



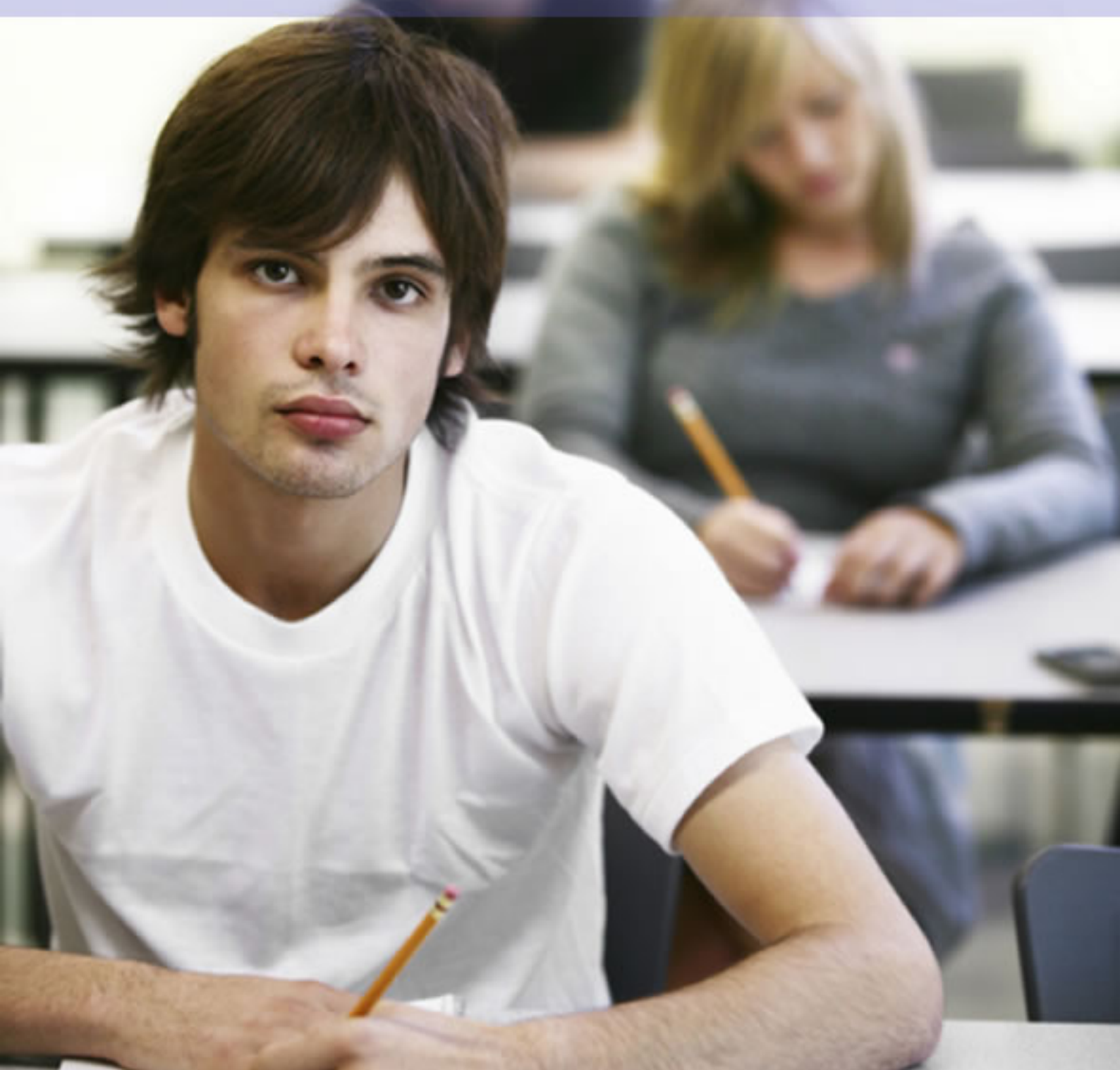
Lumen artis mathematicae

Years 11 & 12

Author: R. M. O'Toole

GCSE Ordinary Level

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Contents List

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GCSE ORDINARY LEVEL: CONTENTS LIST

SECTION 1: NUMBER

Part

Contents

1.

TYPES OF NUMBERS:

Natural Numbers

Whole Numbers

Integers

Rational Numbers

Terminating Decimal Numbers

Non – terminating Repeating Decimal Numbers

Irrational Numbers

Real Numbers

Venn Diagrams for sets of numbers

Prime Numbers

Square Numbers

Cubic Numbers

Triangular Numbers

Fibonacci Numbers

SURDS:

Simplifying Surds

Rationalizing the Denominator of Surds

Highest Common Factor (H.C.F.)

Lowest Common Multiple (L.C.M.)

KEY STAGE 3 MATERIAL:

[N.B. Knowledge of the **following Sections of Key Stage 3** required also.]

Section 3

Section 4

Section 5

Section 6

Section 7

Section 8

Section 9

Section 20

2. NUMBER SEQUENCES & PATTERNS:

Linear Sequences

Quadratic Sequences

Cubic Sequences

3. INDICES & STANDARD FORM:

Laws of Indices

Standard Form: $A \times 10^n$

4. ROUNDING OFF:

Decimal Places

Significant Figures

5. UPPER & LOWER BOUNDS

6. APPROXIMATIONS



SECTION 2: SHAPE: EXPLORATION

Part

Contents

1.

TRIANGLES:

Types of Triangle

Area Formulae: $\frac{1}{2}bh$ and $\frac{1}{2}ab \sin C$

Pythagoras's Theorem

Similar Triangles

Congruent Triangles

2.

QUADRILATERALS:

Types of Quadrilateral:

Square

Rectangle

Parallelogram

Rhombus

Kite

Trapezium

- 3. POLYGONS:**
- Interior Angles of Polygons
 - Exterior Angles of Polygons
 - Names of Polygons
 - Regular Polygon
 - Worked Examples on Polygons
- 4. TESSELLATION OF SHAPES**
- 5. THE CIRCLE:**
- Components of Circle
 - Circle theorems
- 6. LOCUS:**
- Intersecting Loci
- 7. PARALLEL LINES AND TRANSVERSALS:**
- Vertically – Opposite Angles
 - Corresponding Angles
 - Alternate Angles
 - Interior Angles

8. SIMILAR SHAPES:

Similar lengths

Similar areas

Similar volumes



SECTION 3: SHAPES: POSITION AND MOVEMENT

Part

Contents

1.

DIRECTIONS:

Cardinal Points: N, S, E, W, NE, SE, SW, NW

Three – figure Bearings

Use of Co-ordinates: 2-D and 3-D

2.

SHAPES: SYMMETRY

3.

TRANSFORMATIONS:

Reflection in a line: $x = a$; $y = b$; $y = x$; $y = -x$

Rotation about centre (x, y) through $\pm 90^\circ$, 180°

Enlargement (or reduction), using centre (x, y) ,

together with \pm scale factor

Translation through $\begin{pmatrix} x \\ y \end{pmatrix}$

4.

VECTORS:

Magnitude

Direction

Position Vector

Displacement Vector

Addition and Subtraction of Vectors

Multiplication of a Vector by a scalar

SECTION 4: SHAPES: MEASUREMENT

Part

Contents

- 1. WEIGHTS AND MEASURES**
Metric / Imperial Conversions
Temperature Scales: Celsius & Fahrenheit
- 2. LINEAR, SQUARE AND CUBIC MEASURE:**
1 – Dimensional Measure
2 – Dimensional Measure
3 – Dimensional Measure
- 3. PERIMETER, AREA AND VOLUME:**
Rectangle
Triangle
Circle
- 4. NET:**
Cube
Rectangle
Triangular Pyramid

5. IRREGULAR AREA: AREA- APPROXIMATING RULES:

Trapezium Rule

Mid-Ordinate Rule

Simpson's Rule

6. PERIMETERS, SURFACE AREAS AND VOLUMES OF FIGURES

Parallelogram

Circle

Area of Sector of Circle (includes radian formula)

Length of Arc of Circle (includes radian formula)

Cylinder

Cone

Sphere

Prism

Solid of Constant Cross – Section

Pyramid

7. PRACTICE QUESTIONS

Practice Questions: Answers

SECTION 5: SHAPES: TRIGONOMETRY

Part

Contents

1. TRIGONOMETRY: RIGHT ANGLED TRIANGLES

Sin, Cos and Tan

Angle of Elevation

Angle of Depression

Bearings

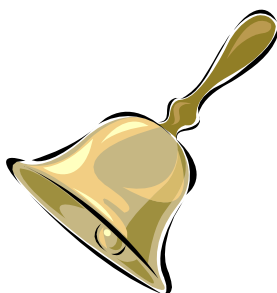
3 – Dimensional Trigonometry

2. TRIGONOMETRY: ANGLES 0° TO 360°

Graphs of $\sin x$, $\cos x$ and $\tan x$

Sine Rule

Cosine Rule



SECTION 6: ALGEBRA 1:

SEQUENCES: n^{th} Term Formula

KEY CONCEPTS; MANIPULATIVE SKILLS

Part

Contents

1. NUMBER SEQUENCES & PATTERNS:

n^{th} term linear

n^{th} term quadratic

n^{th} term cubic

2. ALGEBRA: (KEY CONCEPTS.)

What is Algebra?

The four basic rules: Add +, Subtract -, Multiply \times , Divide \div

Brackets

Powers

Like Terms

Simplifying: (Expanding and collating like terms.)

Worked Examples

Worked Examples: Answers

Practical Applications: (Algebra studied so far.)

3. **FACTORISATION:**

Factor

Highest Common Factor (H.C.F.)

Prime Factor

Simplifying: [Factorising using H.C.F.]

Product of Two Binomial Expressions:

$$(a + b)(c + d)$$

Binomial Expansion:

$$(a + b)(c + d) = ac + bc + ad + bd$$

Factorising Quadratic Functions

Factorising Binomial Expansions: **Grouping:**

$$\begin{aligned}(ac + bc) + (ad + bd) &= c(a + b) + d(a + b) \\ &= (a + b)(c + d)\end{aligned}$$

Indices: Laws of Indices

Worked Examples

4. **ALGEBRAIC FRACTIONS:**

Simplifying

Lowest Common Multiple (L.C.M.)

The four basic rules: Add +, Subtract -, Multiply \times , Divide \div

Worked Examples

SECTION 7: ALGEBRA: GRAPHS

Part

Contents

- 1. CONVERSION GRAPHS:**
Temperatures ($^{\circ}\text{C}$ / $^{\circ}\text{F}$)
Mass (kg / lb)
Currency (€ / £)
Distance (km / miles)
- 2. AVERAGE SPEED, DISTANCE AND TIME:**
TRAVEL GRAPHS: VELOCITY / TIME GRAPHS
Average Speed, Distance and Time
Distance / Time Graphs
Displacement, Velocity and Acceleration
Velocity / Time Graphs
- 3. GRAPHS OF FUNCTIONS:**
Linear Function: $y = mx + c$
Line of **best fit**
Correlation and Scatter Diagram
Mid – Point Theorem: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Distance Formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Quadratic Function: $y = ax^2 + bx + c$
Exponential function: $y = a^x$ or $y = a^{-x}$

Reciprocal Function: $y = \frac{a}{x}; x \neq 0$

Asymptote

Simple Transformations on Graphs

**4. EQUATIONS / INEQUALITIES:
GRAPHICAL SOLUTION**

Simple Equations / Inequalities

Linear Equations / Inequalities

Quadratic Equations / Inequalities

Functions generally: Equations / Inequalities

Linear Programming: Linear Inequalities

Intersecting Graphs: Simultaneous Equations:

Linear Equations

Quadratic Equations

Other Equations

Intersecting Lines / Curves

SECTION 8: ALGEBRA: SOLUTION OF EQUATIONS & INEQUATIONS

Part

Contents

- 1. EQUATIONS AND INEQUATIONS:**
Simple Equations /Inequations

Quadratic Equations / Inequations:
Formula method
Factors method
Trial & Improvement method
Iteration method
Roots
Problems involving Quadratic Equations:
Worked Answers to quadratic problems
- 2. POLYNOMIAL EQUATIONS / INEQUATIONS:**
Factors method
Trial & Improvement method
- 3. SIMULTANEOUS EQUATIONS:**
Linear Equations: **2** variables

Lines and Curves
- 4. ALGEBRAIC FRACTIONAL EQUATIONS / INEQUATIONS**

5. **FORMULAE:**
Transposition / Changing the Subject
6. **VARIATION / PROPORTION:**
Direct Variation / Proportion
Inverse Variation / Proportion



SECTION 9: HANDLING DATA: STATISTICS & PROBABILITY

Part

Contents

1.

STATISTICS:

Frequency:

Raw Data

Frequency Table

Continuous

Discreet

Averages:

Mean

Median

Mode

Bar Graph

Histogram:

Frequency Density: $\left(\frac{\text{Frequency}}{\text{ClassWidth}}\right)$

Interquartile Range

Grouped Distribution:

Mean

Histogram

Cumulative Frequency Graph: (Ogive)

Median

Interquartile Range

Diagrams:

Box & Whisker

Stem & Leaf

Correlation & Scatter

Frequency Polygon

2. PROBABILITY

Definition of Probability

Probability Scale

Laws of Probability

Probability Tree

Worked Examples

PRACTICE PAPER 1: (Based on Section 1.)

SECTION 1: NUMBER

<u>Part</u>	<u>Contents</u>
--------------------	------------------------

1.	TYPES OF NUMBERS:
	Natural Numbers
	Whole Numbers
	Integers
	Rational Numbers
	Terminating Decimal Numbers
	Non – terminating Repeating Decimal Numbers
	Irrational Numbers
	Real Numbers
	Venn Diagrams for sets of numbers
	Prime Numbers
	Square Numbers
	Cubic Numbers
	Triangular Numbers
	Fibonacci Numbers

SURDS:

Simplifying Surds

Rationalizing the Denominator of Surds

Highest Common Factor (H.C.F.)

Lowest Common Multiple (L.C.M.)

KEY STAGE 3 MATERIAL:

[N.B. Knowledge of the **following Sections** of **Key Stage 3** required also.]

Section 3

Section 4

Section 5

Section 6

Section 7

Section 8

Section 9

Section 20

2.

NUMBER SEQUENCES & PATTERNS:

Linear Sequences

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3. INDICES & STANDARD FORM:

Laws of Indices

Standard Form: $A \times 10^n$

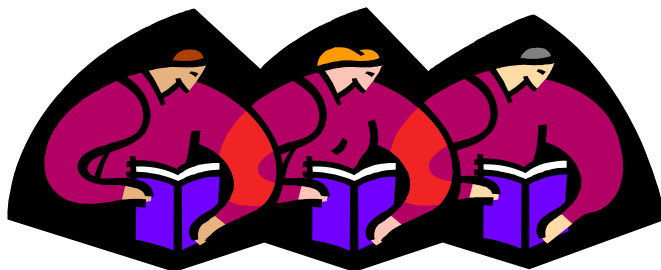
4. ROUNDING OFF:

Decimal Places

Significant Figures

5. UPPER & LOWER BOUNDS

6. APPROXIMATIONS



PRACTICE PAPER 2: (Based on Sections 2, 3, 4 and 5.)

SECTION 2: EXPLORATION OF SHAPE

Part

Contents

1.

TRIANGLES:

Types of Triangle

Area Formulae: $\frac{1}{2}bh$ and $\frac{1}{2}ab \sin C$

Pythagoras's Theorem

Similar Triangles

Congruent Triangles

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QUADRILATERALS:

Types of Quadrilateral:

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Rhombus

Kite

Trapezium

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 - Corresponding Angles
 - Alternate Angles
 - Interior Angles

8. SIMILAR SHAPES:

Similar lengths

Similar areas

Similar volumes

SECTION 3: SHAPES AND SPACE: POSITION AND MOVEMENT

Part Contents

1. DIRECTIONS:

Cardinal Points: N, S, E, W, NE, SE, SW, NW

Three – figure Bearings

Use of Co-ordinates: 2-D and 3-D

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3 – Dimensional Measure

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Rectangle

Triangle

Circle

4. **NET:**
- Cube
 - Rectangle
 - Triangular Pyramid
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- Trapezium Rule
 - Mid-Ordinate Rule
 - Simpson's Rule
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 - Cone
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 - Prism
 - Solid of Constant Cross – Section
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Practice Questions: Answers

SECTION 5: SHAPES: TRIGONOMETRY

Part Contents

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2. TRIGONOMETRY: ANGLES 0° TO 360°

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Sine Rule

Cosine Rule

PRACTICE PAPER 3: (Based on Sections 6 & 8.)

SECTION 6: ALGEBRA 1

SEQUENCES: n^{th} Term Formula

KEY CONCEPTS; MANIPULATIVE SKILLS

Part Contents

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Worked Examples

SECTION 8: ALGEBRA 2:

SOLUTION OF EQUATIONS & INEQUATIONS

Part

Contents

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5. FORMULAE:
Transposition / Changing the Subject

6. VARIATION / PROPORTION:
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Inverse Variation / Proportion

PRACTICE PAPER 4: (Based on Section 7.)

SECTION 7: GRAPHS

Part Contents

1. CONVERSION GRAPHS:

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Distance (km / miles)

2. AVERAGE SPEED, DISTANCE AND TIME:

TRAVEL GRAPHS: VELOCITY / TIME GRAPHS

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Distance / Time Graphs

Displacement, Velocity and Acceleration

Velocity / Time Graphs

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GRAPHICAL SOLUTION

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Linear Equations / Inequalities

Quadratic Equations / Inequalities

Functions generally: Equations / Inequalities

Linear Programming: Linear Inequalities

Intersecting Graphs: Simultaneous Equations:

Linear Equations

Quadratic Equations

Other Equations

Intersecting Lines / Curves

PRACTICE PAPER 5: (Based on Section 9.)

SECTION 9: HANDLING DATA: STATISTICS & PROBABILITY

Part

Contents

1.

STATISTICS:

Frequency:

Raw Data

Frequency Table

Continuous

Discreet

Averages:

Mean

Median

Mode

Bar Graph

Histogram:

Frequency Density: $\left(\frac{\text{Frequency}}{\text{ClassWidth}}\right)$

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Histogram

Cumulative Frequency Graph: (Ogive)

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Interquartile Range

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Stem & Leaf

Correlation & Scatter

Frequency Polygon

2.

PROBABILITY

Definition of Probability

Probability Scale

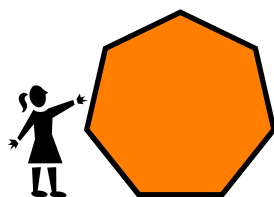
Laws of Probability

Probability Tree

Worked Examples

PRACTICE PAPER 6:

FOUNDATION LEVEL REVISION TEST



PRACTICE PAPER 7:

HIGHER LEVEL REVISION TEST

