_{d}^{\infty} e^{\frac{-(x-m)^2}{2\sigma^2}}dx \tag{15}$$

where d denotes the decision boundary. We can calculate the total error probability for the point $(1+j)$ as the average of inphase and quadrature error probabilities [11].

$$P_{e|(1+j)} = \frac{1}{2}P_e^{\mathcal{R}} + \frac{1}{2}P_e^{Q} \tag{16}$$

Furthermore the symbol errors of the other 16-QAM constellation points $P_{e|d_0+jd_1}, d_0, d_1 \in \pm 1 \pm 3$ can be obtained by following a similar procedure. Assuming all constellation points appear with equal probability, i.e., 1/16, the total average SER is given by $P_e = \frac{1}{16}\sum_{d_0,d_1} P_{e|d_0+jd_1}$. The average bit error rate (BER) of M-QAM is also approximately related to the average SER as $P_b \approx P_e/\log_2 M$.

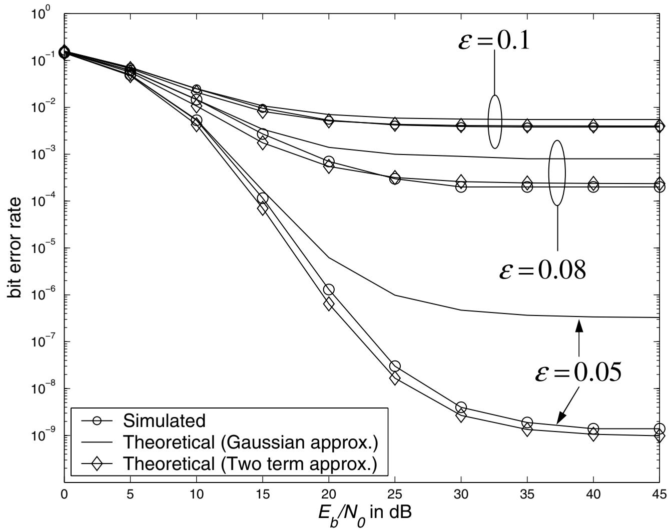

Fig. 3. Theoretical and simulated BER versus E_b/N_0 in the AWGN channel. Gray coded 16-QAM. CPE is corrected.

IV. SER Analysis in Flat Fading Channels

For the flat fading channel noting that all subcarriers experience same level of fading we can omit the frequency index from $H(k)$. That is $H(k) = H$ for all $k = 0, 1, ..., (N-1)$. In this scenario (5) is simplified as

$$Y(k) = S(0)HX(k) + H\left(\sum_{l=0,\, l\neq k}^{N-1} S(l-k)X(l)\right) + W(k) \tag{17}$$

The channel coefficient H common to all subcarriers is factored out of the ICI summation in (17).

Remark 1: In this paper we assume that perfect channel state information is available to the receiver. However in practice the error performance strictly depends on the pilot training and accuracy of the channel estimation algorithm. Analyzing these effects are beyond the scope of this paper.

A. Channel Independent Approach

In the channel independent method the faded OFDM signal and the ICI are assumed to be independent. Also an average ICI power is calculated neglecting the instantaneous fading effects. Hence the total effective noise power σ_T^2 degrading the performance is [12]

$$\sigma_T^2 = E\left\{\left|\sum_{l=0,\, l\neq k}^{N-1} S(l-k)H(l)X(l)\right|^2\right\} + \sigma_{\text{AWGN}}^2 \approx (1-|S(0)|^2)\sigma_h^2\sigma_x^2 + \sigma_{\text{AWGN}}^2 \tag{18}$$

and the SINR $\gamma(h)$ in this case is approximated by $\gamma(h) = |S(0)|^2 h^2 \sigma_x^2/\sigma_T^2$. The Rayleigh distributed channel coefficients are $h = |H|$ and the pdf is given by $p_H(h) = \frac{2h}{\sigma_h^2} e^{-h^2/\sigma_h^2}$. In fading channels the error performance is obtained by averaging (11) over the pdf of h. Hence the average probability of error P_e can be calculated as $P_e = \int_0^\infty P_e(h)p_H(h)\,dh$. The average SER can be written as

$$P_e = \frac{8\alpha_1}{\sigma_h^2}\int_0^\infty Q(\sqrt{\alpha_2\gamma(h)})h\, e^{-h^2/\sigma_h^2}dh - \frac{8\alpha_1^2}{\sigma_h^2}\int_0^\infty Q^2(\sqrt{\alpha_2\gamma(h)})h\, e^{-h^2/\sigma_h^2}dh \tag{19}$$

The only difference between (19) and a standard OFDM error performance analysis for the Rayleigh channel is the additional interference due to CFO. Since the ICI due to CFO is treated as channel independent, (19) can be evaluated in closed form using previously reported results. However simulations not provided here indicate that this theoretical analysis predicts pessimistic results. The error in closed form can be expressed using [13, Eq. 49] as

$$P_e = 2\alpha_1\left(1-\sqrt{\frac{\alpha_2\bar{\gamma}_h}{2+\alpha_2\bar{\gamma}_h}}\right) + \alpha_1^2 \cdot \left[\frac{4}{\pi}\sqrt{\frac{\alpha_2}{2+\alpha_2\bar{\gamma}_h}}\arctan\left(\sqrt{\frac{2+\alpha_2\bar{\gamma}_h}{\alpha_2\bar{\gamma}_h}}\right) - 1\right] \tag{20}$$

However acknowledging the fact that ICI due to CFO also fades along with the useful OFDM signal, an accurate analytical solution can be derived. In this approach rather than estimating a statistical average for the ICI power, ICI is treated as a random variable. That is,

$$\gamma(h) = \frac{|S(0)|^2\sigma_x^2 h^2}{(1-|S(0)|^2)\sigma_x^2 h^2 + \sigma_{\text{AWGN}}^2} \tag{21}$$

The refined average SER is given by

$$P_e = \frac{8\alpha_1}{\sigma_h^2}\int_0^\infty Q(\sqrt{\alpha_2\gamma(h)})h\, e^{-h^2/\sigma_h^2}dh - \frac{8\alpha_1^2}{\sigma_h^2}\int_0^\infty Q^2(\sqrt{\alpha_2\gamma(h)})h\, e^{-h^2/\sigma_h^2}dh \tag{22}$$

The first integral in (22) can be evaluated in closed form using some results reported in [14], [15].

$$I_1 = \frac{8\alpha_1}{\sigma_h^2}\int_0^\infty Q(\sqrt{\alpha_2\gamma_h})h\, e^{-h^2/\sigma_h^2}dh = \alpha_1 - \frac{\alpha_1 c}{\sqrt{2}d^3}e^{\frac{-c^2}{2d^2}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{c^2}{2d^2}\right)^k U\left(k+\frac{3}{2}, k'; \frac{1}{d^2}\right) \tag{23}$$

where $U(\cdot)$ is confluent hypergeometric function of the second kind and $k' = (k+2)$, $c^2 = |S(0)|^2\alpha_2\sigma_x^2/\sigma_{\text{AWGN}}^2$, $d^2 = (1-|S(0)|^2)\sigma_x^2/\sigma_{\text{AWGN}}^2$. The second integral in (22) can be evaluated again as an infinite summation and is given by

$$I_2 = \frac{\alpha_1^2}{2} - \frac{\alpha_1^2 c}{\sqrt{2}d^3}e^{\frac{-c^2}{2d^2}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{c^2}{2d^4}\right)^k U\left(k+\frac{3}{2}, k', \frac{1}{d^2}\right) + \frac{4\sqrt{2}\alpha_1^2}{\sqrt[3]{\pi}}e^{\frac{1-2c^2}{2d^2}}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{d^{-3k}c^{(2k+2n+4)}\,\Gamma(n+2)}{(2n+1)n!(2d)^{2n}} \times (-1)^n\, W_{-n-2-\frac{k}{2},\frac{k+1}{2}}\left(\frac{1}{d^2}\right)$$

Above the Gamma function is $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\ dt$. $W_{\lambda,\mu}(\cdot)$ denotes the Whittaker function [16].

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V. SER Analysis in Frequency Selective Fading Channels

The received signal $Y(k)$ in a frequency selective fading channel is given by (6). The SER can be analyzed similarly as in the case of frequency flat fading. However with frequency selective fading, the channel coefficients $H(l)$ are different from $H(k)$, the channel coefficient of the k-th concerned subcarrier. The ICI is the summation of $(N-1)$ differently channel attenuated $S(l-k)H(l)X(l)$ terms. Hence more randomness in the ICI term is expected compared to the flat fading conditions.

In the frequency selective fading channel, we jointly analyze the OFDM signal and the ICI using the frequency domain channel correlation information. Reference [4] elaborates the significance of using the fading nature of ICI to calculate the BER. Also see the work reported in [8]. In OFDM systems, generally at least several adjacent subcarrier channel coefficients are correlated. This is because the systems are designed with $P < N$. Hence the k-th subcarrier $H(k)$ could be approximately written as

$$H(k) \approx \lambda_{\Delta} H(l) + \left(\sqrt{1-\lambda_{\Delta}^2}\right) Z(l) \tag{24}$$

where the correlation coefficient λ_{Δ} in (24) depends on the distance or separation between k-th and l-th frequency bins. The term $\sqrt{1-\lambda_{\Delta}^2}$ ensures that the power is normalized. $Z(l)$ is modeled as a Gaussian random variable, $Z(k) \sim \mathcal{N}(0,1)$. In other terms given that k-th and l-th bins are sufficiently apart, the channel correlation is negligible.

$$\begin{aligned} \lambda_{\Delta k} &= E\{H(k)H^*(k+\Delta k)\} \qquad (25)\\ &= E\left\{ \left(\sum_{l=0}^{L-1} h(l) e^{\frac{-j2\pi kl}{N}} \right) \sum_{m=0}^{L-1} h^*(m) e^{\frac{-j2\pi(k+\Delta k)m}{N}} \right\} \\ &= \sum_{l=0}^{L-1} \sigma_l^2 e^{\frac{j2\pi\Delta kl}{N}} \end{aligned}$$

In (25) σ_l^2 is the power of the l-th tap. When the channel power delay profile is uniform with $\sigma_l^2 = 1/L$, λ_{Δ} can be further simplified as [17]

$$\lambda_{\Delta} = \frac{1}{L} \frac{\sin\left(\frac{\pi\Delta L}{N}\right)}{\sin\left(\frac{\pi\Delta}{N}\right)} e^{\frac{j\pi\Delta(L-1)}{N}} \tag{26}$$

By substituting (24), into (4) we obtain,

$$\begin{aligned} Y(k) &= S(0)H(k)X(k) \qquad (27)\\ &+ H(k)\left(\sum_{l=0,l\neq k}^{N-1} \lambda_{|l-k|} S(l-k)X(l) \right) \\ &+ \sum_{l=0,l\neq k}^{N-1} \left(\sqrt{1-\lambda_{|l-k|}^2}\right) S(l-k)X(l)Z(l) + W(k) \end{aligned}$$

Let $\alpha = H(k)$, hence the instantaneous SINR for the frequency selective channel can be established as [4]

$$\gamma(h) = \frac{|S(0)|^2 \alpha^2 \sigma_x^2}{\alpha^2 \sigma_{n_1}^2 + \sigma_{n_2}^2 + \sigma_{\text{AWGN}}^2} \tag{28}$$

where $\sigma_{n_1}^2$ and $\sigma_{n_1}^2$ are given by

$$\sigma_{n1}^2 = E\left(\left| \sum_{l=0,l\neq k}^{N-1} \lambda_{|l-k|} S(l-k)X(l) \right|^2 \right) \tag{29}$$

and

$$\sigma_{n2}^2 = E\left(\left| \sum_{l=0,l\neq k}^{N-1} \left(\sqrt{1-\lambda_{|l-k|}^2}\right) S(l-k)Z(l)X(l) \right|^2 \right)$$

Finally the average SER of the OFDM system $\mathcal{P}_e$ in the frequency selective fading channel is obtained as

$$\mathcal{P}_e = \frac{1}{N} \sum_{k=0}^{N-1} P_e(k) \tag{30}$$

Similarly to the two term ICI approach in AWGN channels, significant ICI terms can also be considered for a precise error analysis in fading channels. However simulations indicated that the above analysis (considering the channel correlations and assuming a Gaussian ICI distribution) is capable of providing accurate theoretical results in all cases of practical importance.

A. Simulation Results

We have performed Matlab simulations to investigate the accuracy of the theoretical analysis provided in previous Sections. All cases correspond to $N = 64$ and uncoded 16-QAM. For frequency flat and selective scenarios a block fading channel was assumed. In other words an independent channel realization was generated per transmitted OFDM symbol. A four path uniform power delay profile was assumed for the frequency selective channel and $P = 4$. Tap spacing is in integer multiples of OFDM sample spacing.

Fig. 3 shows the simulated and the theoretical BER performance predictions for three different normalized CFO values in the AWGN channel. The theoretical plots are due to the simple Gaussian analysis and the two term approximation. The modified two term approximation with the tuning parameter k_1 provides results very close to the simulated plots in all cases. The theoretical Gaussian and simulated curves differ substantially at high E_b/N_0. The Gaussian approach only produces accurate results roughly in the 0 dB $< E_b/N_0 <$ 12 dB region for all three CFOs. In this region AWGN is dominant over the ICI and governs the error performance. However at high E_b/N_0, the AWGN becomes insignificant and ICI determines the error rate. An error floor is seen for the theoretical estimates due to the Gaussian analysis indicating that the ICI contribution is overestimated.

Fig. 4 illustrates the theoretical and simulated SER performance in the frequency flat fading channel. Note that in this case the predicted results from theory with ICI modeled as a Gaussian random variable are closer to the actual values (compared to the AWGN channel). In fading channels one has to carefully weigh the joints effects of fading and interference to calculate the error. The error performance of an OFDM

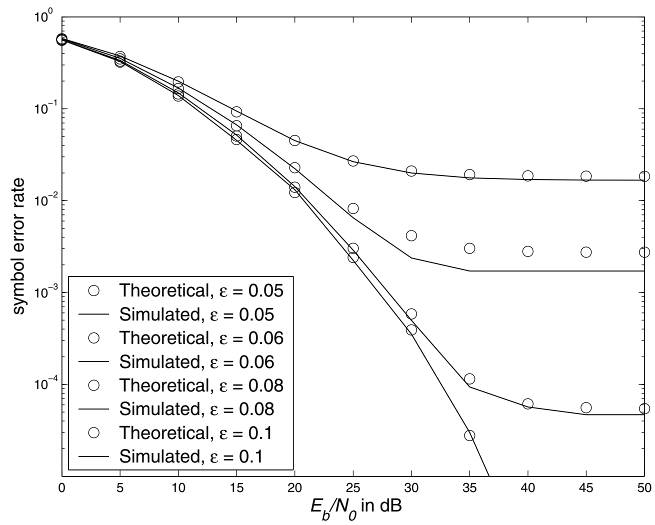

Fig. 4. Theoretical and simulated SER against E_b/N_0 in the frequency flat Rayleigh fading channel. 16-QAM. CPE is corrected.

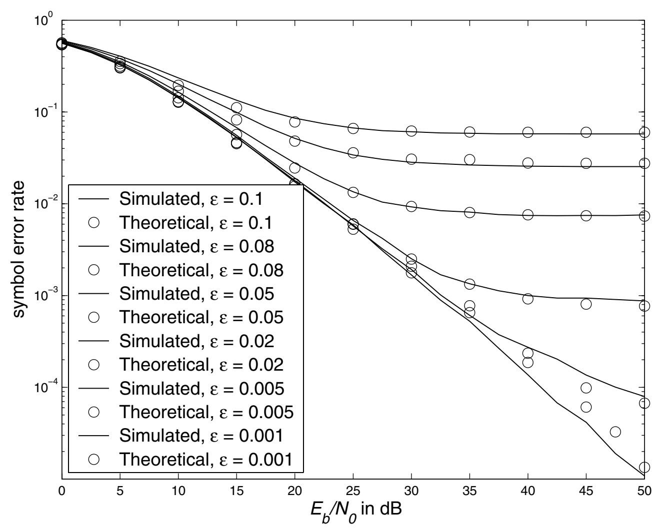

Fig. 5. Theoretical and simulated SER against E_b/N_0 in the 4-tap frequency selective fading channel. 16-QAM and CPE is corrected.

system is mainly influenced by bad subcarriers subjected to deep fading [18]. For these subcarriers, the channel faded effective ICI is also small.

Fig. 5 shows the theoretical and Simulated SER results in the four tap frequency selective fading channel. Again the theoretical results closely agree with the simulated SERs for different values of the normalized CFO.

VI. Conclusions

This paper investigated the SER of OFDM with CFO in AWGN and multipath fading channels. So far the majority of papers have assumed a Gaussian distributed ICI. The validity of such an assumption can be tested by using statistical measures such as the pdf and Kurtosis excess. In fact, the exact ICI distribution deviate from Gaussian characteristics for several normalized values of CFO.

We have shown that the predicted SER due to a Gaussian ICI assumption is not accurate in an AWGN channel. An analysis that incorporates the significant adjacent channel ICI effects can be used to predict accurate theoretical results. However in a fading channel considering the channel correlations between the OFDM signal and the ICI, a Gaussian ICI assumption initiated theoretical analysis is able to predict accurate results. The OFDM subcarriers in deep fades those govern the SER are less affected by the ICI.

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This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2006 proceedings.