IX. Loop Quantum Gravity

- The “new variables” are Yang-Mills (for an SU(2) or SO(3) gauge group).
- Invented by Ashtekar in the early 1980’s.
- Very technical (see Rovelli’s book, “Quantum Gravity”, for a relatively easy introduction).
- The Hamiltonian formalism is dominant (“ADM”).
- It is largely a theory of constraint equations:

Gauss-law: " $\vec{\nabla}\cdot\vec{E}$ " $= 0$

Momentum: " $E \times B$ " $= 0$

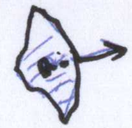
Hamiltonian: $H = 0$

- The form of the Hamiltonian is NOT Maxwell-like:

$$H \propto \left[\frac{-M_{pl}^2}{\sqrt{\det E}} BEE + \mu^4 \sqrt{\det E} \right]$$

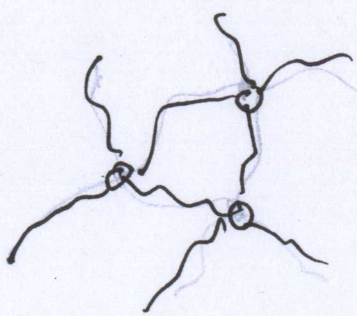
- The electric field is an "area operator".

- Quantization is somewhat unconventional ("Bohr quantization").

$A(\ell) = \int \vec{A} \cdot d\vec{\ell}$  $E(\sigma) = \int \vec{E} \cdot d\vec{S}$  $[A(\ell), E(\sigma)] = \begin{cases} 1 & \text{puncture} \\ 0 & \text{otherwise} \end{cases}$

- Spectrum of area operator is discrete (by construction).

- The Hilbert-space basis states are not particle-like (Fock space), but rather based on holonomies (Wilson lines).



$$\Psi = \prod_{\text{vertices } i} V_i \prod_{\text{lines } j} h_j$$

$$h_i \sim P\left(e^{i\int A\,ds}\right) \equiv \text{holonomies}$$

- Spin networks

$$V_i \sim \text{Wigner coefficients}$$

- Some obstacles to comprehension:

    Hamiltonian-constraint mathematics.

    (Almost) no simple examples.

    Emphasis on the Planck scale and quantization.

    Classical limit often is obscure and implicit (but need not be, in my opinion).

    Practitioners are strong mathematical physicists, but not as conversant with phenomenology per se. (It is a bit like string theory, but with a distinctly different cultural tradition.)

    Dynamics ("spin-foam" theory) not as well developed.

Bojowald Cosmology

- Loop gravity applied to FRW cosmology.

- It is pretty simple:

  Isotropy and homogeneity removes indices.
  Spin connection is trivial (space is flat)
  Only the global coordinates are retained (universe-in a box again).

- Discreteness of the area operator leads to difference equations.

- Big bang singularity at t = 0 can be finessed; “bounce” solutions exist.

- Classical limit can be described.

- Our goals:

Describe LQG cosmology in the classical limit.

Generalize the comoving box description to black hole geometries via the Swiss cheese construction.

Identify LQG variables for the black hole geometry.

- Simplifying features for the LQG cosmology:

$$E_i^A(\vec{x},t) \Rightarrow \delta_i^A E(t)$$

$$A_i^A(x,t) \Rightarrow \delta_i^A A(t)$$

- All indices disappear.

- Some nuances:

  E is smeared over an area $E \Rightarrow \int E\, d\sigma$

  A is smeared along an arc: $A \rightarrow \int A\, dl$

- The commutator of the smeared operators is $[E(\sigma), A(l)] = \begin{cases} 1 & \text{puncture} \\ 0 & \text{otherwise} \end{cases}$

- The covariant derivative usually is written

$$\vec{D} = \vec{\nabla} - \vec{\Gamma} \Leftarrow \text{“spin connection”}$$

- The definition of A contains an extra term

$$\vec{A} = \vec{\Gamma} + \gamma \vec{K}$$

- K is “extrinsic curvature”, part of the ADM Hamiltonian-formalism.

Summary

- For PG metrics the extrinsic curvature is proportional to the gauge potential A and plays a central role.

- For the Schwarzschild black hole, E is huge, A is order unity near the horizon, and B is small except near the singularity.

$$\int A\, ds \sim \gamma\, \mathbb{K} \times (\text{length})$$

$$\int B\, d\sigma \sim \gamma^2\, \mathbb{K}^2 \times (\text{area})$$

$$\int E\, d\sigma \sim M_{pl}^2 \times (\text{area})$$

- For FRW cosmology, A, B, and E are isotropic. A and $\mathbb{K}$ are identified with the Hubble parameter H(t).

- The Hamiltonian has the structure (schematically)

$$\mathbb{H} \sim (\text{volume}) \times \left(\frac{B}{E} - \frac{\Lambda}{3}\right)$$

$$\frac{\Lambda}{3} = H^2 = (\text{cosmological constant}) \times \frac{1}{3}$$

- Also—

- A·E , B/E, and A·B = A A A are interesting.

$$A\cdot E \sim \mathbb{K} \times \text{volume}$$

$$\frac{B}{E} \sim \mathbb{K}^2$$

$$A\cdot B \sim \mathbb{K}^3 \times \text{volume} \qquad (\mathbb{K} = \text{extrinsic curvature})$$

- The determinant of K (or A) vanishes when the deceleration parameter vanishes.

Details for A·B:

$$A\cdot B \sim (\text{Volume})\,\|\det \mathbb{K}\| \sim (\text{volume}) \times \frac{V^2}{r^2}\frac{\partial V}{\partial r} = \text{volume} \times \frac{1}{3r^2}\frac{\partial}{\partial r}V^3$$

X. First-Order Gravity: Cosmology

- Preliminaries: “stupid Lagrangians”

- Introduction to the first-order formalism:

  Cartan
  Palatini
  Ashtekar
  Holst
  Macdowell and Mansouri
  Plebanski
  Freidel and Starodubtsev
  Sciama
  Kibble
  Heyl
  Shapiro

- Revisits of FRW, LQG, PG

- Spinors and their role in the story

  Immirzi parameter
  Torsion

Stupid Lagrangians

- Definition: so simple that they are hard
- Suppose L = - U(q)

  Solution q = constant, at point where U is minimized or maximized.
- Suppose U(q) = q

  Equation of motion is 0 = - 1
- What to do?

  1) abandon action principle
  2) add a small perturbation
  3) compactify the coordinate

  None are especially satisfying choices.
- Suppose

$$L = \dot{q} - q$$

Equation of motion is still 0 = -1

A semistupid Lagrangian

- This one is closer to what we encounter:

$$L = Q\dot{q} - U(Q, q)$$

- Equations of motion:

$$\left.\begin{array}{l} \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \dot{Q} = \frac{\partial L}{\partial q} = -\frac{\partial U}{\partial q} \\ \frac{d}{dt}\frac{\partial L}{\partial \dot{Q}} = 0 = \frac{\partial L}{\partial Q} = \dot{q} - \frac{\partial U}{\partial Q} \end{array}\right\} \Rightarrow \quad \begin{array}{l} \dot{Q} = -\frac{\partial U}{\partial q} \\ \dot{q} = \frac{\partial U}{\partial Q} \end{array}$$

- Identify Q with p and U with H. Then the structure is the same as Hamilton's equations.

- Upon quantization, we would expect q and Q to commute. Should they?

The answer is NO. Just follow the identification as above.

The reasoning is lengthy, however.

One needs the Dirac theory of quantization in presence of first and second class constraints.

See Wikipedia entry "Dirac Bracket" for a good exposition, including an example.

A generalization

- If the number of q's equals the number of Q's, the previous example generalizes.

$$L(Q_i, q_i) = \sum_{i=1}^{n} Q_i \dot{q}_i - U(Q_i, q_i)$$

- If there are more Q's than q's, things are still manageable:

$$L = \sum_{i=1}^{n} \tilde{Q}_i \dot{q}_i - U(\tilde{Q}, q; Q)$$

- Note: this is pretty general; canonical transformations can be invoked to get many candidate Lagrangians into this form.

- Again identify momenta and Hamiltonian before:

$$p_i \equiv \tilde{Q}_i \qquad U \equiv H$$

- Equations of motion:

$$\frac{\partial H}{\partial p_i} = \dot{q}_i \qquad \frac{\partial H}{\partial q_i} = -\dot{p}_i \qquad \frac{\partial H}{\partial Q_j} = 0$$

- But some constraint equations might be stupid.

The Palatini / Holst Action

- The tetrad

$$e_\mu^{\ A} \qquad A = 0\ 1\ 2\ 3 \quad \text{(internal "frame" variables)}$$
$$\mu = t\ x\ y\ z$$

- The connection

$$\omega_\mu^{\ AB}$$ "Gauge potential" (Similar to Christoffel symbols)

- The Riemann Curvature Tensor

$$R_{\mu\nu}^{\ \ AB} = \partial_\mu \omega_\nu^{\ AB} - \partial_\nu \omega_\mu^{\ AB} + [\omega_\mu, \omega_\nu]^{AB} \qquad R = d\omega + \omega \wedge \omega$$

- The internal-symmetry group is SO(3,1)

$$g_{\mu\nu} = \sum_{AB} \eta_{AB}\, e_\mu^{\ A}\, e_\nu^{\ B} \qquad \eta_{AB} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \text{Minkowski metric}$$

The Palatini Action

$$S = \frac{M_{pl}^2}{32\pi} \int d^4x \; e^A \wedge e^B \wedge [R - H^2 e \wedge e]^{CD} \epsilon_{ABCD}$$

$$= \frac{M_{pl}^2}{8\pi} \int d^4x \; e^A_\mu \, e^B_\nu \left[R^{CD}_{\lambda\sigma} - H^2 e^C_\lambda e^D_\sigma \right] \epsilon_{ABCD} \, \epsilon^{\mu\nu\lambda\sigma}$$

- Vary with respect to e and ω:

$$\delta e: \quad e \wedge (R - H^2 e \wedge e) = 0 \qquad \text{Einstein eqns}$$

$$\delta\omega: \quad e \wedge (de + \omega \wedge e) = 0 \qquad \text{Connection constraint}$$

- Solutions (the path back to the metric formalism):

$$\omega \sim \frac{e \partial e}{ee} \sim g^{-1} \partial g \sim \text{Christoffel}$$

- [H = constant = de Sitter Hubble expansion rate:

$$H^2 = \frac{\Lambda}{3}$$]

The Holst Term

$$S = -\frac{M_{pl}^2}{32\pi}\int d^4x \left[\underbrace{R^{AB}}_{\text{Einstein-Hilbert}} - \underbrace{H^2 e^A \wedge e^B}_{\text{Cosmological constant}}\right] \wedge e^C \wedge e^D \epsilon_{ABCD} + \frac{M_{pl}^2}{16\pi\gamma}\int d^4x \underbrace{R^{AB} \wedge e_A \wedge e_B}_{\text{Holst}}$$

- (We also added the cosmological term)
- The parameter γ is the Barbero-Immirzi parameter. It is a coupling constant!!
- The Holst term violates CP (one less Levi-Civita symbol than the Palatini term.)
- Effective field theory suggests that the Holst term should be present, if only via radiative-correction effects.
- We will find that the Holst term does not affect the metric-gravity equations of motion.
- References: gr-qc/9511026; hep-th/0507253

FRW Cosmology

- The tetrad and connection, assuming homogeneity and isotropy, are:

$$e_\mu{}^A = \begin{array}{cc} \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \left(\begin{array}{c|ccc} N & 0 & 0 & 0 \\ \hline 0 & a(t) & 0 & 0 \\ 0 & 0 & a(t) & 0 \\ 0 & 0 & 0 & a(t) \end{array}\right) \\ & \begin{array}{cccc} t & x & y & z \end{array} \end{array}
\qquad
-\omega_\mu{}^{AB} = \begin{array}{cc} \begin{array}{c} 01 \\ 02 \\ 03 \\ 23 \\ 31 \\ 12 \end{array} & \left(\begin{array}{c|ccc} 0 & K(t) & 0 & 0 \\ 0 & 0 & K(t) & 0 \\ 0 & 0 & 0 & K(t) \\ \hline 0 & C(t) & 0 & 0 \\ 0 & 0 & C(t) & 0 \\ 0 & 0 & 0 & C(t) \end{array}\right) \\ & \begin{array}{cccc} t & x & y & z \end{array} \end{array}$$

- N is the “lapse” variable in ADM formalism.
- C(t) is a form of torsion, called “contorsion”.

The Curvature Tensor

- Computation of R involves some index hell, not exhibited here:

$$R^{AB}_{\mu\nu} = \begin{array}{cc} \begin{array}{c} 01 \\ 02 \\ 03 \\ 23 \\ 31 \\ 12 \end{array} & \left(\begin{array}{ccc|ccc} -\dot{K} & 0 & 0 & 2KC & 0 & 0 \\ 0 & -\dot{K} & 0 & 0 & 2KC & 0 \\ 0 & 0 & -\dot{K} & 0 & 0 & 2KC \\ \hline -\dot{C} & 0 & 0 & (-K^2+C^2) & 0 & 0 \\ 0 & -\dot{C} & 0 & 0 & (-K^2+C^2) & 0 \\ 0 & 0 & -\dot{C} & 0 & 0 & (-K^2+C^2) \end{array}\right) \\ & \begin{array}{cccccc} tx & ty & tz & yz & zx & xy \end{array} \end{array}$$

- A warm-up: Omit the Holst term and set C = 0 .

- The Palatini Lagrangian is (after more index hell):

$$L = \frac{3M_{pl}^2}{8\pi}\left[N a K^2 + a^2 \dot{K} - H^2 N a^3 \right]$$

- Equations of motion:

Vary N: $a^2 K - a^3 H^2 = 0$ FRW Energy

Vary K: $2NaK - 2a\dot{a} = 0$ Connection

Vary a: $NK^2 + 2a\dot{K} - 3H^2 N a^2 = 0$ FRW Pressure

- Identify canonical momentum and pass to Hamiltonian (call it U):

$$E \equiv P_K = \frac{3M_{pl}^2}{8\pi} a^2 \qquad U = P_K \dot{K} - L = \frac{3M_{pl}^2}{8\pi} N \left[H^2 a^3 - K^2 a \right].$$

- Note that the Hamiltonian constraint U = 0 is satisfied, via varying N, the lapse.

FRW Including the Holst Term

- The Lagrangian (after more index hell):

$$\frac{8\pi}{3M_{pl}^2} L = Na(K^2 - C^2) + a^2\dot{K} - H^2 N a^3 + \frac{1}{\gamma}\left[2NaKC + a^2\dot{C}\right]$$

- Only one combination of K and C carries a time derivative:

$$A = K + \frac{C}{\gamma}$$

- A and E (same as before) are conjugate variables. Eliminate K in the action:

$$\frac{8\pi}{3M_{pl}^2} L = Na^2 A + a^2\dot{A} - H^2 N a^3 - \frac{(\gamma^2+1)}{\gamma^2} NaC^2$$

- Only one new equation of motion:

Vary C: $$2\left(\frac{\gamma^2+1}{\gamma^2}\right) NaC = 0$$

- If $\gamma = \pm i$, special things happen. But we assume γ is real.

XII. The MacDowell-Mansouri Extension

- Purpose:

  Synthesize Palatini/ Holst / cosmological terms in the action into a single structure.

  Synthesize the connection ω and the tetrad e into a grand SO(4,1) connection A (Sorry, this is NOT the same as the A's in previous chapters).

  The theory becomes even more like Yang-Mills—a theory of the connection A only. (MacDowell-Mansouri; Freidel-Starodubtsev)

- The grand connection A:

$A, B = 01235$

$a, b = 0123$

$$
A_\mu^{AB} = \begin{array}{c} 50 \\ 51 \\ 52 \\ 53 \\ 01 \\ 02 \\ 03 \\ 23 \\ 31 \\ 12 \end{array} \left(\begin{array}{c} 5a \\ m e_\mu \\ \hdashline \\ \omega_\mu^{ab} \\ \\ \end{array} \right) \atop \begin{array}{cccc} t & x & y & z \end{array}
$$

{Append an index to the tetrad (vierbein)

{Multiply by a mass parameter to get the dimensionality right

- Dirac gamma-matrices γ_5 and γ_μ peacefully coexist with O(4,1):

<u>Note</u>: $\gamma_5^2 = -1 \qquad \gamma_0^2 = +1 \qquad \gamma_1^2 = \gamma_2^2 = \gamma_3^2 = -1$

- Contract the connection into gamma matrices:

$$A_\mu \equiv \frac{1}{2} A_\mu^{AB} \gamma_A \gamma_B$$

- Schematically,

$$\boxed{A_\mu = m e_\mu + \omega_\mu}$$

- Field strengths in this notation:

$$O(4,1) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$O(3,1) \qquad R_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu + [\omega_\mu, \omega_\nu]$$

- The MacDowell-Mansouri action:

$$S = C \int d^4x \, \mathrm{Tr}\, \gamma_5 F \wedge F \propto C \int d^4x \, \mathrm{Tr}\, \gamma_5 F_{\mu\nu} F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta}$$

- Generalization to include CP violation is simple:

$$\text{Let} \quad G = e^{\gamma_5 \theta} F = (\cos\theta + \gamma_5 \sin\theta) F$$

- Then

$$S = C \int d^4x \, \mathrm{Tr}\, G \wedge G$$

- When one expands out this action, one finds six terms.

$$\mathcal{L} = C\left[2\cos\theta\sin\theta\left\{\mathcal{L}_{GB} + m^2\mathcal{L}_{EH} + m^4\mathcal{L}_{CC}\right\} + \left\{(\cos^2\theta - \sin^2\theta)(\mathcal{L}_P + m^2\mathcal{L}_{NY}) + 2m^2\sin^2\theta\,\mathcal{L}_H\right\}\right]$$

- Three are familiar:

Einstein-Hilbert $\mathcal{L}_{EH} \propto \mathrm{Tr}\,\gamma_5\, e \wedge e \wedge R$

Holst $\mathcal{L}_H \propto \mathrm{Tr}\; e \wedge e \wedge R$

Cosmological constant $\mathcal{L}_{CC} \propto \mathrm{Tr}\,\gamma_5\, e \wedge e \wedge e \wedge e$

- Three are topological (total derivatives):

Euler (Gauss-Bonnet) $\mathcal{L}_{GB} \propto \mathrm{Tr}\,\gamma_5\, R \wedge R$

Pontryagin $\mathcal{L}_P \propto \mathrm{Tr}\; R \wedge R$

Nieh-Yan $\mathcal{L}_{NY} \propto \mathrm{Tr}\left[d_\omega e \wedge d_\omega e \pm e \wedge e \wedge R\right]$

Note: $d_\omega e \Leftrightarrow \partial_\mu e_\nu - \partial_\nu e_\mu + \omega_\mu e_\nu - \omega_\nu e_\mu$

- Note: topological terms do not affect equations of motion:

$$S_{topological} = 2\pi\int d^4x\, \partial_\mu K_\mu = 2\pi\int dt \frac{\partial Q}{\partial t} \qquad Q = \int d^3x\, K_0(x,t) = \text{integer}$$

- Look at simple FRW form and display A and F in matrix form:

$$A = \begin{pmatrix} mN & 0 \\ \hline 0 & m\,a(t) \\ \hline 0 & k(t) \\ \hline 0 & c(t) \end{pmatrix} \qquad F = \begin{pmatrix} \dot{\theta} & 0 \\ \hline m(\dot{a} - NK) & 2maC \\ \hline (\dot{K} - Nm^2 a) & 2Kc \\ \hline \dot{c} & m^2a^2 - K^2 + c^2 \end{pmatrix}$$

- Except for N, each nonvanishing entry is a multiple of a 3 x 3 unit matrix.
- In Dirac-matrix form,

$$F_{tx} = m(\dot{a} - NK)\gamma_5\gamma_1 + (\dot{K} - Nm^2 a)\gamma_0\gamma_1 + \dot{c}\,\gamma_2\gamma_3$$

$$F_{yz} = (2maC)\gamma_5\gamma_1 + (2Kc)\gamma_0\gamma_1 + (m^2a^2 - K^2 + c^2)\gamma_2\gamma_3$$

- The remaining algebra is now rather straightforward:

$$\mathcal{L} \propto C\,\frac{\mathrm{Tr}}{4}(\cos\theta + \gamma_5 \sin\theta)\,F_{tx}(\cos\theta + \gamma_5 \sin\theta)\,F_{yz}$$

$$F_{tx} = m(\hat{a} - NK)\gamma_5\gamma_1 + (\dot{K} - Nm^2a)\gamma_0\gamma_1 + \dot{c}\gamma_2\gamma_3$$

$$F_{yz} = (2maC)\gamma_5\gamma_1 + (2Kc)\gamma_0\gamma_1 + (m^2a^2 - K^2 + c^2)\gamma_2\gamma_3$$

$$\mathcal{L} \propto C\,\frac{T_r}{4}(\cos\theta + \gamma_5\sin\theta)\,F_{tx}\,(\cos\theta + \gamma_5\sin\theta)\,F_{yz}$$

- Evaluate the traces:

$\cos^2\theta$ terms: $\mathcal{L}_1 = \cos^2\theta\left[-2m^2a\,C(\hat{a} - \cancel{NK}) + 2KC(\dot{K} - \cancel{Nm^2a}) - \dot{c}(m^2a^2 - K^2 + c^2)\right]$

$\sin^2\theta$ terms: $\mathcal{L}_2 = \sin^2\theta\left[-2m^2aC(\hat{a} - NK) - 2Kc(\dot{K} - Nm^2a) + \dot{c}(m^2a^2 - K^2 + c^2)\right]$

$\cos\theta\sin\theta$ terms: $\mathcal{L}_3 = 2\sin\theta\cos\theta\left[(\dot{K} - Nm^2a)(m^2a^2 - K^2 + c^2) + \dot{c}(2Kc)\right]$

$$\mathcal{L}_1+\mathcal{L}_2 = \cos^2\theta\left[-2m^2 a\dot{a}C + 2K\dot{K}C - \dot{C}(m^2a^2-K^2+C^2)\right.$$
$$\left.+\sin^2\theta\left[-2m^2 aC(\dot{a}-NK) - 2KC(\dot{K}-Nm^2a) + \dot{C}(m^2a^2-K^2+C^2)\right]\right.$$
$$= (\cos^2\theta-\sin^2\theta)(\mathcal{L}_P + m^2\mathcal{L}_{NY}) + 2m^2\sin^2\theta\,\mathcal{L}_H$$

- The remaining terms are CP odd:

$$\mathcal{L}_1+\mathcal{L}_2 = (\cos^2\theta-\sin^2\theta)\left[2KC\dot{K}+K^2\dot{C}-C^2\dot{C}\right] + m^2\cos^2\theta\left[-2a\dot{a}C - a^2\dot{C}\right]$$
$$-m^2\sin^2\theta\left\{\left[4NKCa + 2a^2\dot{C}\right] - \left[a^2\dot{C} + 2a\dot{a}C\right]\right\}$$

- The Pontryagin term is similar to the GB term:

$$\mathcal{L}_P = 2K\dot{K}C + K^2\dot{C} - C^2\dot{C} = \frac{d}{dt}\left(-\frac{1}{3}C^3 + CK^2\right)$$

- The Holst term is

$$\mathcal{L}_H = 2NaCK + a^2\dot{C}$$

- The Nieh-Yan term is a total time derivative:

$$\mathcal{L}_{NY} = -a^2\dot{C} - 2a\dot{a}C = -\frac{d}{dt}(a^2C)$$

Determining the coefficients

$$\mathcal{L} = C \sin 2\theta \left[\mathcal{L}_{GB} + m^2 \mathcal{L}_{EH} + m^4 \mathcal{L}_{CC} \right] + 2Cm^2 \sin^2\theta \, \mathcal{L}_H + \ldots$$

- The ratio of the CC to the EH term determines m. It is precisely H, the deSitter Hubble-expansion rate:

$$C \sin 2\theta \, m^2 \sim M_{pl}^2$$

$$C \sin 2\theta \, m^4 \sim M_{pl}^2 \Lambda \equiv M_{pl}^2 \frac{H}{3}$$

$$\boxed{m^2 \sim \Lambda}$$

- The ratio of the Holst term to the Einstein-Hilbert term determines θ as a function of the Barbero-Immirzi parameter γ:

$$\frac{C \sin 2\theta \, m^2}{2 c m^2 \sin^2\theta} \equiv \frac{1}{\gamma}$$

$$\boxed{\frac{1}{\gamma} = \cot\theta}$$

- The overall normalization is now determined by the absolute normalization of the EH term:

$$C \sin 2\theta \, m^2 \sim M_{pl}^2$$

$$\therefore \quad C \sin 2\theta \sim \frac{M_{pl}^2}{m^2} \sim \frac{M_{pl}^2}{\Lambda} = \frac{M_{pl}^2}{3H^2} \sim 10^{120}$$

- The Gauss-Bonnet coefficient is huge!!
- The MacDowell-Mansouri coefficient likewise is huge!!

Comments

- We reach the same conclusion regarding the overall normalization, even if CP violating terms are excised from the beginning.
- There are nontrivial solutions of the equation $F = 0$. They correspond to pure deSitter space.
- When applied to FRW cosmology, this implies that the wave function of an expanding, dark-energy-driven comoving box has no semiclassical phase.
- The above statement is in conflict with the calculations done in Chapter IV, which give a phase which is proportional to volume in QCD units.
- The resolution of this conflict lies in the presence of the GB term in the action, which generates a compensating phase.

- Review of the metric gravity situation (Chapter V):

- For a deSitter box, the phase accumulation is proportional to the exponentially increasing volume:

$$S \propto (H M_{pl}^2) V(t) \sim \Lambda_{QCD}^3 V(t)$$

- What to do? Include the Gauss-Bonnet term. In metric gravity language, it is

$$\mathcal{L}_{GB} \propto \int d^4x \sqrt{-g}\, R_{\mu\nu}^{\alpha\beta} R_{\lambda\sigma}^{\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \epsilon^{\mu\nu\lambda\sigma}$$

- For FRW cosmology, evaluation is easy:

$$\mathcal{L}_{GB} \propto \int dt\, d^3\xi\, a^3(t) \left(\frac{\ddot{a}}{a}\right)\left(\frac{\dot{a}}{a}\right)^2 = \int dt \frac{d}{dt}(\dot{a})^3 d^3\xi$$

$$= H^3 \int d^3\xi\, a^3(t) = H^3 V(t)$$

- --

- The Gauss-Bonnet action is again proportional to the exponentially increasing volume:

$$\mathcal{L}_{GB} \sim H^3 \, V(t)$$

- But the natural size of the term as we have defined it is too small.
- A cancellation can be arranged provided the coefficient in front of the Gauss-Bonnet term is of order 10^120.
- The Gauss-Bonnet factor is what is called in LQG "the Kodama wave function."
- In this interpretation, this is not the wave function of the universe, as sometimes stated in LQG. But there has to be a connection somehow.
- Is there an "inaction principle" that states that deSitter space does not evolve quantum-mechanically?

One Final Comment

- The MacDowell-Mansouri extension suggests in a very natural way further extensions:

  1) Expand the internal gauge group beyond O(4,1) in order to incorporate standard-model internal symmetries.

  2) Increase the number of spacetime dimensions, keeping a similar structure for the action, again with an eye to incorporating standard model internal symmetries.

Relatively little has been done so far in these directions.

XIII. Conclusions

- The real conclusion is no conclusion.
- The dark energy problem is out there to be solved.
- I look for a synthesis of some of the ideas scattered through these notes.
- The topological terms in the action perhaps should be not ignored—it does not take much to put some dynamics in them.

Regrettable Omissions

- Phenomenology of inflation.
- Bounce cosmologies.
- Effects of space curvature.
- Rotating black holes.
- Gravitational optics.
- More on spinors

Thanks for looking at all this!!

- Comments and criticisms are most welcome.

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- Thanks again!