

# Simulation of Hygrothermal Effects on Laminated Composite Plates Using Classical Laminated Plate Theory (CLPT)

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**Abstract-** Fiber reinforced composites offer light weight and high stiffness which attracts largely by the aerospace industry for structural applications. In particular, carbon composites are extensively used in space applications where dimensional stability is a prime requirement. Most of the structural composites are made of with polymers as matrix material. Composite structures may be exposed to humidity, water immersion, salt spray, jet fuel, hydraulic fluid, stack gas (includes sulfur dioxide), and the combined effects temperature and moisture for a long time which reduces the strength and stiffness of the structure. In the design of composites, it is important to consider the effects of moisture absorption and temperature distribution, since polymers are more vulnerable to environment. Therefore a detailed study is necessary to study environmental effects in design of composites. An attempt is made to study the environmental effects using classical laminate plate theory (CLPT). In the CLPT, the peel stresses and transverse shear stress are not considered as the laminate thickness is very small compared to other dimensions of the plate. A systematic procedure is developed for the analysis of laminated composite plates using CLPT. The bending behavior is analyzed for simply supported plates subjected to sinusoidal load. The computed numerical results are compared with the published results the accuracy of procedure. Having tested the procedure, the procedure is extended further to include the stress components which are not considered in CLPT. The procedure is known to be the first order shear deformation theory (FSDT). To study the effects of hygrothermal critically it is necessary to evaluate all the stress components. Once again the procedure is being tested for the accuracy with the published results. Tsai-Wu failure criterion is employed to do study the first ply failure analysis. The developed procedure is implemented in MATLAB. Transverse deflections, stresses and first ply failure loads are computed when a simply supported plate is subjected to Mechanical, hygrothermal, and both mechanical and hygrothermal loads.

**Keywords-** CLPT, FSDT.

## I. INTRODUCTION

A composite is structural material consists of two or more combined constituents that are combined at macroscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. It is well known that composite materials are being widely used in recent years in such diverse industries as aero-

space, chemical, automobile, ship building and construction. Very often structures made of composite materials are expected to perform under widely varying temperature and moisture loadings. This has led to an increased in the hygrothermo-elastic response of composite structures in recent years.

### • FIRST ORDER SHEAR DEFORMATION THEORY

The classical laminate plate theory is based on the Kirchhoff assumptions, in which transverse normal and shear stresses are neglected. Although such stresses can be post computed through 3-D elasticity equilibrium equations, they are not always accurate. The equilibrium-derived transverse stress field is sufficiently accurate for homogeneous and thin plates; they are not accurate when plates are relatively thick (i.e.,  $a/h < 20$ ). In the first-order shear deformation theory (FSDT), a constant state of transverse shear stresses is accounted for, and often the transverse normal stress is neglected. The FSDT allows the computation of interlaminar shear stresses through constitutive equations, which is quite simpler than deriving them through equilibrium equations. It should be noted that the interlaminar stresses derived from constitutive equations do not match, in general, those derived from equilibrium equations. In fact, the transverse shear stresses derived from the equilibrium equations are quadratic through lamina thickness, as was shown in CLPT, whereas those computed from constitutive equations are constant. The more significant difference between the classical and first-order theories is the effect of including transverse shear deformation on the predicted deflections, frequencies, and buckling loads. As noted in the classical laminate theory under predicts deflections and over predicts frequencies as well as buckling loads with plate side-to-thickness ratios of the order of 20 or less. For this reason alone it is necessary to use the first-order theory in the analysis of relatively thick laminated plates.

In CLPT the peel stresses are zero. To overcome this disadvantage First Order Shear Deformation theory is formulated.

### • CLASSICAL LAMINATE PLATE THEORY

The classical laminated plate theory is an extension of the classical plate theory to composite laminates. In the classical laminated plate theory (CLPT) it is assumed that the Kirchhoff hypothesis holds:

An assumption is that which is necessary for the development of the mathematical model, whereas a restriction is not a necessary condition for the development of the theory.

(1) Straight lines perpendicular to the midsurface (i.e., transverse normals) before deformation remain straight after deformation.

(2) The transverse normals do not experience elongation (i.e., they are inextensible).

(3) The transverse normals rotate such that they remain perpendicular to the midsurface after deformation. The first two assumptions imply that the transverse displacement is independent of the transverse (or thickness) coordinate and the transverse normal strain  $\epsilon_{zz}$  is zero. The third assumption results in zero transverse shear strains,  $\epsilon_{xz} = 0$ ,  $\epsilon_{yz} = 0$ .

#### Boundary Conditions:

The following are the different types of simply supported boundary conditions on all four edges of the plate.

$$\text{BC1a: } u=w=\Phi_y=0 \text{ at } x=0, a \text{ and } v=w=\Phi_x=0 \text{ at } y=0, b$$

$$\text{BC1b: } v=w=\Phi_y=0 \text{ at } x=0, a \text{ and } u=w=\Phi_x=0 \text{ at } y=0, b$$

$$\text{BC1c: } u=v=w=\Phi_y=0 \text{ at } x=0, a \text{ and } u=v=w=\Phi_x=0$$

$$y=0, b.$$

## II. STRENGTH THEORIES AND HYGROTHERMAL EFFECTS

### Failure Criterion

#### Strength Ratio:

$$SR = \frac{\text{Maximum Load Which Can Be Applied}}{\text{Load Applied}}$$

### Tsai-Wu Failure Theory:

This theory is based on the total strain energy failure theory of Beltrami, According to this theory a lamina is to be failed if

$$f_1\sigma_1 + f_2\sigma_2 + f_6\sigma_{12} + f_{22}\sigma_{22} + f_{66}\sigma_{12}^2 + 2f_{12}\sigma_1\sigma_2 < 1 \quad (1)$$

Where

$$f_1 = (1/(F_1^T)_{ult}) - (1/F_1^C)_{ult},$$

$$f_{11} = (1/(F_1^T)_{ult})(F_1^C)_{ult}$$

$$f_2 = (1/(F_1^T)_{ult}) - (1/F_2^C)_{ult},$$

$$f_{22} = (1/(F_2^T)_{ult}) - (F_2^C)_{ult}$$

$$f_6 = 0, \quad f_{66} = (1/(F_{12}^T)_{ult})^2$$

$$f_{12} = \frac{1}{2} \sqrt{1/((F_1^T)_{ult}(F_1^C)_{ult}(F_2^T)_{ult}(\sigma_2^C)_{ult})}$$

### HYGROTHERMAL EFFECTS:

- The fabrication process of composite materials may introduce reversible and irreversible effects due to processing thermal cycle and chemical changes and due to the mismatch in thermal properties of the constituents. The most common manifestation of these effects are residual stresses and warpage.
- After fabrication, composite structures operate in a variety of thermal and moisture environments that may have a pronounced impact on their performance. These hygrothermal effects are a result of the temperature and mois-

ture content variations and are related to the difference in thermal and hygric properties of the constituents.

- Processing and environmental effects are similar in nature. They can be viewed and analyzed from the microscopic point of view, on the scale of the fiber diameter, or from the macroscopic point of view, by considering the overall effects on the lamina, which is treated as a homogeneous material.
- Analysis of the processing and hygrothermal effects is an important component of the overall structural design and analysis. The performance of a composite structure is a function of its environmental history, temperature and moisture distributions, processing and hygrothermal stresses, and property variations with temperature and moisture.

Hygrothermal effects can be categorized as follows:

#### Physical and Chemical Effects:

Moisture absorption and desorption process in polymer/matrix composites depend on the current hygrothermal state and on the environment. The glass transition temperature of the polymeric matrix varies with the moisture content. Polymeric processes are a function of the hygrothermal properties of the constituent materials and the composite and the current hygrothermal state. Material degradation and corrosion can be related to hygrothermal factors.

#### Effects on Mechanical Properties:

Elastic and viscoelastic properties may vary with temperature and moisture concentration. Failure and strength characteristics, especially interfacial and matrix-dominated ones, may vary with temperature and moisture content.

#### Hygrothermoelastic (HTE) Effects:

The composite material undergoes reversible deformations related to thermal expansion ( $\alpha$ ) and moisture expansion ( $\beta$ ) coefficients. Intralaminar and interlaminar stresses are developed as a result of the thermoelastic and hygroelastic inhomogeneity and anisotropy of the material.

#### Hygrothermal Effects on Mechanical Behavior:

The hygrothermal state affects the stress-strain behavior of composite materials in two ways; the properties of the constituents may vary with temperature and moisture concentration, and fabrication residual stresses are altered by the hygrothermal state. Since the fibers are usually the least sensitive to environment, hygrothermal effects are most noticeable in matrix dominated properties, e.g., transverse tensile, transverse compressive and in-plane shear properties. The influence of moisture concentration is similar to that of temperature on polymer matrix composites, and it is more pronounced at elevated temperatures.

#### Hygrothermal Strains in Unidirectional Lamina:

A lamina undergoes hygrothermal deformation when subjected to a uniform change in temperature  $\Delta T = T - T_0$  and uniform change in moisture concentration  $\Delta c = c - c_0$ , where  $(T_0, c_0)$  is a reference hygrothermal state. Assuming the thermal and moisture deformations to be uncoupled and the thermal and moisture expansion coefficients to be constant (which is a good approximation for most composites under normal Service

conditions), the hygrothermal strains referred to the principal material axes of the lamina are

**Stress Analysis and Safety Factors for First-Ply Failure of Symmetric Laminates:**

For the FPF approach the selected failure criterion is applied to the state of each layer separately. Thus, for a state of stress  $(\sigma_1, \sigma_2, \tau_6)_k$  in layer k, the state of stress at Failure of the same layer is  $S_{fk} (\sigma_1, \sigma_2, \tau_6)$ , where  $S_{fk}$  is the safety factor for layer k.

According to Tsai-Wu criterion:  $a S_{fk}^2 + b S_{fk} + c = 0$

Where  $a = f_{11} (\sigma_1^2)_k + f_{22} (\sigma_2^2)_k + f_{66} (\tau_6^2)_k + 2f_{12} (\sigma_1 \sigma_2)_k$ ;  
 $b = f_{11} (\sigma_1)_{1e} + f_{22} (\sigma_2)_{1e} + 2f_{11} (\sigma_1 \sigma_{1e})_k + 2f_{22} (\sigma_2 \sigma_{2e})_k + 2f_{66} (\tau_6 \tau_{6e})_k + 2f_{12} (\sigma_1 \sigma_{1e} + \sigma_2 \sigma_{2e})_k$ ;  
 $c = f_{11} (\sigma_1^e)_{1e} + f_{22} (\sigma_2^e)_{1e} + f_{11} (\sigma_{1e}^2)_k + f_{22} (\sigma_{2e}^2)_k + 2f_{66} (\tau_{6e}^2)_k + 2f_{12} (\sigma_{1e} \sigma_{2e})_k - 1$ .

$$S_{fka} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{2}$$

$$\& S_{fkr} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{3}$$

Where  $S_{fka}$  is the safety factor of layer k for the actual state of stress  $(\sigma_1, \sigma_2, \tau_6)_k$  and  $S_{fkr}$  is the safety factor of the same layer k for a state of stress with reversed sign, i.e.,  $(-\sigma_1, -\sigma_2, -\tau_6)_k$ . The procedure above is carried out repeatedly for all layers of the laminate to find the minimum values of  $S_{fka}, S_{fkr}$ . These minimum values are the safety factors of the Laminate based on the FPF approach, for the actual and reverse loadings. Thus

$$\bar{S}_{fa} = (S_{fka})_{min}, \quad \bar{S}_{fr} = (S_{fkr})_{min}$$

The axial tensile and compressive strengths of the laminate are obtained as

$$\bar{F}_{xt} = (S_{fka})_{min}, \quad \bar{F}_{xc} = (S_{fkr})_{min}$$

**The deflection of a plate subjected to temperature:**

The temperature variation in the shell structure is either uniform everywhere, i.e.,  $T(x,y,z)=T_0$ , or linearly varying across the thickness, i.e.,  $T(x, y, z)=T_1 z/h$ . The nondimensionalized deflection values are given as

$$w = h w / \alpha_L T_1 a^2 \quad (\text{or}) \quad w^* = h w / \alpha_L T_0 a^2$$

where

$\alpha_L$  is the longitudinal coefficient of thermal expansion, h is the laminate thickness, and a is the length of the plate.

**III. MATLAB**

The name MATLAB stands for MATrix LABoratory. MATLAB was written originally to provide easy access to matrix software developed by the LINPACK (linear system package) and EISPACK (Eigen system package) projects. MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming environment. Furthermore, MATLAB is a modern programming language environment: it has sophisticated data structures, contains built-in editing and debugging tools, and supports object-oriented programming. These factors make MATLAB an excellent tool for teaching and research. MATLAB has many advantages compared to conventional computer languages (e.g., C, FORTRAN) for solving technical

problems. MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. The software package has been commercially available since 1984 and is now considered as a standard tool at most universities and industries worldwide. It has powerful built-in routines that enable a very wide variety of computations. It also has easy to use graphics commands that make the visualization of results immediately available. Specific applications are collected in packages referred to as toolbox. There are toolboxes for signal processing, symbolic computation, control theory, simulation, optimization, and several other fields of applied science and engineering.

**IV. RESULTS**

The following properties are used in the analysis [14]:

$$E_1/E_2=25, G_{12}=G_{13}=0.5E_2, \nu_{12}=0.25, \alpha_1=3\alpha_2$$

Table 4.1 Non dimensional transverse deflections and stresses of simply supported plate subjected to Sinusoidal loading.

Lami-nate	Source	$\hat{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xz}$
0°	Refer-ence [14]	0.4312	0.5387	0.0267	0.0213
	Present	0.4312 5	0.5387 0	0.0266 6	0.0212 8
(0°/90°/0°)	Refer-ence [14]	0.4312	0.5387	0.0267	0.0213
	Present	0.4312 5	0.5387 0	0.0266 6	0.0212 8

Table 4.2 Non dimensional transverse deflections and stresses of simply supported plate

a/h	Loa-d	Source	$\hat{w} * 10^2$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{xz}$
2	SSL	Refer-ence [14]	1.107 0	0.715 7	0.052 5	0.272 8
		Present	1.121 8	0.678 4	0.051 1	0.239 8
100	SSL	Refer-ence [14]	1.065 3	0.715 7	0.052 5	0.272 8
		Present	1.038 5	0.678 4	0.051 1	0.239 8
CLP T (a/b = 1)	SSL	Refer-ence [14]	1.063 6	0.715 7	0.052 5	-
		Present	1.683 5	0.678 4	0.051 1	-

The failure of a multidirectional laminated plate [0/90/0] subjected to mechanical loading.

The following properties are used in the analysis [18];

$$E_1=181GPa, E_2=10.3Gpa, G_{12}=7.17GPa, a/b = 1, a/h = 100.$$

Table 4.3.1 the layer safety factors at the midplane of each layer for mechanical loading only

Layer safety factor	Surface	0°	90°	0°
S <sub>fka</sub>	Middle	6.8318*10 <sup>5</sup>	1/0	1.0525*10 <sup>5</sup>
S <sub>fkr</sub>	Middle	1.0525*10 <sup>5</sup>	1/0	6.8318*10 <sup>5</sup>

Table 4.3.2 the laminated plate safety factors and the axial tensile and compressive strengths of a simply supported plate subjected to mechanical loading only

The failure of a multidirectional laminated plate [0/90/0] subjected to both mechanical and hygrothermal loading. The following properties are used in the analysis [18];  $E_1=181GPa$ ,  $E_2=10.3GPa$ ,  $G_{12}=7.17GPa$ ,  $a/b=1$ ,  $a/h=100$ ,  $\Delta T=45^\circ c$ ,  $\Delta c=0.01$ .

Table 4.3.3 the layer safety factors at the midplane of each layer for mechanical loading and hygrothermal loadings.

Layer safety factor	Surface	0°	90°	0°
S <sub>fka</sub>	Middle	1.0525*10 <sup>5</sup>	1/0	1.0525*10 <sup>5</sup>
S <sub>fkr</sub>	Middle	1.0525*10 <sup>5</sup>	1/0	1.0525*10 <sup>5</sup>

Table 4.3.4 the laminated plate safety factors and the axial tensile and compressive strengths for mechanical loading and hygrothermal loadings.

Laminate	$\bar{S}_{fa}$	$\bar{S}_{fr}$	$\bar{F}$	$\bar{F}_{xc}$
[0/90/0]	1.0525*10 <sup>5</sup>	1.0525*10 <sup>5</sup>	1.0525*10 <sup>5</sup>	1.0525*10 <sup>5</sup>

Table 4.4 Comparison of strength components of a [0/90/0] laminated plate subjected pure mechanical loading and combined mechanical and hygrothermal loading.

Mechanical loading		Mechanical+Hygrothermal loading	
$\bar{F}_{xt}$	$\bar{F}_{xc}$	$\bar{F}_{xt}$	$\bar{F}_{xc}$
1.0525*10 <sup>5</sup>	1.0525*10 <sup>5</sup>	1.0525*10 <sup>5</sup>	1.0525*10 <sup>5</sup>

The Hygrothermoelastic stresses of all the plies of [0/90] laminated plate subjected to a temperature change. The following properties are given for the analysis [18]:

$E_1=181GPa$ ,  $E_2=10.3GPa$ ,  $G_{12}=7.17GPa$ ,  $\nu_{12}=0.28$ ,  $\alpha_1=0.02\mu m/m^\circ c$ ,  $\alpha_2=22.5\mu m/m^\circ c$ ,  $\alpha_{12}=0$ ,  $\beta_1=0$ ,  $\beta_2=0.6m/m/kg/kg$ ,  $\beta_{12}=0$ ,  $\Delta T=45^\circ c$ ,  $\Delta c=0.01$ .

Table 4.5.1 The Hygrothermoelastic stresses at the Top of each ply of a simply supported plate

Ply	$\sigma_{1e}$	$\sigma_{2e}$	$\sigma_{12e}$
0°	-1.1069e+004	-0.1802e+004	0.0000
90°	0.3704e+003	-8.8971e+003	0.0000

Table 4.5.2 The Hygrothermoelastic stresses at the Bottom of each ply of a simply supported plate.

Ply	$\sigma_{1e}$	$\sigma_{2e}$	$\sigma_{12e}$
0°	8.8971e+003	-0.3704e+003	0.0000
90°	0.1802e+004	1.1069e+004	0.0000

The nondimensionalized central deflection ( $w^*$ ) values of

Laminate	$\bar{S}_{fa}$	$\bar{S}_{fr}$	$\bar{F}_{xt}$	$\bar{F}_{xc}$
[0/90/0]	1.0525*10 <sup>5</sup>	1.0525*10 <sup>5</sup>	1.0525*10 <sup>5</sup>	1.0525*10 <sup>5</sup>

four- layer simply supported plates (BC 1b) with uniform temperature field are presented in table 5.8. The generated results are found to be in good agreement with the published results. The following properties are given for the analysis [16]:

$E_1 = 53.8GPa$ ,  $E_2 = 17.9GPa$ ,  $G_{12} = G_{23} = G_{13} = 8.62GPa$ ,  $\nu_{12} = 0.25$ ,  $\alpha_1 = 6.3*10^{-6} m/m^\circ c$ ,  $\alpha_2 = 22.5*10^{-6} m/m^\circ c$ ,  $a/b=1$ ,  $a/h = 10$ ,  $T=T_0=45^\circ c$ .

Table 4.6 the non dimensionalized central deflection ( $w^*$ ) of simply supported plate subjected to uniform temperature field.

Laminate	[0/90/90/0]	[0/90/0/90]	[45/-45/-45/45]	[45/-45/45/-45]	[30/50/30/50]
Reference [16]	0.0	0.0	0.0	0.0	0.0
Present	7.5363*10 <sup>-8</sup>	7.5363*10 <sup>-8</sup>	6.012*10 <sup>-8</sup>	6.012*10 <sup>-8</sup>	6.29*10 <sup>-8</sup>

The Strength Ratio's of [0/90/0] laminate subjected to tensile normal load in the X-direction are presented in Table 4.7.1, 2,3.

The generated results are found to be in good agreement with the published results.

The following properties are given for the analysis [18]:  $E_1=181GPa$ ,  $E_2=10.3GPa$ ,  $G_{12}=7.17GPa$ ,  $\nu_{12}=0.28$  and thickness of each ply is 5mm,  $f_1=0 Pa^{-1}$ ,  $f_2=2.093*10^{-8} Pa^{-1}$ ,  $f_{11}=4.4444*10^{-19} Pa^{-2}$ ,  $f_{22}=1.0162*10^{-16} Pa^{-2}$ ,  $f_6=0 Pa^{-1}$ ,  $f_{66}=2.1626*10^{-16} Pa^{-2}$ ,  $f_{12}=3.360*10^{-18} Pa^{-2}$ .

Table 4.7.1 The Local Stresses (Pa) of each layer of the laminate [0/90/0]

Ply No.	Position	Source	$\sigma_1$	$\sigma_2$	$\tau_{12}$
1 (0°)	Top	Reference[18]	9.726*10 <sup>1</sup>	1.313*10 <sup>0</sup>	0
		Present	97.2639	1.3131	0
	Middle	Reference[18]	9.726*10 <sup>1</sup>	1.313*10 <sup>0</sup>	0
		Present	97.2639	1.3131	0

2 (90°)	Bot-tom	Refer-ence[18] Present	9.726*10 <sup>1</sup> 97.2639	1.313*10 <sup>0</sup> 1.3131	0. 0 0
	Top	Refer-ence[18] Present	-2.626*10 <sup>0</sup> -2.6263	5.472*10 <sup>0</sup> 5.4721	0. 0 0
	Middle	Refer-ence[18] Present	-2.626*10 <sup>0</sup> -2.6263	5.472*10 <sup>0</sup> 5.4721	0. 0 0
3) (0°)	Top	Refer-ence[18] Present	9.726*10 <sup>1</sup> 97.2639	1.313*10 <sup>0</sup> 1.3131	0. 0 0
	Middle	Refer-ence[18] Present	9.726*10 <sup>1</sup> 97.2639	1.313*10 <sup>0</sup> 1.3131	0. 0 0
	Bot-tom	Refer-ence[18] Present	9.726*10 <sup>1</sup> 97.2639	1.313*10 <sup>0</sup> 1.3131	0. 0 0

3(0°)	Mid-dle	Refer-ence[18] Present	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	0. 0 0
	Bot-tom	Refer-ence[18] Present	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	0. 0 0
	Top	Refer-ence[18] Present	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	0. 0 0
3(0°)	Mid-dle	Refer-ence[18] Present	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	0. 0 0
	Bot-tom	Refer-ence[18] Present	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	0. 0 0
	Top	Refer-ence[18] Present	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	0. 0 0

Table 4.7.2 The Local Strains of each layer of the laminate [0/90/0]

Ply No.	Posi-tion	Source	$\epsilon_1$	$\epsilon_2$	$\gamma_{12}$
1(0°)	Top	Refer-ence[18] Present	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	0. 0 0
	Mid-dle	Refer-ence[18] Present	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	0. 0 0
	Bot-tom	Refer-ence[18] Present	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	0. 0 0
2 (90°)	Top	Refer-ence[18] Present	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	0. 0 0
	Middle	Refer-ence[18] Present	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	0. 0 0
	Bot-tom	Refer-ence[18] Present	-2.297*10 <sup>-11</sup> -0.023*10 <sup>-9</sup>	5.353*10 <sup>-10</sup> 0.5353*10 <sup>-9</sup>	0. 0 0

Table 4.7.3 The Strength Ratios of each layer of the laminate [0/90/0]

Ply No.	Source	S.R. at Top	S.R. at Middle	S.R. at Bottom
1(0°)	Refer-ence[18]	1.339*10 <sup>7</sup>	1.339*10 <sup>7</sup>	1.339*10 <sup>7</sup>
	Present	1.3395*10 <sup>7</sup>	1.3395*10 <sup>7</sup>	1.3395*10 <sup>7</sup>
2(90°)	Refer-ence[18]	7.2799*10 <sup>6</sup>	7.2799*10 <sup>6</sup>	7.2799*10 <sup>6</sup>
	Present	7.2773*10 <sup>6</sup>	7.2773*10 <sup>6</sup>	7.2773*10 <sup>6</sup>
3 (0°)	Refer-ence[18]	1.339*10 <sup>7</sup>	1.339*10 <sup>7</sup>	1.339*10 <sup>7</sup>
	Present	1.3395*10 <sup>7</sup>	1.3395*10 <sup>7</sup>	1.3395*10 <sup>7</sup>

The global strains and stresses of all the plies of [0/90] graphite/epoxy laminate subjected to a temperature change are presented in Table 4.8.1, 2. The generated results are found to be in good agreement with the published results.

The following properties are given for the analysis [18]:

$$E_1=181\text{GPa}, E_2=10.3\text{GPa}, G_{12}=7.17\text{GPa}, \nu_{12}=0.28, \alpha_1=0.02\text{m/m/}^\circ\text{C}, \alpha_2=22.5\text{m/m/}^\circ\text{C}, \Delta T= -75^\circ\text{C}.$$

Table 4.8.1 The Global Strains of each layer of a laminate.

Ply No.	Posi-tion	Source	$\epsilon_x$	$\epsilon_y$	$\gamma_{xy}$
	Top	Refer-	2.475*10	-	0.0

1(0°)		ence[18] Present	<sup>-4</sup> 0.0002	<sup>-3</sup> 1.029*10 -0.0010	0 0
	Mid- dle	Refer- ence[18] Present	- <sup>-5</sup> 7.160*10 - 0.0716*1 <sup>-3</sup>	- <sup>-4</sup> 7.098*10 - 0.7098*1 <sup>-3</sup>	0.0 0
	Bot- tom	Refer- ence[18] Present	- <sup>-4</sup> 3.907*10 - 0.3907*1 <sup>-3</sup>	- <sup>-4</sup> 3.907*10 - 0.3907*1 <sup>-3</sup>	0.0 0
2(90°)	Top	Refer- ence[18] Present	- <sup>-4</sup> 3.907*10 - 0.3907*1 <sup>-3</sup>	- <sup>-4</sup> 3.907*10 - 0.3907*1 <sup>-3</sup>	0.0 0
	Mid- dle	Refer- ence[18] Present	- <sup>-4</sup> 7.098*10 - 0.7098*1 <sup>-3</sup>	- <sup>-5</sup> 7.160*10 - 0.0716*1 <sup>-3</sup>	0.0 0
	Bot- tom	Refer- ence[18] Present	- <sup>-3</sup> 1.029*10 -0.0010	<sup>-4</sup> 2.475*10 0.0002	0.0 0

				<sup>07</sup>	
Mid- dle	Refer- ence[18] Present	<sup>6</sup> 9.912*10 9.9120*1 <sup>06</sup>	- <sup>6</sup> 9.912*10 - 9.9120*1 <sup>06</sup>	0. 0 0	
Bot- tom	Refer- ence[18] Present	<sup>6</sup> 7.535*10 0.7535*1 <sup>07</sup>	<sup>7</sup> 4.718*10 4.7183*1 <sup>07</sup>	0. 0 0	

V. CONCLUSIONS

Graphite composites have exceptional mechanical properties. These materials are strong, stiff, and lightweight. Graphite composite is the material of choice for applications where lightweight & superior performance is important, such as components for spacecraft, fighter aircrafts, and racecars.

The three greatest advantages of graphite composites are:

- High specific stiffness (stiffness divided by density)
  - High specific strength (strength divided by density)
  - Extremely low coefficient of thermal expansion (CTE)
1. The nondimensionalized transverse deflections and stresses in specially orthotropic square plates subjected to various types of mechanical loadings under Classical Lamination Plate Theory are obtained with negligible errors.
  2. The nondimensionalized maximum transverse deflections and stresses of simply supported antisymmetric cross-ply square plates using First Order Shear Deformation Theory are obtained with in 0 and 2% errors
  3. The failure of multidirectional laminated plates subjected to purely mechanical and combined mechanical and hygro-thermal loadings are presented in this work are validated.
  4. The nondimensionalized central deflection values of four layer plates with uniform temperature field for one boundary condition are obtained with negligible error.
  5. The residual stresses at the top, middle, bottom surfaces of all the plies of [0/90] laminate subjected to temperature change are obtained with negligible error.
  6. The hygrothermoelastic stresses are calculated.
  7. The strength ratios for all the plies of [0/90/0] laminate using Tsai-Wu failure theory are obtained with negligible errors.
- Tsai-Wu failure criterion is achieved in association with CLPT to predict the failure of a simply supported plate subjected to both mechanical and hygrothermal loads.

4.8.2 The Global Stresses (Pa) of each layer of a laminate.

Ply No.	Position	Source	$\sigma_x$	$\sigma_y$	$\tau_{xy}$
1(0°)	Top	Refer- ence[18] Present	<sup>7</sup> 4.718*10 4.7183*1 <sup>07</sup>	<sup>6</sup> 7.535*10 0.7535*1 <sup>07</sup>	0. 0 0
	Mid- dle	Refer- ence[18] Present	- <sup>6</sup> 9.912*10 - 9.9120*1 <sup>06</sup>	<sup>6</sup> 9.912*10 9.9120*1 <sup>06</sup>	0. 0 0
	Bot- tom	Refer- ence[18] Present	- <sup>7</sup> 6.701*10 - 6.7007*1 <sup>07</sup>	<sup>7</sup> 1.229*10 1.2289*1 <sup>07</sup>	0. 0 0
2(90°)	Top	Refer- ence[18] Present	<sup>7</sup> 1.229*10 1.2289*1 <sup>07</sup>	- <sup>7</sup> 6.701*10 - 6.7007*1 <sup>07</sup>	0. 0 0

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