Study of an Expanding Bubble in the Rayleigh Model

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Abstract: In this study we investigate the expanding bubble in the rayleigh model. A bubble expands adiabatically in an incompressible, inviscid liquid. The variation of its radius R with time is given by the Rayleigh's equation. We find that the bubble is stable at the equilibrium point in this model.

Keywords*: *Bubble, Rayleigh's model and stability

I. INTRODUCTION :

When a bubble expands adiabatically in an inviscid liquid at rest at infinity, the variation of its radius R with time is given by Rayleigh's equation, which is a highly non - linear equation. This equation has been generalized including viscosity of surrounding liquid and surface tension by Noltingk and Neppiras and Poritsky. Progress in the study of this equation is generally expected numerically. However, when the liquid out side the bubble is inviscid, Rayleigh's modified equation can be transformed suitably and we can prove, analytically, that the expanding bubble in this model is stable at the equilibrium point . .

II. MATHEMATICAL ANALYSIS :

As the bubble epands adiabatically, its radius R is given by Rayleigh's equation, modified by surface tension σ. This equation can be written as –

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2+\frac{1}{\rho}\left\{p_e-p_{g0}\left(\frac{R_0}{R}\right)^{3\gamma}+\frac{2\sigma}{R}\right\}=0, \tag{1}$$

Where ρ is the density of the outside inviscid liquid, ρ_{go} and Ro are the gas pressure and radius of the bubble initially, p_e is the pressure, taken as constant, in the liquid at a large distance from the bubble and γ is the ratio of two specific heats of the gas. Than find that equation (1) can be written in the form-

$$\frac{d^2}{dt^2}(R^{5/2})+\frac{5}{2}\frac{R^{1/2}}{\rho}\left\{p_e-p_{g0}\left(\frac{R_0}{R}\right)^{3\gamma}+\frac{2\sigma}{R}\right\}=0. \tag{2}$$

Defning

$$r = R^{5/2}$$

and taking $\gamma = 4/3$ for simplicity, we finally get from (2) the equation

$$\frac{d^2r}{dt^2}+\frac{5}{2\rho}r^{1/5}\left\{p_e-p_{g0}\left(\frac{r_0}{r}\right)^{8/5}+2\sigma r^{-2/5}\right\}=0, \tag{3}$$

where $r_0 = R_0^{5/2}$

We use ρ_{go} and U0 as characteristic pressure and speed respectively and ro as the characteristic value of r and To as that of time, where To = Ro/Uo. Than define the dimensionless quantities r', t' and p'e as

$$R' = r/ro,\ t' = t/To,\ p'e = pe/p_{go}$$

We may now write eq. (3) as

$$\frac{d^2r'}{dt'^2}=\frac{-5p_{g0}}{2\rho U_0^2}\left\{p'_e r'^{1/5}+\frac{2\sigma}{p_{g0}R_0}r'^{-1/5}-r'^{-7/5}\right\}. \tag{4}$$

Omitting the dashes from r', t' and p'e and using from now on these undashed symbols for the corresponding quantities, than find that eq (4) can be written in dimensionless form as

$$\frac{d^2r}{dt^2}=\frac{-5p_{g0}}{2\rho U_0^2}\left\{p_e r^{1/5}+\frac{2\sigma}{p_{g0}R_0}r^{-1/5}-r^{-7/5}\right\}. \tag{5}$$

III. CONCLUSION

We find that F (x) vanishes at the equilibrium point. Also H and its partial derivatives are continuous at all points except when x = 0. If ά is a positive real root, then shows that x = ά and y = 0 is the equilibrium point. Let us choose C in such a say that at this equilibrium point, the value of H vanishes. Which is always satisfied as x is real and positive. Therefore, H is minimum at the equilibrium point x = ά , y = 0, but H vanishes at (ά, o). Thus H is always positive near (ά, o) and is therefore positive definite in the neighbourhood of the equilibrium point.

REFERENCES:

[1] Chakraborty B B, Effect of a viscous fluid flow past a spherical gas bubble on the growth o f its radius. Proc. Ind. Acad. Sci. (Math. Sci.) 100 (1990) 185-188

[2] Chakraborty B B and Tuteja G S, Motion o f an expanding gas bubble in a viscous liquid under gravity, Phys. Fluids 5 (1993) 187

[3] Drazin P G, Nonlinear Systems (Cambridge: Cambridge University Press) (1992)

[4] Lord Rayleigh, On the pressure developed in a liquid during the collapse of a spherical void. Philos. Mag. 34 (1917) 94

[5] Noltingk B E and Neppiras E A, Proc. Phys. Soc. London Sec. B63 (1950) 674-685; 64 (1951) 1032-1038

[6] Poritsky H , The collapse or growth o f a spherical bubble or cavity in a viscous fluid, Proceedings o f the first U.S. National Congress on Applied Mechanics (ed.) E Sternberg, Am. Soc. Mech. Eng. (1952) (New York) pp. 813-821