

Math 2471 - Calc 3 - HW12

1. Are the following fields conservative?

$$(i) \quad \vec{F} = \langle 9x^2y^2, 6x^3y - 1 \rangle$$

$$(ii) \quad \vec{F} = \langle 3x^2yz, x^3z + 2y, x^3y - 3 \rangle$$

If so, evaluate the following

$$(i) \quad \int_c 9x^2y^2dx + (6x^3y - 1)dy \text{ along } y = x^2 \text{ from } x = 0 \text{ to } x = 1.$$

$$(ii) \quad \int_c 3x^2yzdx + (x^3z + 2y)dy + (x^3y - 3)dz \text{ from } P(1, 1, 0) \text{ to } Q(2, -1, 1).$$

2. Verify Green's Theorem

$$\oint_c Pdx + Qdy = \iint_R (Q_x - P_y) dydx$$

for the following:

$$\vec{F} = \langle y^2, x^2 \rangle \text{ where } c \text{ is the square with vertices } (0, 0), (1, 0), (1, 1), (0, 1)$$

$$\vec{F} = \langle 2xy, x + y \rangle \text{ where } c \text{ is the curve } y = 0, y = 1 - x^2$$

3. Evaluate the following surface integrals

$$(i) \quad \iint_S (x + 2y + z)dS, \quad S : z = 2 - x - y, x, y, z \geq 0$$

$$(ii) \quad \iint_S (x^2 + y^2 + z^2)dS, \quad S : z = x + y, x^2 + y^2 \leq 1$$

4. Evaluate the flux integral

$$\iint_S \vec{F} \cdot \hat{n}dS$$

where

$$(i) \quad \vec{F} = \langle 3z, -4, y \rangle \quad S : z = 1 - x - y, x, y, z \geq 0.$$

$$(ii) \quad \vec{F} = \langle x, y, z \rangle, \quad S : z = 1 - x^2 - y^2, z \geq 0$$

Ans

1. (i) 2 (ii) -11

2. (i) 0 (ii) 4/3

3. (i) 16/3 (ii) $\sqrt{3}\pi$

4. (i) -4/3 (ii) $3\pi/2$