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Structurally constrained 3D anisotropic inversion of marine CSEM data using a crossgradient approach

Max A. Meju[1] and Aslam B A Fatah[1]
[1]PETRONAS Upstream, Kuala Lumpur

SUMMARY

Marine controlled-source electromagnetic (CSEM) methods now find applications in integrated exploration for oil, gas and hydrates, but robust 3D geological interpretation of the field data faces major challenges in complex geological terrains. Where electrical anisotropy is known or assumed to be present, the challenge is how to reduce the non-uniqueness in inversion while resolving the unknown horizontal and vertical resistivities (ρ_h and ρ_v) with limited data. The typically reconstructed ρ_h and ρ_v models often have conflicting depth structures, crossing major formational boundaries and hence are difficult to explain in terms of subsurface geology. In integrated hydrocarbon exploration campaigns, it is highly desirable to reduce ambiguity in depth interpretation of ρ_h and ρ_v models and also achieve comparability with other collocated subsurface models. Here, we propose that an objective structure-guided inversion can be achieved by requiring in the inverse problem formulation that the cross-product of the gradient of horizontal resistivity and the gradient of the vertical resistivity is equal to zero at significant geological boundaries. This will result in ρ_h and ρ_v models that are structurally similar in depth and thus reduce interpretational ambiguity as well as facilitate comparability with other geophysical results. We provide a mathematical solution to the cross-gradient coupled 3D anisotropic inverse problem. The use of a priori information and structural constraints to resolve interpretational non-uniqueness differentiates our formulation from the conventional regularised 3D CSEM inversion approaches.

Keywords: Marine CSEM, electrical anisotropy, 3D resistivity inversion, cross-gradient constraint

INTRODUCTION

The focus of oil and gas exploration has now moved into frontier regions where structural complexity, heterogeneous overburden and hydrocarbons system fundamentals are significant challenges. Marine controlled-source electromagnetic (CSEM) methods find routine applications in such environments (e.g., Gabrielsen et al., 2013; Zhdanov et al., 2014; McKay et al., 2015). These methods have the potential to increase the probability of finding hydrocarbons and to reduce the risk of drilling dry wells in promising prospects previously identified by seismic and surface geochemical exploration. Three-dimensional (3D) CSEM surveying (Fig 1) and anisotropic inversion are emerging as the techniques of choice in such frontier regions. The challenge in 3D CSEM inversion is to derive true formation resistivities unbiased by heterogeneous overburden or formation anisotropy so that the usual follow-up estimation of hydrocarbon or water saturation (e.g. Gao et al., 2012; Liang et al., 2016) accurately represents the actual fluid content of the reservoir. This is a difficult task when the geology is complex.

Heterogeneous overburden causes undesirable geological noise in electromagnetic field measurements and rock formations may be electrically anisotropic at a scale influencing CSEM measurements (e.g., Tompkins et al., 2004; Newman et al., 2010; Sasaki, 2011; Davydycheva & Frenkel, 2013; Zhu et al., 2013; Gribenko & Zhdanov, 2014). Where electrical macro-anisotropy is known or assumed to be present, the challenge is how to reduce the non-uniqueness in inversion while resolving the unknown horizontal and vertical resistivities (ρ_h and ρ_v) with incomplete, insufficient or inconsistent data. For the most common type of macro-anisotropy in sedimentary basin exploration (transverse isotropy with vertical symmetry), Sasaki (2013) showed that ρ_h and ρ_v sensitivities are decoupled and that the resolution of anisotropic 3D targets is significantly improved by imposing equality constraint on the horizontal and vertical resistivities in 3D anisotropic inversion, instead of the inequality constraint previously suggested by Newman et al. (2010, equations 1 & 2), but there is no guarantee that the recovered resistivity values are physically correct, especially given the possible compensating relationships between the two resistivities in inversion. The typically reconstructed ρ_h and ρ_v models often have conflicting depth structures that are difficult to explain in terms of subsurface geology. It is thus practically expedient to reduce subjectivity in depth interpretation of ρ_h and ρ_v models and also achieve comparability with other subsurface models in hydrocarbon exploration. This paper presents a new theoretical framework for achieving this goal in 3D anisotropic inversion.

Geological intuition suggests that the level and form of anisotropy would vary from one rock unit to another but a given reservoir or rock mass would have the same boundary for ρ_h and ρ_v. Prospective hydrocarbon-bearing finite sedimentary reservoirs are stratigraphically bound or structure-conformable, even where tilted transverse isotropy (Davydycheva & Frenkel, 2013) is present (Fig 1). The presence of tilted or folded geological structures inferred from collocated seismic data can be accommodated as a priori structural information in the initial model for CSEM inversion ($\mathbf{m}_{ref}$), but the key operational question here is: what can be done to assure structural robustness in areas of poor-quality seismic data or frontier regions where there are no pre-existing wells**?** Brown et al. (2012) suggest from forward modelling that the resolution of reservoir level anisotropy for 3D targets will require good a priori knowledge of the background sediment resistivity and structural boundaries. Objective inverse modelling is required in conventional exploration for hydrocarbons where large data volumes are involved and (drill or drop) decisions have to be made on a short time frame. We propose in this paper that structural requirements can be satisfied by use of an inverse problem formulation incorporating geologically appropriate structural constraints in 3D anisotropic inversion. A new algorithm is presented here for 3D inversion of CSEM data using objective structural constraints to reduce the current ambiguity in the interpretation of ρ_h and ρ_v models.

METHODOLOGY FOR 3D ANISOTROPIC RESISTIVITY INVERSION WITH CROSS-GRADIENT CONSTRAINT

The challenge here is to develop a fit-for-purpose structure-coupled 3D anisotropic resistivity inversion method for CSEM data so as to reduce uncertainty in subsurface resistivity characterisation. A well-known approach to improve structural similarity between different coincidentally-located models of the subsurface and the accuracy of their actual physical property estimates is the cross-gradient coupled joint inversion method (Gallardo & Meju, 2003, 2004). Let the model parameters be $\mathbf{m}_h$ and $\mathbf{m}_v$, which are the logarithm of ρ_h and ρ_v respectively. An objective structure-guided inversion can be achieved by requiring in the problem formulation that the cross-product of the gradient of horizontal resistivity ($\nabla\mathbf{m}_h$) and the gradient of the vertical resistivity ($\nabla\mathbf{m}_v$) is equal to zero at significant geological boundaries. We minimize the function

$$U(\mathbf{m}_h,\mathbf{m}_v) = \left\|\mathbf{W}\left[\mathbf{d}_o - f(\mathbf{m}_h,\mathbf{m}_v)\right]\right\|^2 + \alpha^2\left\|\mathrm{R}\mathbf{m}_h\right\|^2 + \alpha^2\left\|\mathrm{R}\mathbf{m}_v\right\|^2 + C_m^{-1}\left\|(\mathbf{m}_h - \mathbf{m}_{ref,h})\right\|^2 + C_m^{-1}\left\|(\mathbf{m}_v - \mathbf{m}_{ref,v})\right\|^2 \qquad (1)$$

subject to the condition

$$\tau(x,y,z) = \nabla\mathbf{m}_h(x,y,z) \times \nabla\mathbf{m}_v(x,y,z) = 0 \qquad (2)$$

and fit the observed data ($\mathbf{d}_o$) to a specified tolerance, q_{ls}. Here, $\mathbf{d}_o$ are the conventional in-line and broadside multi-offset and multi-frequency CSEM data which can be the real and imaginary parts of the recorded electric and magnetic fields or their corresponding geometry-normalised apparent resistivities ρ_a and phases (Meju, 2013). **W** is a diagonal matrix whose elements are the reciprocals of the standard deviations of the noise in the field data. The functionals $f(\mathbf{m}_h,\mathbf{m}_v)$ are the CSEM data predicted by 3D forward theory. The roughness matrix **R** is a second-order finite-difference operator. $\boldsymbol{C}_m$ is the model covariance matrix. $\mathbf{m}_{ref}$ and C_m^{-1} serve to honour the trusted *a priori* information either explicitly or otherwise and, together with **Rm**, also stabilize the inversion process in zones of poor data coverage, while α is an enforcement factor. Initially $\mathbf{m}_{ref,h}$ is set equal to $\mathbf{m}_{ref,v}$ but both can take on different values where resistivity well logs are available.

The inverse problem of 3D resistivity and depth determination is non-linear but nonlinearity can be reduced by incorporating realistic target depths (constrained by coincident seismic depth imaging) in $\mathbf{m}_{ref}$, and solving for refinements necessary to fit the CSEM data especially in areas not constrained *ab initio*. Linearizing about an initial model ($\mathbf{m}^0$), we obtain the equivalent of equation (1) as,

$$\begin{aligned} U(\mathbf{m}_h,\mathbf{m}_v) &= \left\|\mathbf{W}\left[\Delta\mathbf{d} - \mathbf{A}_h\Delta\mathbf{m}_h - \mathbf{A}_v\Delta\mathbf{m}_v\right]\right\|^2 \\ &+ \alpha^2\left\|\mathrm{R}(\mathbf{m}_h^0 + \Delta\mathbf{m}_h)\right\|^2 + \alpha^2\left\|\mathrm{R}(\mathbf{m}_v^0 + \Delta\mathbf{m}_v)\right\|^2 \\ &+ C_m^{-1}\left\|((\mathbf{m}_h^0 + \Delta\mathbf{m}_h) - \mathbf{m}_{ref,h})\right\|^2 \\ &+ C_m^{-1}\left\|((\mathbf{m}_v^0 + \Delta\mathbf{m}_v) - \mathbf{m}_{ref,v})\right\|^2 \end{aligned} \qquad (3)$$

where $\Delta\mathbf{d} = \mathbf{d}_0 - f(\mathbf{m}_h^0,\mathbf{m}_v^0)$, $\mathbf{A}_h$ and $\mathbf{A}_v$ are the matrix of partial derivatives with respect to the horizontal and vertical resistivities respectively which can be calculated using reciprocity principle, and $\Delta\mathbf{m}_{h,v}$ are the desired updates for $\mathbf{m}^0_{h,v}$.

Equation (2) can be linearised using Taylor series expansion noting that

$$\begin{aligned} \nabla\mathbf{m}_h \times \nabla\mathbf{m}_v &\cong \nabla\mathbf{m}_h^0 \times \nabla\mathbf{m}_v^0 + [\nabla\mathbf{m}_h^0 \times B_v + B_h \times \nabla\mathbf{m}_v^0]\Delta\mathbf{m}_{h,v} \\ &\cong \tau_0 + [b_h(\mathbf{m}_h - \mathbf{m}_h^0) + b_v(\mathbf{m}_v - \mathbf{m}_v^0)] = 0 \end{aligned} \qquad (4)$$

where $B_h = \frac{d(\nabla\mathbf{m}_h^0)}{d\mathbf{m}_h}$ and $B_v = \frac{d(\nabla\mathbf{m}_v^0)}{d\mathbf{m}_v}$ while $b_h = B_h \times \nabla\mathbf{m}_v^0$ and $b_v = -B_v \times \nabla\mathbf{m}_h^0$ are the structural similarity projection vectors for both models. The cross-gradient constraint (4) reduces to $\tau_0 = -[b_h(\mathbf{m}_h - \mathbf{m}_h^0) + b_v(\mathbf{m}_v - \mathbf{m}_v^0)]$ and is added to the problem formulation (3). The solution to the inverse problem is then equivalent to obtaining the least-squares solution of the rectangular linear system of equations

$$\begin{bmatrix} \mathbf{WA}_h & \mathbf{WA}_v \\ \alpha \mathrm{R} & 0 \\ 0 & \alpha \mathrm{R} \\ C_m^{-0.5} & 0 \\ 0 & C_m^{-0.5} \\ b_h & b_v \end{bmatrix} \begin{Bmatrix} \Delta \mathbf{m}_h \\ \Delta \mathbf{m}_v \end{Bmatrix} = \begin{bmatrix} \mathbf{W}\Delta\mathbf{d} \\ -\alpha \mathrm{R}\mathbf{m}_h^0 \\ -\alpha \mathrm{R}\mathbf{m}_v^0 \\ -C_m^{-0.5}(\mathbf{m}_{ref,h} - \mathbf{m}_h^0) \\ -C_m^{-0.5}(\mathbf{m}_{ref,v} - \mathbf{m}_v^0) \\ -\tau_0 \end{bmatrix}, \quad (5)$$

which can be solved using Lagrange multiplier method as in Gallardo & Meju (2004). The desired structurally-coupled solution can be written in compact form as

$$\mathbf{m}_{h,v} = \mathbf{m}_{h,v}^0 + \left[A_{h,v}^T W^T W A_{h,v} + \alpha_{h,v}^2 \mathrm{R}^T \mathrm{R} + C_m^{-1}\right]^{-1} \left\{A_{h,v}^T W^T W \Delta d - \alpha_{h,v}^2 \mathrm{R}^T \mathrm{R}\mathbf{m}_{h,v}^0 - b_{h,v}\right\} \quad (6)$$

where $A_{h,y}$ = [$\mathbf{A}_h$: $\mathbf{A}_v$] and the other symbols are as previously defined. The components of the sensitivity matrix **A** are calculated using additional forward modelling and reciprocity principle as in Sasaki & Meju (2006a) and Sasaki (2013), whereby a fictitious source is placed at each receiver position following Newman & Alumbaugh (1995). Where only electric field data are being inverted, then an electric dipole is assumed to be located at each receiver position. In the case involving orthogonal components of electric and magnetic fields, both electric and magnetic dipole sources with two polarisation directions are assumed at each receiver position. In the iterative process, the rms value of the normalised residual of the fitted data ($\mathbf{d}_0$) is computed after the k[th] iteration as $q_{ls} = \sqrt{\frac{\| \mathbf{W}[\mathbf{d}_o - f(\mathbf{m}_h^k, \mathbf{m}_v^k)] \|^2}{n}}$, where there are n observed data and the updated model $\mathbf{m}^k$ is accepted if the value is close to unity. With the cross-gradient constraint, at each iteration step a line-search procedure is used to find the most feasible iterate having the minimum value of $\tau(\mathbf{m}_h,\mathbf{m}_v)$. Ideally, at the minimum, $\nabla \mathbf{m}_h^{k-1} \times \nabla \mathbf{m}_v^{k-1} = -[B_h \times \nabla \mathbf{m}_v^{k-1} - B_v \times \nabla \mathbf{m}_h^{k-1}]\Delta \mathbf{m}_{h,v}$ for the k[th] iteration.

DISCUSSION

Although significant progress has been made in 3D anisotropic CSEM inversion (e.g., Newman et al., 2010; Sasaki, 2013; Gribenko & Zhdanov, 2014), robust inversion in complex geological terrains is still a difficult proposition. It is proposed here that a necessary geological condition to satisfy is that the reconstructed horizontal and vertical resistivity models must have a common depth structure or volume. An objective formulation of this geological condition is presented based on the cross-gradient criterion emphasizing directional similarity. Since the cross product is related to the dot product, an alternative structural constraint to use for this purpose is

$$\|\nabla \mathbf{m}_h \times \nabla \mathbf{m}_v\|^2 = \|\nabla \mathbf{m}_h\|^2 \, \|\nabla \mathbf{m}_v\|^2 - (\nabla \mathbf{m}_h \cdot \nabla \mathbf{m}_v)^2, \quad (7)$$

which emphasises the magnitude of the cross product of the two property gradients. A comparable constraint was suggested for joint geophysical inversion by *Haber and Gazit* [2013]. The use of the structural constraints as proposed here should lead to horizontal and vertical resistivity models with similar geological structure and hence facilitate easier comparison or integration with other geophysical and geological models in conventional integrated subsurface exploration for hydrocarbons in frontier regions.

CONCLUSION

Robust interpretation of CSEM surveys requires accurate estimation of heterogeneous background resistivities and the lateral boundaries and burial depth of 3D resistive targets. Currently, the typically reconstructed ρ_h and ρ_v models often have conflicting depth structures that are difficult to explain in terms of subsurface geology. In the conventional integrated hydrocarbon exploration approach, it is highly desirable to reduce ambiguity or subjectivity in depth interpretation of ρ_h and ρ_v models and also achieve comparability with other coincidentally-located subsurface models. Geological intuition suggests that the level and form of anisotropy would vary from one rock unit to another but a given reservoir or anomalous rock mass would have the same boundary for ρ_h and ρ_v. This geological condition is met by incorporating an objective cross-gradient constraint in this new formulation of the 3D anisotropic inverse problem. The new inverse solution will yield models that have a common depth structure and hence reduce ambiguity in geological interpretation. The use of a priori information and structural constraints to resolve interpretational non-uniqueness differentiates our approach from the more conventional smoothness-constrained 3D electromagnetic inversion approaches. The structural-coupling procedure described here can also be applied to the inversion of land CSEM survey data or any other geophysical tensor property measurements.

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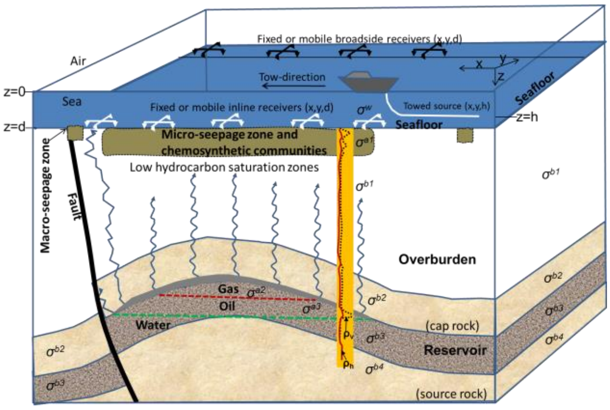

Figure 1. CSEM field set-up. A typical leaky structural trap is shown together with seepage-induced alterations. Localised 3D zones of chemosynthetic communities and patches of small-size bodies abound in the near-surface. Background (b) and anomalous (a) conductivity distribution is indicated. Inset is a hypothetical well log showing the expected depth variation in anisotropic resistivities (ρ_h and ρ_v).

REFERENCES

Brown, V., Hoversten M, Key K, and Chen J.S., 2012, Resolution of reservoir scale electrical anisotropy from marine CSEM data, *Geophysics*, **77**, E147-E158.

Davydycheva, S. & M.A. Frenkel, 2013. The impact of 3D tilted resistivity anisotropy on marine CSEM measurements. *The Leading Edge*, **32**, 1374-1381. Nov. 2013.

Gabrielsen, P.T., Abrahamson, P., Panzner, M., Fanavoll, S. & Ellingsrud, S. , 2013, Exploring frontier areas using 2D seismic and 3D CSEM data, as exemplified by multi-client data over the Skrugard and Havis discoveries in the Barents sea, *First Break*, **31**, 65-73.

Gallardo, L.A. & Meju, M.A., 2004. Joint two-dimensional dc resistivity and seismic travel-time inversion with cross-gradients constraints. *J. Geophys. Res.*, **109**, B03311, doi:10.1029/2003JB0022717.

Gallardo, L.A. and Meju, M.A., 2003. Characterisation of heterogeneous near-surface materials by joint 2D inversion of dc resistivity and seismic data. *Geophys. Res. Letters*, **30**, (13),1658-1661.

Gao, G., A. Abubakar and T.M. Habashy, 2012. Joint petrophysical inversion of electromagnetic and full-waveform seismic data. Geophysics, **77** (3), WA3-WA18. doi: 10.1190/geo2011-0157.1.

Gribenko, A. & M.S. Zhdanov, 2014. Anisotropic inversion of MCSEM data based on the integral equation method. SEG Denver 2014 Annual meeting, Expanded Abstract, pp.792- 797.

Haber, E., and M.H. Gazit, 2013, Model fusion and joint inversion, *Surv. Geophys*., **34**, 675- 695.

McKay, A., J. Mattsson and Z. Du, 2015. Towed streamer EM – reliable recovery of sub-surface resistivity. First Break, **33**, 75-85, April 2015.

Newman G. A., M. Commer & J.J. Carazzone, 2010, Imaging CSEM data in the presence of electrical anisotropy. *Geophysics* **75**, 51-61.

Newman G. A. & D.L. Alumbaugh, 1995, Frequency-domain modelling of airborne electromagnetic responses using staggered finite differences. *Geophysical Prosp.* **43**, 1021-1042.

Sasaki, Y., 2013. Anisotropic 3D inversion of marine CSEM data. 5th International Symposium on Three-Dimensional Electromagnetics, Sapporo, Japan, May 7-9, 2013, Expanded abstracts.

Sasaki, Y., 2011. Anisotropic, joint 3D inversion of marine CSEM and MT data. 81st Annual International Meeting, SEG, Expanded Abstracts, 589-593.

Sasaki, Y. & M.A. Meju, 2009, Some useful characteristics of shallow and deep marine CSEM responses inferred from 3D finite-difference modelling, *Geophysics*, **74**, F67-F76.

Sasaki, Y. & M.A. Meju, 2006b. A multidimensional horizontal-loop controlled-source electromagnetic inversion method and its use to characterize aquiferous fractured crystalline rocks. *Geophys. J. International*, **166**, 59-66.

Sasaki, Y. & M.A. Meju, 2006a, Three-dimensional joint inversion for magnetotelluric resistivity and static shift distributions in complex media, *J. Geophys. Research*, **111**, B05101, doi:10.1029/2005JB004009.

Tompkins, M., R. Weaver and L. MacGregor, 2004. Effects of vertical anisotropy on marine active source electromagnetic data and inversions: 66th Conference and Exhibition, EAGE, Extended Abstracts, E026.

Zhdanov, M.S., M. Endo, D. Yoon, M. Cuma, J. Mattsson and J. Midgley, 2014. Anisotropic 3D inversion of towed streamer electromagnetic data: Case study from the Troll West Oil Province. *Interpretation*, **2**(3), p.SH01-SH17.

Zhu, Y., X. Lu, S. Xu, E. Liu, M.A. Payne and C.X. Zhan, 2013. Method for generating anisotropic resistivity volumes from seismic and log data using a rock physics model. United States Patent US008473213B2, June 25, 2013.