Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Algebraic fractions

Exercise A, Question 1

Question:

Factorise completely

(a) $2x^3 - 13x^2 - 7x$

(b) $9x^2 - 16$

(c) $x^4 + 7x^2 - 8$

Solution:

(a)

$2x^3 - 13x^2 - 7x$ — x is a common factor

$= x(2x^2 - 13x - 7)$ — So take x outside the bracket.

$= x(2x^2 + x - 14x - 7)$ — For the quadratic, $ac = -14$ and $1 - 14 = -13 = b$

$= x[x(2x + 1) - 7(2x + 1)]$ — Factorise

$= x(2x + 1)(x - 7)$

(b)

$9x^2 - 16$ — This is a difference of two squares, $(3x)^2$ and 4^2

$= (3x)^2 - 4^2$

$= (3x + 4)(3x - 4)$ — Use $x^2 - y^2 = (x + y)(x - y)$

(c)

$x^4 + 7x^2 - 8$ — Let $y = x^2$

$= y^2 + 7y - 8$

$= y^2 - y + 8y - 8$ — $ac = -8$ and $-1 + 8 = +7 = b$

$= y(y - 1) + 8(y - 1)$ — Factorise

$= (y - 1)(y + 8)$

$= (x^2 - 1)(x^2 + 8)$ — Replace y by x^2

$= (x + 1)(x - 1)(x^2 + 8)$ — $x^2 - 1$ is a difference of two squares, so use $x^2 - y^2 = (x + y)(x - y)$

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Algebraic fractions
Exercise A, Question 2

Question:

Find the value of

(a) $81^{\frac{1}{2}}$

(b) $81^{\frac{3}{4}}$

(c) $81^{\frac{3}{4}}$.

Solution:

(a)

$81^{1/2}$ Use $a^{\frac{1}{m}} = \sqrt[m]{a}$, so $a^{\frac{1}{2}} = \sqrt{a}$

$= \sqrt{81}$

$= 9$

(b)

$81^{\frac{3}{4}}$ $a^{\frac{n}{m}} = \sqrt[m]{(a^n)}$ or $(\sqrt[m]{a})^n$

$= (\sqrt[4]{81})^3$ It is easier to find the fourth root, then cube this

$= 3^3$ $\sqrt[4]{81} = 3$ because $3 \times 3 \times 3 \times 3 = 81$

$= 27$

(c)

$81^{-\frac{3}{4}} = \frac{1}{81^{3/4}}$ Use $a^{-m} = \frac{1}{a^m}$

$= \frac{1}{27}$ Use the answer from part (b)

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Algebraic fractions
Exercise A, Question 3

Question:

(a) Write down the value of $8^{\frac{1}{3}}$.

(b) Find the value of $8^{-\frac{2}{3}}$.

Solution:

(a)

$8^{\frac{1}{3}}$
$= \sqrt[3]{8}$
$= 2$

Use $a^{\frac{1}{m}} = \sqrt[m]{a}$, so $a^{\frac{1}{3}} = \sqrt[3]{a}$

$\sqrt[3]{8} = 2$ because $2 \times 2 \times 2 = 8$

(b)

$8^{-\frac{2}{3}}$

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2$

First find $8^{\frac{2}{3}}$ $a^{\frac{n}{m}} = \sqrt[m]{(a^n)}$ or $(\sqrt[m]{a})^n$

$= 2^2 = 4$

$8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}}$

Use $a^{-m} = \frac{1}{a^m}$

$= \frac{1}{4}$

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Algebraic fractions
Exercise A, Question 4

Question:

(a) Find the value of $125^{\frac{4}{3}}$.

(b) Simplify $24x^2 \div 18x^{\frac{4}{3}}$.

Solution:

(a)

$125^{\frac{4}{3}}$ — $a^{\frac{n}{m}} = \sqrt[m]{(a^n)}$ or $(\sqrt[m]{a})^n$

$= (\sqrt[3]{125})^4$ — It is easier to find the cube root, then the fourth power

$\sqrt[3]{125} = 5$ because $5 \times 5 \times 5 = 125$

$= 5^4$

$= 625$

(b)

$24x^2 \div 18x^{\frac{4}{3}}$

$= \frac{24x^2}{18x^{\frac{4}{3}}} = \frac{4x^2}{3x^{\frac{4}{3}}}$ — Divide by 6

$= \frac{4x^{\frac{2}{3}}}{3}$ — Use $a^m \div a^n = a^{m-n}$

(or $\frac{4}{3}x^{\frac{2}{3}}$)

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Algebraic fractions
Exercise A, Question 5

Question:

(a) Express $\sqrt{80}$ in the form $a\sqrt{5}$, where a is an integer.

(b) Express $(4-\sqrt{5})^2$ in the form $b+c\sqrt{5}$, where b and c are integers.

Solution:

(a)

$\sqrt{80} = \sqrt{16} \times \sqrt{5}$ — Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

$= 4\sqrt{5} \quad (a=4)$

(b)

$(4-\sqrt{5})^2 = (4-\sqrt{5})(4-\sqrt{5})$

$= 4(4-\sqrt{5}) - \sqrt{5}(4-\sqrt{5})$ — Multiply the brackets.

$= 16 - 4\sqrt{5} - 4\sqrt{5} + 5$ — $\sqrt{5} \times \sqrt{5} = 5$

$= 21 - 8\sqrt{5}$

$(b = 21$ and $c = -8)$

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Algebraic fractions
Exercise A, Question 6

Question:

(a) Expand and simplify $(4+\sqrt{3})(4-\sqrt{3})$.

(b) Express $\frac{26}{4+\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are integers.

Solution:

(a)

$(4+\sqrt{3})(4-\sqrt{3})$

$=4(4-\sqrt{3})+\sqrt{3}(4-\sqrt{3})$ Multiply the brackets.

$=16-4\sqrt{3}+4\sqrt{3}-3$ $\sqrt{3}\times\sqrt{3}=3$

$=13$

(b)

$\frac{26}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}}$ To rationalise the denominator, multiply top and bottom by $4-\sqrt{3}$

$=\frac{26(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})}$

$=\frac{26(4-\sqrt{3})}{13}$ Use the answer from part (a)

$=2(4-\sqrt{3})$ Divide by 13

$=8-2\sqrt{3}$

$(a=8$ and $b=-2)$

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Algebraic fractions
Exercise A, Question 7

Question:

(a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(b) Express $(2-\sqrt{3})^2$ in the form $b+c\sqrt{3}$, where b and c are integers to be found.

Solution:

(a)

$\sqrt{108} = \sqrt{36} \times \sqrt{3}$ Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

$= 6\sqrt{3} \quad (a = 6)$

(b)

$(2-\sqrt{3})^2 = (2-\sqrt{3})(2-\sqrt{3})$

$= 2(2-\sqrt{3}) - \sqrt{3}(2-\sqrt{3})$ Multiply the brackets

$= 4 - 2\sqrt{3} - 2\sqrt{3} + 3$ $\sqrt{3} \times \sqrt{3} = 3$

$= 7 - 4\sqrt{3}$

$(b = 7$ and $c = -4)$

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Algebraic fractions
Exercise A, Question 8

Question:

(a) Express $(2\sqrt{7})^3$ in the form $a\sqrt{7}$, where a is an integer.

(b) Express $(8+\sqrt{7})(3-2\sqrt{7})$ in the form $b+c\sqrt{7}$, where b and c are integers.

(c) Express $\frac{6+2\sqrt{7}}{3-\sqrt{7}}$ in the form $d+e\sqrt{7}$, where d and e are integers.

Solution:

(a)

$$\begin{aligned}(2\sqrt{7})^3 &= 2\sqrt{7}\times 2\sqrt{7}\times 2\sqrt{7}\\ &= 8(\sqrt{7}\times\sqrt{7}\times\sqrt{7})\\ &= 8(7\sqrt{7})\\ &= 56\sqrt{7} \quad (a=56)\end{aligned}$$

Multiply the 2s.

$\sqrt{7}\times\sqrt{7}=7$

(b)

$$\begin{aligned}&(8+\sqrt{7})(3-2\sqrt{7})\\ &= 8(3-2\sqrt{7})+\sqrt{7}(3-2\sqrt{7})\\ &= 24-16\sqrt{7}+3\sqrt{7}-14\\ &= 10-13\sqrt{7}\end{aligned}$$

$(b=10 \text{ and } c=-13)$

$\sqrt{7}\times 2\sqrt{7}=2\times 7$

(c)

$$\frac{6+2\sqrt{7}}{3-\sqrt{7}}\times\frac{3+\sqrt{7}}{3+\sqrt{7}}$$

To rationalise the denominator, multiply top and bottom by $3+\sqrt{7}$

$$=\frac{(6+2\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$$

$$=\frac{6(3+\sqrt{7})+2\sqrt{7}(3+\sqrt{7})}{3(3+\sqrt{7})-\sqrt{7}(3+\sqrt{7})}$$

$$=\frac{18+6\sqrt{7}+6\sqrt{7}+14}{9+3\sqrt{7}-3\sqrt{7}-7}$$

$$=\frac{32+12\sqrt{7}}{2}=16+6\sqrt{7}$$

Divide by 2

$(d=16 \text{ and } e=6)$

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Algebraic fractions
Exercise A, Question 9

Question:

Solve the equations

(a) $x^2 - x - 72 = 0$

(b) $2x^2 + 7x = 0$

(c) $10x^2 + 9x - 9 = 0$

Solution:

(a)

$x^2 - x - 72 = 0$

$(x + 8) (x - 9) = 0$ Factorise

$x + 8 = 0 , x - 9 = 0$

$x = - 8 , x = 9$

Although the equation could be solved using the quadratic formula or 'completing the square', factorisation is quicker.

(b)

$2x^2 + 7x = 0$

$x (2x + 7) = 0$ Use the factor x.

$x = 0 , 2x + 7 = 0$ Don't forget the $x = 0$ solution.

$x = 0 , x = - \frac{7}{2}$

(b)

$2x^2 + 7x = 0$

$x (2x + 7) = 0$ Use the factor x.

$x = 0 , 2x + 7 = 0$ Don't forget the $x = 0$ solution.

$x = 0 , x = - \frac{7}{2}$

(c)

$10x^2 + 9x - 9 = 0$

$(2x + 3) (5x - 3) = 0$ Factorise

$2x + 3 = 0 , 5x - 3 = 0$

$x = - \frac{3}{2} , x = \frac{3}{5}$

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Algebraic fractions
Exercise A, Question 10

Question:

Solve the equations, giving your answers to 3 significant figures

(a) $x^2 + 10x + 17 = 0$

(b) $2x^2 - 5x - 1 = 0$

(c) $(2x - 3)^2 = 7$

Solution:

(a)

$x^2 + 10x + 17 = 0$ Since the question requires answers to

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ 3 significant figures, you know that the

quadratic will not factorise.

$a = 1$, $b = 10$, $c = 17$

$x = \frac{-10 \pm \sqrt{(100 - 68)}}{2}$ Use the quadratic formula, quoting

the formula first.

$= \frac{-10 \pm \sqrt{32}}{2}$

$= \frac{-10 \pm 5.656\ldots}{2}$ Intermediate working should be to

at least 4 sig. figs.

$= \frac{-10 + 5.656\ldots}{2}$, $\frac{-10 - 5.656\ldots}{2}$

$x = -2.17$, $x = -7.83$ Divide by 2, and round to 3 sig. figs.

Alternative method:

$x^2 + 10x + 17 = 0$

$x^2 + 10x = -17$ Subtract 17 to get LHS in the required form.

$(x + 5)^2 - 25 = -17$ Complete the square for $x^2 + 10x$

$(x + 5)^2 = -17 + 25$ Add 25 to both sides

$(x + 5)^2 = 8$

$x + 5 = \pm \sqrt{8}$ Square root both sides.

$x = -5 \pm \sqrt{8}$ Subtract 5 from both sides.

$x = -5 + \sqrt{8}$, $x = -5 - \sqrt{8}$

$x = -2.17$, $x = -7.83$

(b)

$2x^2 - 5x - 1 = 0$

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$a = 2$, $b = -5$, $c = -1$

$x = \frac{5 \pm \sqrt{(-5)^2 - (4 \times 2 \times -1)}}{4}$ Use the quadratic formula, quoting the formula first.

$= \frac{5 \pm \sqrt{(25 + 8)}}{4} = \frac{5 \pm \sqrt{33}}{4}$

$= \frac{5 + 5.744\ldots}{4}$, $\frac{5 - 5.744\ldots}{4}$

$x = 2.69$, $x = -0.186$ Divide by 4, and round to 3 sig. figs.

(c)

$(2x - 3)^2 = 7$

$2x - 3 = \pm\sqrt{7}$ The quickest method is to take the square root of both sides.

$2x = 3 \pm \sqrt{7}$ Add 3 to both sides.

$x = \frac{3 + \sqrt{7}}{2}$, $x = \frac{3 - \sqrt{7}}{2}$ Divide both sides by 2

$x = 2.82$, $x = 0.177$

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Algebraic fractions
Exercise A, Question 11

Question:

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$
where a and b are constants.

(a) Find the value of a and the value of b.

(b) Hence, or otherwise, show that the roots of
$$x^2 - 8x - 29 = 0$$
are $c \pm d\sqrt{5}$, where c and d are integers to be found.

Solution:

(a)

$x^2 - 8x = (x - 4)^2 - 16$ — Complete the square for $x^2 - 8x$

$x^2 - 8x - 29 = (x - 4)^2 - 16 - 29$

$= (x - 4)^2 - 45$

$(a = -4$ and $b = -45)$

(b)

$x^2 - 8x - 29 = 0$

$(x - 4)^2 - 45 = 0$ — Use the result from part (a)

$(x - 4)^2 = 45$

$x - 4 = \pm\sqrt{45}$ — Take the square root of both sides.

$x = 4 \pm \sqrt{45}$

$\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$ — Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

Roots are $4 \pm 3\sqrt{5}$

$(c = 4$ and $d = 3)$

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Edexcel Modular Mathematics for AS and A-Level

Algebraic fractions
Exercise A, Question 12

Question:

Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(a) express f(x) in the form $(x - a)^2 + b$, where a and b are integers.

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(b) Sketch the graph of C, showing the coordinates of P and Q.

The line $y = 41$ meets C at the point R.

(c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

Solution:

(a)

$f(x) = x^2 - 6x + 18$

$x^2 - 6x = (x - 3)^2 - 9$ Complete the square for $x^2 - 6x$

$x^2 - 6x + 18 = (x - 3)^2 - 9 + 18$

$= (x - 3)^2 + 9$

$(a = 3$ and $b = 9)$

(b)

$y = x^2 - 6x + 18$

$y = (x - 3)^2 + 9$

$(x - 3)^2 \geq 0$ Squaring a number cannot give a negative result

The minimum value of $(x - 3)^2$ is zero, when $x = 3$.

So the minimum value of y is $0 + 9 = 9$, when $x = 3$.

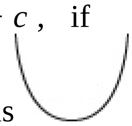
Q is the point (3, 9)

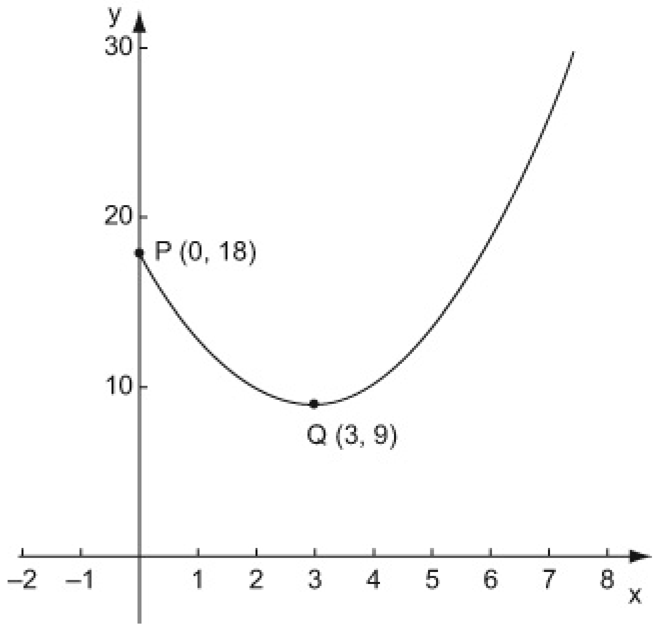
The curve crosses the y-axis where $x = 0$.

For $x = 0$, $y = 18$

P is the point (0, 18)

The graph of $y = x^2 - 6x + 18$ is a  shape.

For $y = ax^2 + bx + c$, if $a > 0$, the shape is 



Use the information about P and Q to sketch the curve $x \geq 0$, so the part where $x < 0$ is not needed.

(c)

$y = (x - 3)^2 + 9$

$41 = (x - 3)^2 + 9$ — Put $y = 41$ into the equation of C.

$32 = (x - 3)^2$ — Subtract 9 from both sides.

$(x - 3)^2 = 32$

$x - 3 = \pm\sqrt{32}$ — Take the square root of both sides.

$x = 3 \pm \sqrt{32}$

$\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$ — Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

$x = 3 \pm 4\sqrt{2}$

x-coordinate of R is $3 + 4\sqrt{2}$ — The other value $3 - 4\sqrt{2}$ is less than 0, so not needed

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Algebraic fractions
Exercise A, Question 13

Question:

Given that the equation $kx^2 + 12x + k = 0$, where k is a positive constant, has equal roots, find the value of k.

Solution:

$Kx^2 + 12x + K = 0$

$a = K$, $b = 12$, $c = K$ — Write down the values of a , b and c

For equal roots, $b^2 = 4ac$ (or $b^2 - 4ac = 0$) — for the quadratic equation.

$12^2 = 4 \times K \times K$

$4K^2 = 144$

$K^2 = 36$

$K = \pm 6$

So $K = 6$ — The question says that K is a positive constant.

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Algebraic fractions
Exercise A, Question 14

Question:

Given that

$$x^2 + 10x + 36 \equiv (x + a)^2 + b,$$

where a and b are constants,

(a) find the value of a and the value of b.

(b) Hence show that the equation $x^2 + 10x + 36 = 0$ has no real roots.

The equation $x^2 + 10x + k = 0$ has equal roots.

(c) Find the value of k.

(d) For this value of k, sketch the graph of $y = x^2 + 10x + k$, showing the coordinates of any points at which the graph meets the coordinate axes.

Solution:

(a)

$x^2 + 10x + 36$

$x^2 + 10x = (x + 5)^2 - 25$ Complete the square for $x^2 + 10x$

$x^2 + 10x + 36 = (x + 5)^2 - 25 + 36$

$= (x + 5)^2 + 11$

$a = 5$ and $b = 11$

(b)

$x^2 + 10x + 36 = 0$

$(x + 5)^2 + 11 = 0$ 'Hence' implies that part (a) must be used

$(x + 5)^2 = -11$

A real number squared cannot

be negative, ∴ no real roots

(c)

$x^2 + 10x + K = 0$

$a = 1, b = 10, c = K$

For equal roots, $b^2 = 4ac$

$10^2 = 4 \times 1 \times K$

$4K = 100$

$K = 25$

(d)

The graph of $y = x^2 + 10x + 25$ is a  shape.

For $y = ax^2 + bx + c$, if $a > 0$, the shape is 

$x = 0$: $y = 0 + 0 + 25 = 25$

Meets y-axis at $(0, 25)$

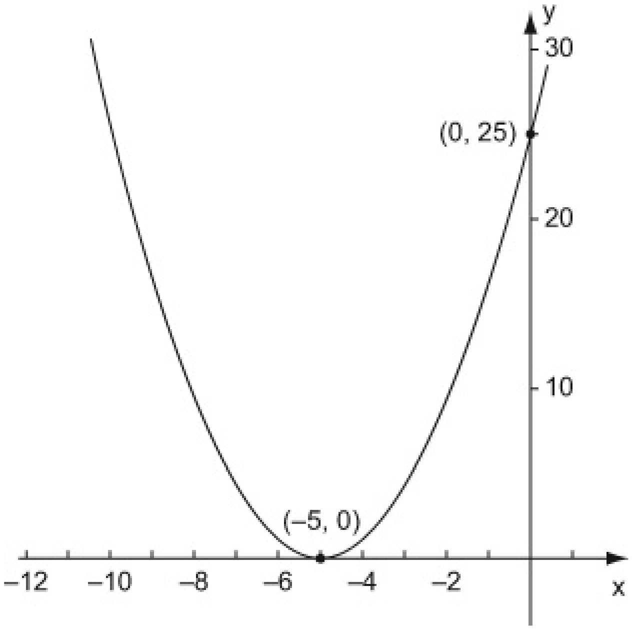
$y = 0$: $x^2 + 10x + 25 = 0$

$(x + 5)(x + 5) = 0$

$x = -5$

Meets x-axis at $(-5, 0)$

Put $x = 0$ to find intersections with the y-axis, and $y = 0$ to find intersections with the x-axis.



The graph meets the x-axis at just one point, so it ‘touches’ the x-axis

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Algebraic fractions
Exercise A, Question 15

Question:

$$x^2 + 2x + 3 \equiv (x + a)^2 + b .$$

(a) Find the values of the constants a and b.

(b) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(c) Find the value of the discriminant of $x^2 + 2x + 3$.
Explain how the sign of the discriminant relates to your sketch in part (b).

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(d) Find the set of possible values of k, giving your answer in surd form.

Solution:

(a)

$x^2 + 2x + 3$

$x^2 + 2x = (x + 1)^2 - 1$ for $x^2 + 2x$ — Complete the square

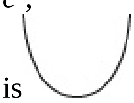
$x^2 + 2x + 3 = (x + 1)^2 - 1 + 3$

$= (x + 1)^2 + 2$

$a = 1$ and $b = 2$

(b)

The graph of $y = x^2 + 2x + 3$ is a  shape

For $y = ax^2 + bx + c$, if $a > 0$, the shape is 

$x = 0 : \quad y = 0 + 0 + 3$

Meets y-axis at $(0, 3)$

Put $x = 0$ to find intersections with the y-axis,

$y = 0 : \quad x^2 + 2x + 3 = 0$

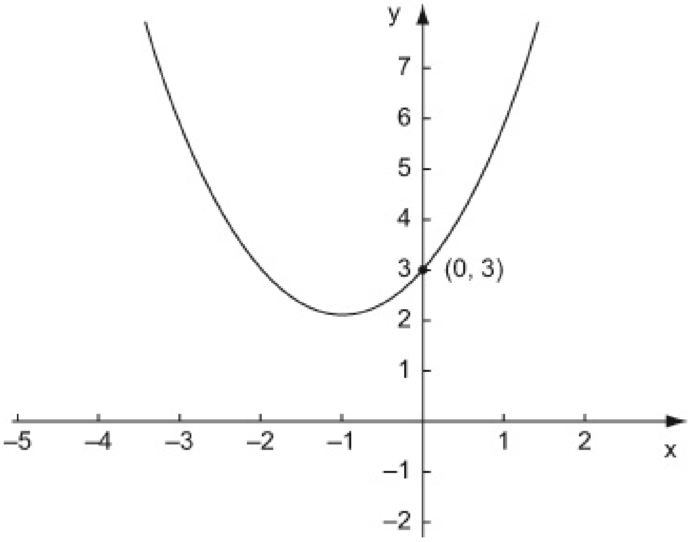
$(x + 1)^2 + 2 = 0$

$(x + 1)^2 = -2$

and $y = 0$ to find intersections with the x-axis.

A real number squared cannot be negative, ∴

no real roots, so no intersection with x-axis.



The minimum value of $(x + 1)^2$ is zero, when $x = -1$, so the minimum point on the graph is at $x = -1$

(c)

$x^2 + 2x + 3$

$a = 1$, $b = 2$, $c = 3$

$b^2 - 4ac$ $= 2^2 - 4 \times 1 \times 3$

$= -8$

The discriminant is $b^2 - 4ac$

Since the discriminant is negative $(b^2 - 4ac < 0)$, $x^2 + 2x + 3 = 0$ has no real roots, so the graph does not cross the x-axis.

No real roots: $b^2 < 4ac$

(d)

$x^2 + kx + 3 = 0$

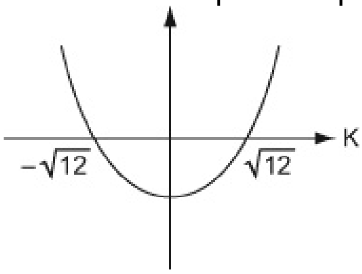
$a = 1$, $b = k$, $c = 3$

For no real roots, $b^2 < 4ac$

$K^2 < 12$

$K^2 - 12 < 0$

This is a quadratic inequality with critical values $-\sqrt{12}$ and $\sqrt{12}$



$(K + \sqrt{12})(K - \sqrt{12}) < 0$

Critical values:

$K = -\sqrt{12}$, $K = \sqrt{12}$

$-\sqrt{12} < K < \sqrt{12}$

($\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$
$-2\sqrt{3} < K < 2\sqrt{3}$)

The surds can be simplified using

$\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

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Algebraic fractions
Exercise A, Question 16

Question:

Solve the simultaneous equations

$$x + y = 2$$
$$x^2 + 2y = 12$$

Solution:

$y = 2 - x$ Rearrange the linear equation to get $y = \ldots$

$x^2 + 2(2 - x) = 12$ Substitute into the quadratic equation

$x^2 + 4 - 2x = 12$

$x^2 - 2x + 4 - 12 = 0$

$x^2 - 2x - 8 = 0$

$(x + 2)(x - 4) = 0$ Solve for x using factorisation

$x = -2$ or $x = 4$

$x = -2: \quad y = 2 - (-2) = 4$ Substitute the x values back into $y = 2 - x$

$x = 4: \quad y = 2 - 4 = -2$

Solution: $x = -2$, $y = 4$ and $x = 4$, $y = -2$

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Algebraic fractions
Exercise A, Question 17

Question:

(a) By eliminating y from the equations

$$y = x - 4 ,$$
$$2x^2 - xy = 8 ,$$

show that

$$x^2 + 4x - 8 = 0.$$

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4 ,$$
$$2x^2 - xy = 8 ,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

Solution:

(a)

$2x^2 - x (x - 4) = 8$ Substitute $y = x - 4$ into the quadratic equation.

$2x^2 - x^2 + 4x = 8$

$x^2 + 4x - 8 = 0$

(b)

(a). The $\sqrt{3}$ factorisation quadratic

Solve the equation found in part (a). The $\sqrt{3}$ in the given answer suggests that factorisation will not be possible, so use the quadratic formula, or complete the square.

$x^2 + 4x - 8$	$= 0$	
$x^2 + 4x$	$= (x + 2)^2 - 4$	Complete the square for $x^2 + 4x$
$(x + 2)^2 - 4 - 8$	$= 0$	
$(x + 2)^2$	$= 12$	
$x + 2$	$= \pm\sqrt{12}$	
x	$= -2 \pm \sqrt{12}$	
$\sqrt{12}$	$= \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$	Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$
x	$= -2 \pm 2\sqrt{3}$	
$(a = -2$ and $b = 2)$		
Using y	$= x - 4$,	
y	$= (-2 \pm 2\sqrt{3}) - 4$	
	$= -6 \pm 2\sqrt{3}$	
Solution: x	$= -2 \pm 2\sqrt{3}$	
y	$= -6 \pm 2\sqrt{3}$	

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Algebraic fractions
Exercise A, Question 18

Question:

Solve the simultaneous equations

$$2x - y - 5 = 0$$
$$x^2 + xy - 2 = 0$$

Solution:

$y = 2x - 5$ — Rearrange the linear equation to get $y = \ldots$

$x^2 + x(2x - 5) - 2 = 0$ — Substitute into the quadratic equation.

$x^2 + 2x^2 - 5x - 2 = 0$

$3x^2 - 5x - 2 = 0$

$(3x + 1)(x - 2) = 0$ — Solve for x using factorisation

$x = -\frac{1}{3}$ or $x = 2$

$x = -\frac{1}{3}: \ y = -\frac{2}{3} - 5 = -\frac{17}{3}$ — Substitute the x values back into $y = 2x - 5$

$x = 2: \ y = 4 - 5 = -1$

Solution $x = -\frac{1}{3}, \ y = -\frac{17}{3}$
and $x = 2, \ y = -1$

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Algebraic fractions
Exercise A, Question 19

Question:

Find the set of values of x for which

(a) $3(2x+1) > 5 - 2x$,

(b) $2x^2 - 7x + 3 > 0$,

(c) both $3(2x+1) > 5 - 2x$ and $2x^2 - 7x + 3 > 0$.

Solution:

(a)

$3(2x+1) > 5 - 2x$

$6x + 3 > 5 - 2x$ Multiply out

$6x + 2x + 3 > 5$ Add $2x$ to both sides.

$8x > 2$ Subtract 3 from both sides

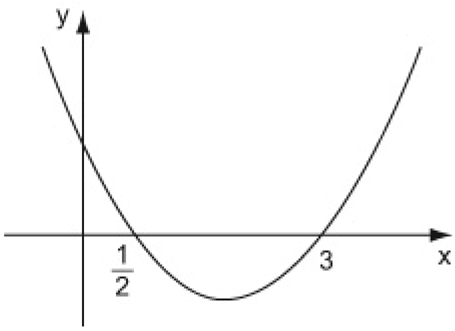
$x > \frac{1}{4}$ Divide both sides by 8

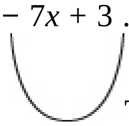
(b)

$2x^2 - 7x + 3 = 0$

$(2x-1)(x-3) = 0$ Factorise to solve the quadratic equation.

$x = \frac{1}{2}$, $x = 3$

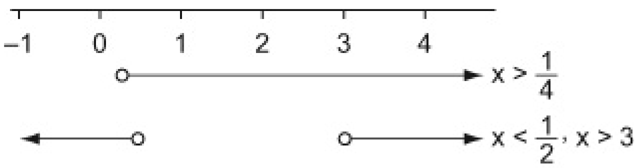


Sketch the graph of $y = 2x^2 - 7x + 3$. The shape is  The sketch does not need to be accurate.

$2x^2 - 7x + 3 > 0$ where $x < \frac{1}{2}$ or $x > 3$

$2x^2 - 7x + 3 > 0$ $(y > 0)$ for the part of the graph above the x-axis

(c)



$\frac{1}{4} < x < \frac{1}{2}$, $x > 3$

Use a number line. The two sets of values (from part (a) and part (b)) overlap for $\frac{1}{4} < x < \frac{1}{2}$ and $x > 3$

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Algebraic fractions
Exercise A, Question 20

Question:

Find the set of values of x for which

(a) $x(x-5) < 7x - x^2$

(b) $x(3x+7) > 20$

Solution:

(a)
$x(x-5) < 7x - x^2$

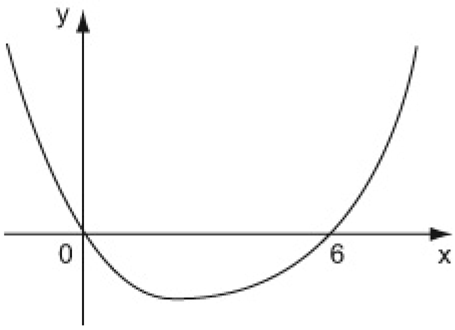
$x^2 - 5x < 7x - x^2$ — Multiply out

$2x^2 - 12x < 0$

$2x(x-6) < 0$ — Factorise using the common factor $2x$

$2x(x-6) = 0$ — Solve the quadratic equation

$x = 0, x = 6$ — to find the critical values



Sketch the graph of $y = 2x^2 - 12x$

$2x^2 - 12x < 0$ where $0 < x < 6$

$2x^2 - 12x < 0 \ (y < 0)$ for the part of the graph below the x-axis

(b)

$x(3x+7) > 20$

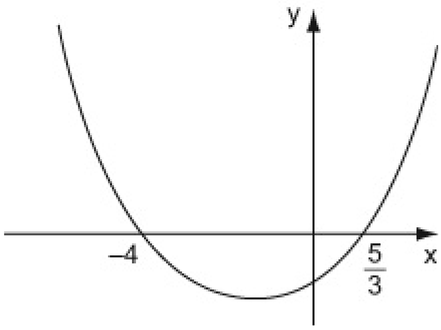
$3x^2 + 7x > 20$ — Multiply out

$3x^2 + 7x - 20 > 0$

$(3x-5)(x+4) > 0$ — Factorise

$(3x-5)(x+4) = 0$ — Solve the quadratic equation to find the critical values

$x = \frac{5}{3}, x = -4$



Sketch the graph of
$y = 3x^2 + 7x - 20$

$3x^2 + 7x - 20 > 0$ where
$x < -4$ or $x > \frac{5}{3}$

$3x^2 + 7x - 20 > 0$ ($y > 0$)
for the part of the graph
above the x-axis.

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Algebraic fractions
Exercise A, Question 21

Question:

(a) Solve the simultaneous equations

$$y + 2x = 5\ ,$$
$$2x^2 - 3x - y = 16.$$

(b) Hence, or otherwise, find the set of values of x for which

$$2x^2 - 3x - 16 > 5 - 2x\ .$$

Solution:

(a)

$y = 5 - 2x$ — Rearrange the linear equation to get $y = \ldots$

$2x^2 - 3x - (5 - 2x) = 16$ — Substitute into the quadratic equation.

$2x^2 - 3x - 5 + 2x = 16$

$2x^2 - x - 21 = 0$

$(2x - 7)(x + 3) = 0$ — Solve for x using factorisation.

$x = 3\frac{1}{2}$ or $x = -3$

$x = 3\frac{1}{2}$: $y = 5 - 7 = -2$ — Substitute the x-values back into $y = 5 - 2x$

$x = -3$: $y = 5 + 6 = 11$

Solution $x = 3\frac{1}{2}$, $y = -2$

and $x = -3$, $y = 11$

(b)

The equations in (a) could be written as

$y = 5 - 2x$ and $y = 2x^2 - 3x - 16$.

The solutions to $2x^2 - 3x - 16 = 5 - 2x$ are the x solutions from (a). These are the critical values for $2x^2 - 3x - 16 > 5 - 2x$.

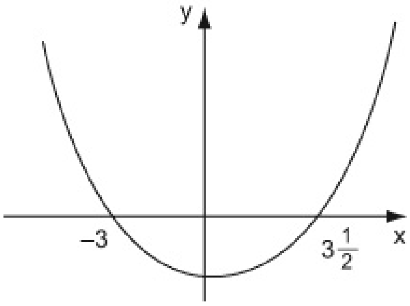
Critical values

$x = 3\frac{1}{2}$ and $x = -3$.

$2x^2 - 3x - 16 > 5 - 2x$

$(2x^2 - 3x - 16 - 5 + 2x > 0)$

$2x^2 - x - 21 > 0$



Sketch the graph of
$y = 2x^2 - x - 21$

$x < -3$ or $x > 3\frac{1}{2}$

$2x^2 - x - 21 > 0$ ($y > 0$) for the part of the graph above the x-axis.

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Algebraic fractions
Exercise A, Question 23

Question:

Given that the equation $kx^2 + 3kx + 2 = 0$, where k is a constant, has no real roots, find the set of possible values of k.

Solution:

$kx^2 + 3kx + 2 = 0$

$a = k$, $b = 3k$, $c = 2$ Write down a, b and c for the equation.

$b^2 < 4ac$

$(3k)^2 < 4 \times k \times 2$ For no real roots, $b^2 < 4ac$.

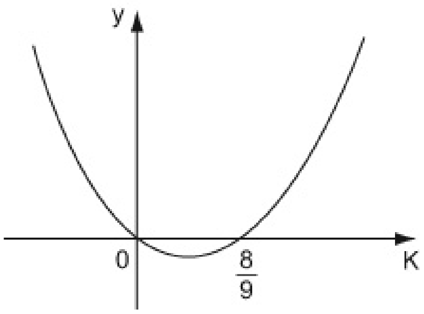
$9k^2 < 8k$

$9k^2 - 8k < 0$

$9k^2 - 8k = 0$

$k(9k - 8) = 0$ Factorise to solve the quadratic equation

$k = 0$, $k = \frac{8}{9}$



Sketch the graph of $y = 9k^2 - 8k$. The shape is . The sketch does not need to be accurate.

$9k^2 - 8k < 0$ where $0 < k < \frac{8}{9}$

$9k^2 - 8k < 0$ $(y < 0)$ for the part of the graph below the k-axis.

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Algebraic fractions
Exercise A, Question 24

Question:

The equation $(2p + 5) x^2 + px + 1 = 0$, where p is a constant, has different real roots.

(a) Show that $p^2 - 8p - 20 > 0$

(b) Find the set of possible values of p.

Given that $p = -3$,

(c) find the exact roots of $(2p + 5) x^2 + px + 1 = 0$.

Solution:

(a)

$(2p + 5) x^2 + px + 1 = 0$

$a = 2p + 5 , b = p , c = 1$ Write down a, b and c for the equation.

$b^2 > 4ac$

$p^2 > 4 (2p + 5)$ For different real roots, $b^2 > 4ac$

$p^2 > 8p + 20$

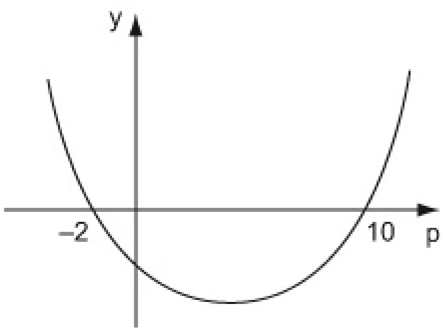
$p^2 - 8p - 20 > 0$

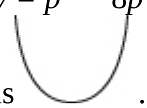
(b)

$p^2 - 8p - 20 = 0$

$(p + 2) (p - 10) = 0$ Factorise to solve the quadratic equation.

$p = -2 , p = 10$



Sketch the graph of $y = p^2 - 8p - 20$ The shape is . The sketch does not need to be accurate

$p^2 - 8p - 20 > 0$ where $p < -2$ or $p > 10$

$p^2 - 8p - 20 > 0$ ($y > 0$) for the part of the graph above the p-axis

(c)

For $p = -3$

$(-6+5)x^2 - 3x + 1 = 0$ Substitute $p = -3$ into the equation.

$-x^2 - 3x + 1 = 0$ Multiply by -1

$x^2 + 3x - 1 = 0$ The equation does not factorise,

$a = 1, b = 3, c = -1$ so use the quadratics formula.

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ Quote the formula.

$x = \frac{-3 \pm \sqrt{9+4}}{2}$

$x = \frac{1}{2}(-3 \pm \sqrt{13})$ Exact roots are required.

$\sqrt{13}$ cannot be simplified.

$x = \frac{1}{2}(-3 + \sqrt{13})$ or $x = \frac{1}{2}(-3 - \sqrt{13})$

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Algebraic fractions
Exercise A, Question 25

Question:

(a) Factorise completely $x^3 - 4x$

(b) Sketch the curve with equation $y = x^3 - 4x$, showing the coordinates of the points where the curve crosses the x-axis.

(c) On a separate diagram, sketch the curve with equation

$$y = (x - 1)^3 - 4(x - 1)$$

showing the coordinates of the points where the curve crosses the x-axis.

Solution:

(a)

$x^3 - 4x$ — x is a common factor

$= x(x^2 - 4)$ — $(x^2 - 4)$ is a difference of squares

$= x(x + 2)(x - 2)$

(b)

Curve crosses x-axis where $y = 0$

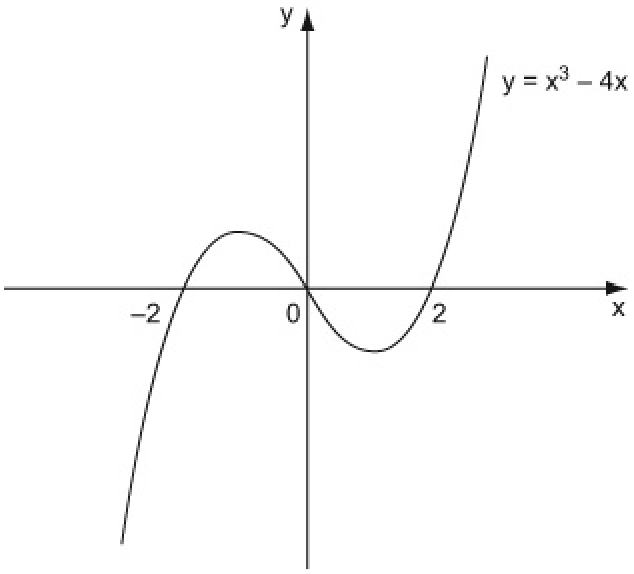
$x(x + 2)(x - 2) = 0$ — Put $y = 0$ and solve for x

$x = 0$, $x = -2$, $x = 2$

When $x = 0$, $y = 0$ — Put $x = 0$ to find where the curve crosses the y-axis.

When $x \to \infty$, $y \to \infty$ — Check what happens to y for large positive and negative values of x

When $x \to -\infty$, $y \to -\infty$



Crosses at (0, 0)
Crosses x-axis at (−2, 0), (2, 0).

(c)

$y = x^3 - 4x$

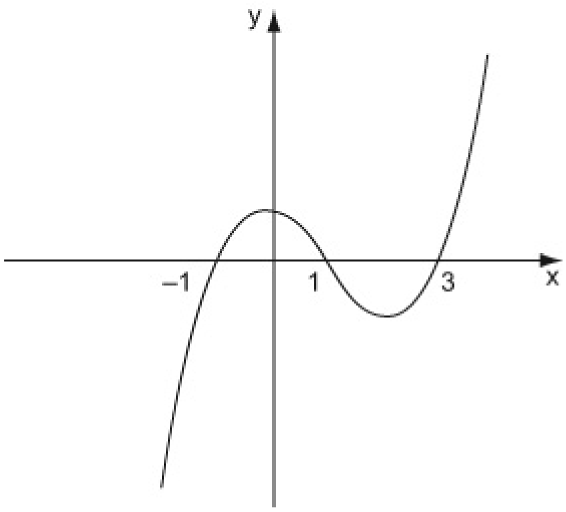
$y = (x-1)^3 - 4(x-1)$

This is a translation of $+1$ in the x-direction.

Compare with the equation from part (b).

x has been replaced by $x - 1$.

$f(x + a)$ is a translation of $-a$ in the x-direction.



The shape is the same as in part (b).

Crosses x-axis at $(-1, 0)$, $(1, 0)$, $(3, 0)$

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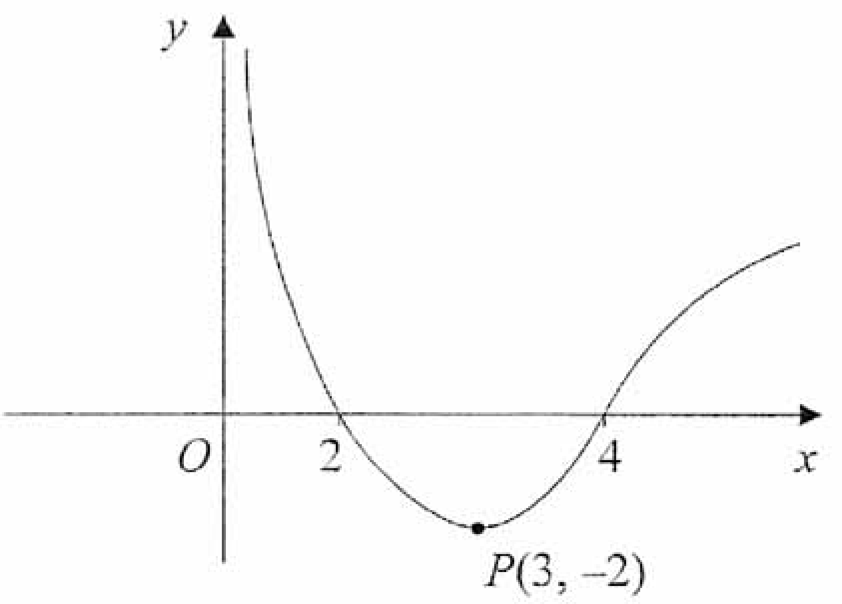
Algebraic fractions
Exercise A, Question 26

Question:



The figure shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points (2, 0) and (4, 0). The minimum point on the curve is $P(3, -2)$.

In separate diagrams, sketch the curve with equation

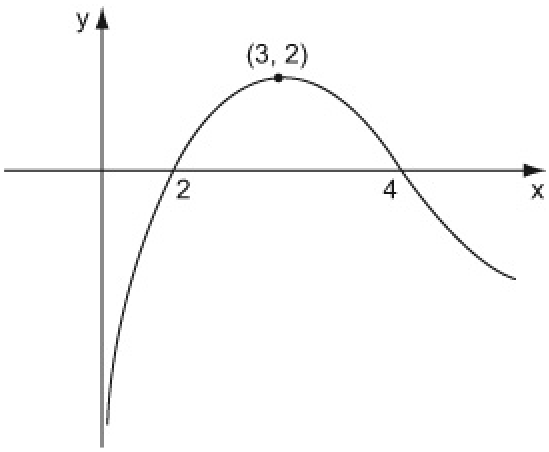
(a) $y = -f(x)$

(b) $y = f(2x)$

On each diagram, give the coordinates of the points at which the curve crosses the x-axis, and the coordinates of the image of P under the given transformation.

Solution:

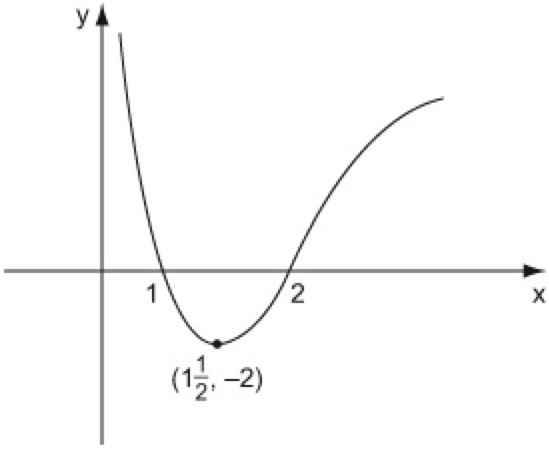
(a)



The transformation $-f(x)$ multiplies the y-coordinates by -1. This turns the graph upside-down.

Crosses the x-axis at $(2, 0)$, $(4, 0)$
Image of P is $(3, 2)$

(b)



$f(2x)$ is a stretch of $\frac{1}{2}$ in the x-direction. (Multiply x-coordinates by $\frac{1}{2}$.)

Crosses the x-axis at $(1, 0)$, $(2, 0)$

Image of P is $\left(1\frac{1}{2}, -2\right)$

y-coordinates are unchanged.

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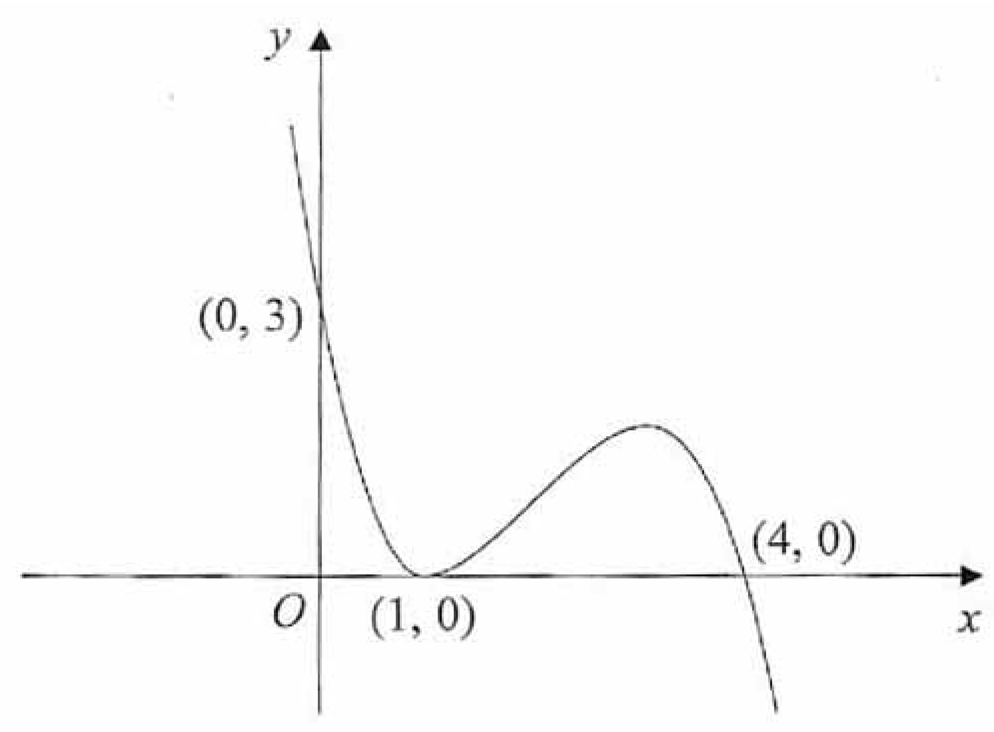
Algebraic fractions
Exercise A, Question 27

Question:



The figure shows a sketch of the curve with equation $y = f(x)$. The curve passes through the points (0, 3) and (4, 0) and touches the x-axis at the point (1, 0).

On separate diagrams, sketch the curve with equation

(a) $y = f(x + 1)$

(b) $y = 2f(x)$

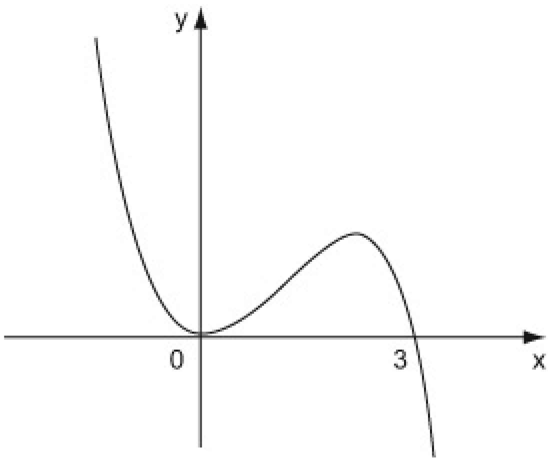
(c) $y = f\left(\frac{1}{2}x\right)$

On each diagram, show clearly the coordinates of all the points where the curve meets the axes.

Solution:

(a)

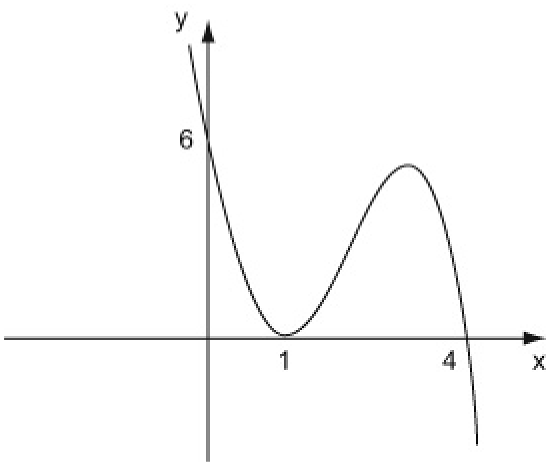
$f(x + 1)$ is a translation of



–1 in the x-direction.

Meets the x-axis at $(0, 0)$, $(3, 0)$
Meets the y-axis at $(0, 0)$

(b)

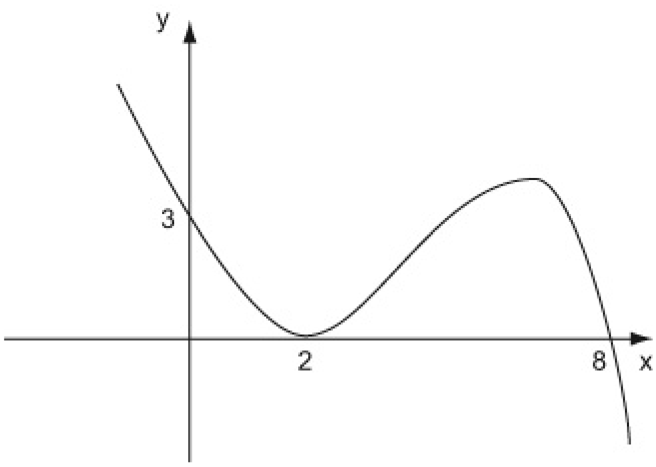


$2f(x)$ is a stretch of scale factor 2 in the y-direction (Multiply y-coordinates by 2)

Meets the x-axis at $(1, 0)$, $(4, 0)$
Meets the y-axis at $(0, 6)$

x-coordinates are unchanged.

(c)



$f\left(\frac{1}{2}x\right)$ is a stretch of scale factor $\frac{1}{\left(\frac{1}{2}\right)} = 2$ in the x-direction. (Multiply x-coordinates by 2)

Meets the x-axis at $(2, 0)$, $(8, 0)$

Meets the y-axis at $(0, 3)$

y-coordinates are unchanged.

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Algebraic fractions
Exercise A, Question 28

Question:

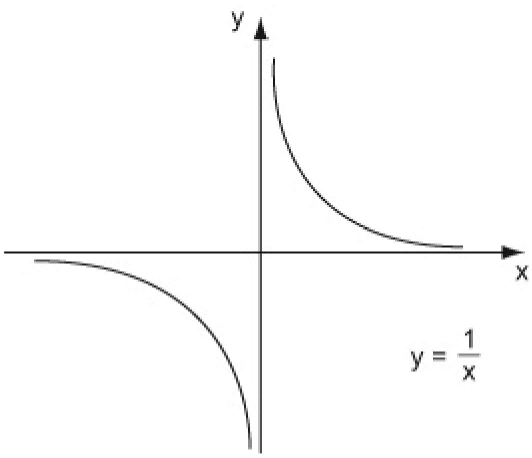
Given that $f\left(x\right)=\frac{1}{x}$, $x\neq0$,

(a) sketch the graph of $y=f\left(x\right)+3$ and state the equations of the asymptotes.

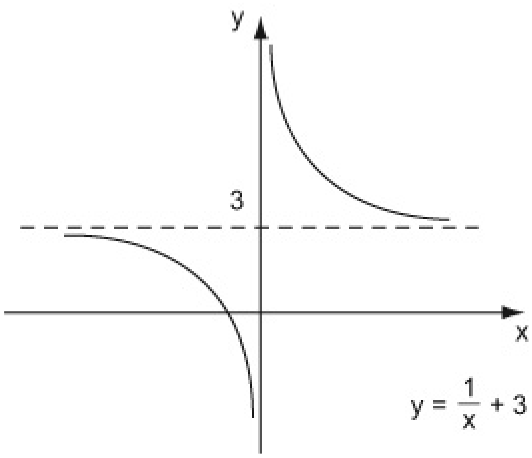
(b) Find the coordinates of the point where $y=f\left(x\right)+3$ crosses a coordinate axis.

Solution:

(a)



You should know the shape of this curve.



$f\left(x\right)+3$ is a translation of $+3$ in the y-direction.

$y=3$ is an asymptote

$x=0$ is an asymptote

The equation of the y-axis is $x=0$

(b)

The graph does not cross the y-axis (see sketch in (a)) .

If you used $x = 0$ you would get $y = \frac{1}{0} + 3$ but $\frac{1}{0}$ is undefined , or infinite.

Crosses the x-axis where $y = 0$:

$$\frac{1}{x} + 3 = 0$$

$$\frac{1}{x} = -3$$

$$x = -\frac{1}{3} \quad \left(-\frac{1}{3}, 0 \right)$$

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Algebraic fractions
Exercise A, Question 29

Question:

Given that $f(x) = (x^2 - 6x)(x - 2) + 3x$,

(a) express f(x) in the form $x(ax^2 + bx + c)$, where a, b and c are constants

(b) hence factorise f(x) completely

(c) sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes

Solution:

(a)

$f(x) = (x^2 - 6x)(x - 2) + 3x$ Multiply out the bracket

$= x^2(x - 2) - 6x(x - 2) + 3x$

$= x^3 - 2x^2 - 6x^2 + 12x + 3x$

$= x^3 - 8x^2 + 15x$ x is a common factor

$= x(x^2 - 8x + 15)$

$(a = 1, b = -8, c = 15)$

(b)

$x(x^2 - 8x + 15)$ Factorise the quadratic

$f(x) = x(x - 3)(x - 5)$

(c)

Curve meets x-axis where $y = 0$.

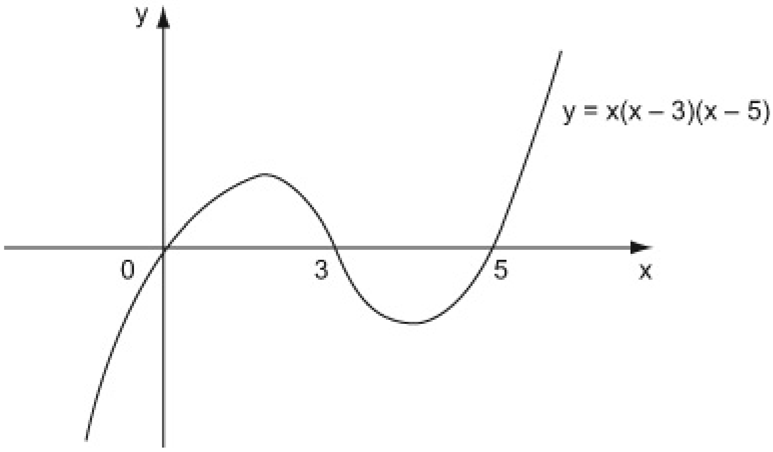
$x(x - 3)(x - 5) = 0$ Put $y = 0$ and solve for x

$x = 0, x = 3, x = 5$

When $x = 0, y = 0$ Put $x = 0$ to find where the curve crosses the y-axis

When $x \to \infty, y \to \infty$ Check what happens to y for large positive and negative values of x.

When $x \to -\infty, y \to -\infty$



Meets x-axis at (0 , 0) , (3 , 0) , (5 , 0)
Meets y-axis at (0 , 0)

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Algebraic fractions
Exercise A, Question 30

Question:

(a) Sketch on the same diagram the graph of $y = x(x+2)(x-4)$ and the graph of $y = 3x - x^2$, showing the coordinates of the points at which each graph meets the x-axis.

(b) Find the exact coordinates of each of the intersection points of $y = x(x+2)(x-4)$ and $y = 3x - x^2$.

Solution:

(a)

$y = x(x+2)(x-4)$

Curve meets x-axis where $y = 0$.

$x(x+2)(x-4) = 0$ — Put $y = 0$ and solve for x.

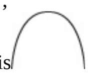
$x = 0, x = -2, x = 4$

When $x = 0, y = 0$ — Put $x = 0$ to find where the curve crosses the y-axis

When $x \to \infty, y \to \infty$

When $x \to -\infty, y \to -\infty$ — Check what happens to y for large positive and negative values of x.

$y = 3x - x^2$

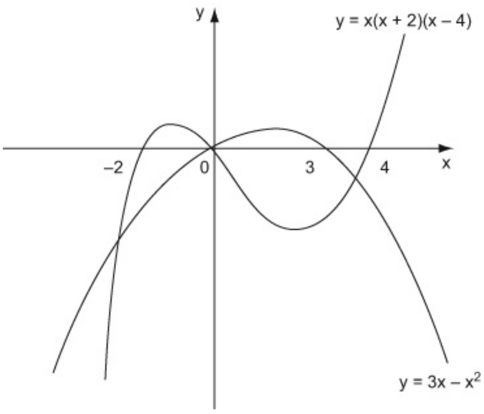
The graph of $y = 3x - x^2$ is a  shape — For $y = ax^2 + bx + c$, if $a < 0$, the shape is 

$3x - x^2 = 0$ — Put $y = 0$ and solve for x.

$x(3 - x) = 0$

$x = 0, x = 3$

When $x = 0, y = 0$ — Put $x = 0$ to find where the curve crosses the y-axis.



$y = x(x+2)(x-4)$ meets the x-axis at $(-2, 0)$, $(0, 0)$, $(4, 0)$

$y = 3x - x^2$ meets the x-axis at $(0, 0)$, $(3, 0)$

(b)

$x(x+2)(x-4) = 3x - x^2$ — To find where the graphs intersect, equate the two expressions for y to give an equation in x.

$x(x+2)(x-4) = x(3-x)$

$(x+2)(x-4) = 3 - x$ — If you divide by x, remember that $x = 0$ is a solution.

One solution is $x = 0$

$x^2 - 2x - 8 = 3 - x$

$x^2 - 2x + x - 8 - 3 = 0$

$x^2 - x - 11 = 0$ — The equation does not factorise, so use the quadratic formula.

$a = 1, b = -1, c = -11$

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ Quote the formula

$x = \frac{1 \pm \sqrt{(-1)^2 - (4 \times 1 \times -11)}}{2}$

$= \frac{1 \pm \sqrt{45}}{2}$ Exact values are required, not rounded

$\sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$ decimals, so leave the answers in surd form.

$x = \frac{1}{2}(1 \pm 3\sqrt{5})$

$x = \frac{1}{2}(1 + 3\sqrt{5})$ or $x = \frac{1}{2}(1 - 3\sqrt{5})$

$x = 0 : y = 0$ The y-coordinates for the intersection

$x = \frac{1}{2}(1 + 3\sqrt{5})$ points are also needed.

$y = \frac{3(1 + 3\sqrt{5})}{2} - \frac{(1 + 3\sqrt{5})^2}{4}$ Use $y = 3x - x^2$, the simpler equation

$(1 + 3\sqrt{5})^2 = (1 + 3\sqrt{5})(1 + 3\sqrt{5})$

$= 1(1 + 3\sqrt{5}) + 3\sqrt{5}(1 + 3\sqrt{5})$

$= 1 + 3\sqrt{5} + 3\sqrt{5} + 45$ $\sqrt{5} \times \sqrt{5} = 5$

$= 46 + 6\sqrt{5}$

$y = \frac{6(1 + 3\sqrt{5})}{4} - \frac{46 + 6\sqrt{5}}{4}$ Use a common denominator 4.

$= \frac{6 + 18\sqrt{5} - 46 - 6\sqrt{5}}{4}$

$= \frac{-40 + 12\sqrt{5}}{4} = -10 + 3\sqrt{5}$

$x = \frac{1}{2}(1 - 3\sqrt{5})$

$y = \frac{3(1 - 3\sqrt{5})}{2} - \frac{(1 - 3\sqrt{5})^2}{4}$

$y = \frac{6(1 - 3\sqrt{5})}{4} - \frac{46 - 6\sqrt{5}}{4}$ The working will be similar to that for $1 + 3\sqrt{5}$, so need not be fully repeated.

$= \frac{6 - 18\sqrt{5} - 46 + 6\sqrt{5}}{4}$

$= \frac{-40 - 12\sqrt{5}}{4} = -10 - 3\sqrt{5}$

Intersection points are : Finally, write down the coordinates of all the points you have found. You can compare these with your sketch, as a rough check.

$(0, 0)$, $\left(\frac{1}{2}(1 + 3\sqrt{5}), -10 + 3\sqrt{5}\right)$

and $\left(\frac{1}{2}(1 - 3\sqrt{5}), -10 - 3\sqrt{5}\right)$