Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

The binomial expansion
Exercise A, Question 1

Question:

Write down the expansion of:

(a) $(x+y)^4$

(b) $(p+q)^5$

(c) $(a-b)^3$

(d) $(x+4)^3$

(e) $(2x-3)^4$

(f) $(a+2)^5$

(g) $(3x-4)^4$

(h) $(2x-3y)^4$

Solution:

(a) $(x+y)^4$ would have coefficients and terms

1	4	6	4	1
x^4	x^3y	x^2y^2	xy^3	y^4

$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

(b) $(p+q)^5$ would have coefficients and terms

1	5	10	10	5	1
p^5	p^4q	p^3q^2	p^2q^3	pq^4	q^5

$(p+q)^5 = 1p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + 1q^5$

(c) $(a-b)^3$ would have coefficients and terms

1	3	3	1
a^3	$a^2(-b)$	$a(-b)^2$	$(-b)^3$

$(a-b)^3 = 1a^3 - 3a^2b + 3ab^2 - 1b^3$

(d) $(x+4)^3$ would have coefficients and terms

1 3 3 1
x^3 $x^2 4$ $x 4^2$ 4^3

$(x+4)^3 = 1x^3 + 12x^2 + 48x + 64$

(e) $(2x-3)^4$ would have coefficients and terms

1 4 6 4 1
$(2x)^4$ $(2x)^3(-3)$ $(2x)^2(-3)^2$ $(2x)(-3)^3$ $(-3)^4$

$(2x-3)^4 = 1(2x)^4 + 4(2x)^3(-3) + 6(2x)^2(-3)^2 + 4(2x)(-3)^3 + 1(-3)^4$
$(2x-3)^4 = 16x^4 - 96x^3 + 216x^2 - 216x + 81$

(f) $(a+2)^5$ would have coefficients and terms

1 5 10 10 5 1
a^5 $a^4 2$ $a^3 2^2$ $a^2 2^3$ $a 2^4$ 2^5

$(a+2)^5 = 1a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32$

(g) $(3x-4)^4$ would have coefficients and terms

1 4 6 4 1
$(3x)^4$ $(3x)^3(-4)$ $(3x)^2(-4)^2$ $(3x)(-4)^3$ $(-4)^4$

$(3x-4)^4 = 1(3x)^4 + 4(3x)^3(-4) + 6(3x)^2(-4)^2 + 4(3x)(-4)^3 + 1(-4)^4$
$(3x-4)^4 = 81x^4 - 432x^3 + 864x^2 - 768x + 256$

(h) $(2x-3y)^4$ would have coefficients and terms

1 4 6 4 1
$(2x)^4$ $(2x)^3(-3y)$ $(2x)^2(-3y)^2$ $(2x)(-3y)^3$ $(-3y)^4$

$(2x-3y)^4 = 1(2x)^4 + 4(2x)^3(-3y) + 6(2x)^2(-3y)^2 + 4(2x)(-3y)^3 + 1(-3y)^4$
$(2x-3y)^4 = 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$

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The binomial expansion
Exercise A, Question 2

Question:

Find the coefficient of x^3 in the expansion of:

(a) $(4 + x)^{4}$

(b) $(1 - x)^{5}$

(c) $(3 + 2x)^{3}$

(d) $(4 + 2x)^{5}$

(e) $(2 + x)^{6}$

(f) $\left(4 - \frac{1}{2}x \right)^{4}$

(g) $(x + 2)^{5}$

(h) $(3 - 2x)^{4}$

Solution:

(a) $(4 + x)^{4}$ would have coefficients 1 4 6 ④ 1
The circled number is the coefficient of the term 4^1x^3.
Term is $4 \times 4^1x^3 = 16x^3$
Coefficient $= 16$

(b) $(1 - x)^{5}$ would have coefficients 1 5 10 ⑩ 5 1
The circled number is the coefficient of the term $1^2 (- x)^{3}$.
Term is $10 \times 1^2 (- x)^{3} = - 10x^3$
Coefficient $= - 10$

(c) $(3 + 2x)^{3}$ would have coefficients 1 3 3 ①
The circled number is the coefficient of the term $(2x)^{3}$.
Term is $1 \times (2x)^{3} = 8x^3$
Coefficient $= 8$

(d) $(4 + 2x)^{5}$ would have coefficients 1 5 10 ⑩ 5 1
The circled number is the coefficient of the term $4^2 (2x)^{3}$.
Term is $10 \times 4^2 (2x)^{3} = 1280x^3$
Coefficient $= 1280$

(e) $(2 + x)^{6}$ would have coefficients 1 6 15 ⑳ 15 6 1
The circled number is the coefficient of the term 2^3x^3.
Term is $20 \times 2^3x^3 = 160x^3$
Coefficient $= 160$

(f) $\left(4 - \frac{1}{2}x\right)^4$ would have coefficients 1 4 6 ④ 1

The circled number is the coefficients of the term $4\left(-\frac{1}{2}x\right)^3$.

Term is $4 \times 4\left(-\frac{1}{2}x\right)^3 = -2x^3$

Coefficient $= -2$

(g) $(x + 2)^5$ would have coefficients 1 5 ⑩ 10 5 1
The circled number is the coefficient of the term $x^3 2^2$.
Term is $10 \times x^3 2^2 = 40x^3$
Coefficient $= 40$

(h) $(3 - 2x)^4$ would have coefficients 1 4 6 ④ 1
The circled number is the coefficient of the term $3^1(-2x)^3$.
Term is $4 \times 3^1(-2x)^3 = -96x^3$
Coefficient $= -96$

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The binomial expansion
Exercise A, Question 3

Question:

Fully expand the expression $(1 + 3x)(1 + 2x)^3$.

Solution:

$(1 + 2x)^3$ has coefficients and terms

$$\begin{matrix} 1 & 3 & 3 & 1 \\ 1^3 & 1^2(2x) & 1(2x)^2 & (2x)^3 \end{matrix}$$

Hence $(1 + 2x)^3 = 1 + 6x + 12x^2 + 8x^3$

$(1 + 3x)(1 + 2x)^3$
$= (1 + 3x)(1 + 6x + 12x^2 + 8x^3)$
$= 1(1 + 6x + 12x^2 + 8x^3) + 3x(1 + 6x + 12x^2 + 8x^3)$
$= 1 + 6x + 12x^2 + 8x^3 + 3x + 18x^2 + 36x^3 + 24x^4$
$= 1 + 9x + 30x^2 + 44x^3 + 24x^4$

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The binomial expansion

Exercise A, Question 4

Question:

Expand $(2+y)^3$. Hence or otherwise, write down the expansion of $(2+x-x^2)^3$ in ascending powers of x.

Solution:

$(2+y)^3$ has coefficients and terms

1	3	3	1
2^3	2^2y	$2y^2$	y^3

Therefore, $(2+y)^3 = 8 + 12y + 6y^2 + 1y^3$

Substitute $y = x - x^2$

$\Rightarrow \quad (2+x-x^2)^3 = 8 + 12(x-x^2) + 6(x-x^2)^2 + 1(x-x^2)^3$

$\Rightarrow \quad (2+x-x^2)^3 = 8 + 12x(1-x) + 6x^2(1-x)^2 + x^3(1-x)^3$

Now

$(1-x)^2 = (1-x)(1-x) = 1 - 2x + x^2$

and

$(1-x)^3 = (1-x)(1-x)^2$

$(1-x)^3 = (1-x)(1-2x+x^2)$

$(1-x)^3 = 1 - 2x + x^2 - x + 2x^2 - x^3$

$(1-x)^3 = 1 - 3x + 3x^2 - x^3$

Or, using Pascal's Triangles

$(1-x)^3 = 1(1)^3 + 3(1)^2(-x) + 3(1)(-x)^2 + 1(-x)^3$

$(1-x)^3 = 1 - 3x + 3x^2 - x^3$

So $(2+x-x^2)^3 = 8 + 12x(1-x) + 6x^2(1-2x+x^2) + x^3(1-3x+3x^2-x^3)$

$\Rightarrow \quad (2+x-x^2)^3 = 8 + 12x - 12x^2 + 6x^2 - 12x^3 + 6x^4 + x^3 - 3x^4 + 3x^5 - x^6$

$\Rightarrow \quad (2+x-x^2)^3 = 8 + 12x - 6x^2 - 11x^3 + 3x^4 + 3x^5 - x^6$

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The binomial expansion
Exercise A, Question 5

Question:

Find the coefficient of the term in x^3 in the expansion of $(2+3x)^3(5-x)^3$.

Solution:

$(2+3x)^3$ would have coefficients and terms

1	3	3	1
2^3	$2^2(3x)$	$2(3x)^2$	$(3x)^3$

$(2+3x)^3 = 1\times 2^3 + 3\times 2^2(3x) + 3\times 2(3x)^2 + 1\times(3x)^3$
$(2+3x)^3 = 8 + 36x + 54x^2 + 27x^3$

$(5-x)^3$ would have coefficients and terms

1	3	3	1
5^3	$5^2(-x)$	$5(-x)^2$	$(-x)^3$

$(5-x)^3 = 1\times 5^3 + 3\times 5^2(-x) + 3\times 5(-x)^2 + 1\times(-x)^3$
$(5-x)^3 = 125 - 75x + 15x^2 - x^3$

$(2+3x)^3(5-x)^3 = (8+36x+54x^2+27x^3)(125-75x+15x^2-x^3)$

Term in x^3 is
$8\times(-x^3) + 36x\times 15x^2 + 54x^2\times(-75x) + 27x^3\times 125$
$= -8x^3 + 540x^3 - 4050x^3 + 3375x^3$
$= -143x^3$
Coefficient of $x^3 = -143$

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The binomial expansion
Exercise A, Question 6

Question:

The coefficient of x^2 in the expansion of $(2 + ax)^3$ is 54. Find the possible values of the constant a.

Solution:

$(2 + ax)^3$ has coefficients 1 3 ③ 1
The circled number is the coefficient of the term $2^1 (ax)^2$.
Term in x^2 is $3 \times 2^1 \times (ax)^2 = 6a^2x^2$
Coefficient of x^2 is $6a^2$.
Hence
$6a^2 = 54 \quad (\div 6)$
$a^2 = 9$
$a = \pm 3$

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The binomial expansion
Exercise A, Question 7

Question:

The coefficient of x^2 in the expansion of $(2 - x)(3 + bx)^3$ is 45. Find possible values of the constant b.

Solution:

$(3 + bx)^3$ has coefficients and terms

1	3	3	1
3^3	$3^2(bx)$	$3(bx)^2$	$(bx)^3$

$(3 + bx)^3 = 1 \times 3^3 + 3 \times 3^2 bx + 3 \times 3(bx)^2 + 1 \times (bx)^3$
$(3 + bx)^3 = 27 + 27bx + 9b^2x^2 + b^3x^3$

So $(2 - x)(3 + bx)^3 = (2-x)(27+27bx + 9b^2x^2 + b^3x^3)$

Term in x^2 is $2 \times 9b^2x^2 - x \times 27bx = 18b^2x^2 - 27bx^2$
Coefficient of x^2 is $18b^2 - 27b$
Hence
$18b^2 - 27b = 45 \quad (\div 9)$
$2b^2 - 3b = 5$
$2b^2 - 3b - 5 = 0$
$(2b - 5)(b + 1) = 0$
$b = \frac{5}{2}, -1$

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The binomial expansion
Exercise A, Question 8

Question:

Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{2x} \right)^3$.

Solution:

$\left(x^2 - \frac{1}{2x} \right)^3$ has coefficients and terms

1	3	3	1
$(x^2)^3$	$(x^2)^2\left(-\frac{1}{2x}\right)^1$	$(x^2)\left(-\frac{1}{2x}\right)^2$ ↑	$\left(-\frac{1}{2x}\right)^3$

This term would be independent of x as the x's cancel.

Term independent of x is $3\left(x^2 \right)\left(-\frac{1}{2x} \right)^2 = 3x^2 \times \frac{1}{4x^2} = \frac{3}{4}$

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The binomial expansion
Exercise B, Question 1

Question:

Find the values of the following:

(a) $4!$

(b) $6!$

(c) $\frac{8!}{6!}$

(d) $\frac{10!}{9!}$

(e) ${}^4C_2$

(f) ${}^8C_6$

(g) ${}^5C_2$

(h) ${}^6C_3$

(i) ${}^{10}C_9$

(j) ${}^6C_2$

(k) ${}^8C_5$

(l) ${}^nC_3$

Solution:

(a) $4! = 4 \times 3 \times 2 \times 1 = 24$

(b) $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

(c) $\frac{8!}{6!} = \frac{8 \times 7 \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}} = 8 \times 7 = 56$

(d) $\frac{10!}{9!} = \frac{10 \times \cancel{9!}}{\cancel{9!}} = 10$

(e) ${}^4C_2 = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{4 \times 3 \times \cancel{2} \times \cancel{1}}{2 \times 1 \times \cancel{2} \times \cancel{1}} = \frac{12}{2} = 6$

(f) ${}^{8}C_{6} = \frac{8!}{(8-6)!6!} = \frac{8!}{2!6!} = \frac{8\times7\times\cancel{6!}}{2!\ \cancel{6!}} = \frac{56}{2} = 28$

(g) ${}^{5}C_{2} = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!} = \frac{5\times4\times\cancel{3!}}{\cancel{3!}\ 2!} = \frac{20}{2} = 10$

(h) ${}^{6}C_{3} = \frac{6!}{(6-3)!3!} = \frac{6!}{3!3!} = \frac{6\times5\times4\times\cancel{3!}}{3!\times\cancel{3!}} = \frac{\cancel{6}\times5\times4}{\cancel{6}} = 20$

(i) ${}^{10}C_{9} = \frac{10!}{(10-9)!9!} = \frac{10!}{1!9!} = \frac{10\times\cancel{9!}}{1!\ \cancel{9!}} = \frac{10}{1} = 10$

(j) ${}^{6}C_{2} = \frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = \frac{6\times5\times\cancel{4!}}{\cancel{4!}\ 2!} = \frac{30}{2} = 15$

(k) ${}^{8}C_{5} = \frac{8!}{(8-5)!5!} = \frac{8\times7\times6\times\cancel{5!}}{3!\ \cancel{5!}} = \frac{8\times7\times\cancel{6}}{\cancel{6}} = 56$

(l) ${}^{n}C_{3} = \frac{n!}{(n-3)!3!} = \frac{n\times(n-1)\times(n-2)\times\cancel{(n-3)!}}{\cancel{(n-3)!}\ 3!} = \frac{n(n-1)(n-2)}{6}$

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The binomial expansion
Exercise B, Question 2

Question:

Calculate:

(a) ${}^4C_0$

(b) $\binom{4}{1}$

(c) ${}^4C_2$

(d) $\binom{4}{3}$

(e) $\binom{4}{4}$

Now look at line 4 of Pascal's Triangle. Can you find any connection?

Solution:

(a) ${}^4C_0 = \frac{4!}{(4-0)!0!} = \frac{4!}{4!\,0!} = 1$

(b) $\binom{4}{1} = \frac{4!}{(4-1)!1!} = \frac{4!}{3!1!} = \frac{4\times3!}{3!\,1!} = 4$

(c) ${}^4C_2 = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = \frac{4\times3\times2!}{2!\,2!} = \frac{12}{2} = 6$

(d) $\binom{4}{3} = \frac{4!}{(4-3)!3!} = \frac{4!}{1!3!} = \frac{4\times3!}{1!\,3!} = \frac{4}{1} = 4$

(e) $\binom{4}{4} = \frac{4!}{(4-4)!4!} = \frac{4!}{0!\,4!} = \frac{1}{0!} = 1$

The numbers 1, 4, 6, 4, 1 form the fourth line of Pascal's Triangle.

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The binomial expansion
Exercise B, Question 3

Question:

Write using combination notation:

(a) Line 3 of Pascal's Triangle.

(b) Line 5 of Pascal's Triangle.

Solution:

(a) Line 3 of Pascal's Triangle is

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

(b) Line 5 of Pascal's Triangle is

$$\binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$$

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The binomial expansion

Exercise B, Question 4

Question:

Why is $^6C_2$ equal to $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$?

(a) Answer using ideas on choosing from a group.

(b) Answer by calculating both quantities.

Solution:

(a) $^6C_2$ or $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ is the number of ways of choosing 2 items from a group of 6 items.

$\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ or $^6C_4$ is the number of ways of choosing 4 items from a group of 6 items.

These have to be the same.
For example, if you have a group of six people and want to pick a team of four, you have automatically selected a team of two.

(b) $^6C_2 = \frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = \frac{6 \times 5 \times \cancel{4!}}{\cancel{4!}\ 2!} = 15$

$\begin{pmatrix} 6 \\ 4 \end{pmatrix} = \frac{6!}{(6-4)!4!} = \frac{6!}{2!4!} = \frac{6 \times 5 \times \cancel{4!}}{2!\ \cancel{4!}} = 15$

Hence $^6C_2 = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

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The binomial expansion
Exercise C, Question 1

Question:

Write down the expansion of the following:

(a) $(2x+y)^4$

(b) $(p-q)^5$

(c) $(1+2x)^4$

(d) $(3+x)^4$

(e) $\left(1-\frac{1}{2}x\right)^4$

(f) $(4-x)^4$

(g) $(2x+3y)^5$

(h) $(x+2)^6$

Solution:

(a) $(2x+y)^4$
$= {}^4C_0 (2x)^4 + {}^4C_1 (2x)^3 (y) + {}^4C_2 (2x)^2 (y)^2 + {}^4C_3 (2x)^1 (y)^3 + {}^4C_4 (y)^4$
$= 1 \times 16x^4 + 4 \times 8x^3y + 6 \times 4x^2y^2 + 4 \times 2xy^3 + 1 \times y^4$
$= 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$

(b) $(p-q)^5$
$= {}^5C_0\ p^5 + {}^5C_1\ p^4(-q) + {}^5C_2\ p^3(-q)^2 + {}^5C_3\ p^2(-q)^3 + {}^5C_4\ p(-q)^4 + {}^5C_5(-q)^5$
$= 1 \times p^5 + 5 \times (-p^4q) + 10 \times p^3q^2 + 10 \times (-p^2q^3) + 5 \times pq^4 + 1 \times (-q^5)$
$= p^5 - 5p^4q + 10p^3q^2 - 10p^2q^3 + 5pq^4 - q^5$

(c) $(1+2x)^4$
$= {}^4C_0 (1)^4 + {}^4C_1 (1)^3 (2x)^1 + {}^4C_2 (1)^2 (2x)^2 + {}^4C_3 (1)(2x)^3 + {}^4C_4 (2x)^4$
$= 1 \times 1 + 4 \times 2x + 6 \times 4x^2 + 4 \times 8x^3 + 1 \times 16x^4$
$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$

(d) $(3+x)^4$
$= {}^4C_0 (3)^4 + {}^4C_1 (3)^3 (x) + {}^4C_2 (3)^2 (x)^2 + {}^4C_3 (3)(x)^3 + {}^4C_4 (x)^4$
$= 1 \times 81 + 4 \times 27x + 6 \times 9x^2 + 4 \times 3x^3 + 1 \times x^4$
$= 81 + 108x + 54x^2 + 12x^3 + x^4$

(e) $\left(1-\frac{1}{2}x\right)^4$

$= {}^4C_0(1)^4 + {}^4C_1(1)^3\left(-\frac{1}{2}x\right) + {}^4C_2(1)^2\left(-\frac{1}{2}x\right)^2 + {}^4C_3\left(1\right)\left(-\frac{1}{2}x\right)^3 + {}^4C_4\left(-x\right)^4$

$= 1 \times 1 + 4 \times \left(-\frac{1}{2}x\right) + 6 \times \frac{1}{4}x^2 + 4 \times \left(-\frac{1}{8}x^3\right) + 1 \times \frac{1}{16}x^4$

$= 1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{16}x^4$

(f) $(4 - x)^4$
$= {}^4C_0(4)^4 + {}^4C_1(4)^3(-x) + {}^4C_2(4)^2(-x)^2 + {}^4C_3(4)^1(-x)^3 + {}^4C_4(-x)^4$
$= 1 \times 256 + 4 \times (-64x) + 6 \times 16x^2 + 4 \times (-4x^3) + 1 \times x^4$
$= 256 - 256x + 96x^2 - 16x^3 + x^4$

(g) $(2x + 3y)^5$
$= {}^5C_0(2x)^5 + {}^5C_1(2x)^4(3y) + {}^5C_2(2x)^3(3y)^2 + {}^5C_3(2x)^2(3y)^3 + {}^5C_4(2x)(3y)^4 + {}^5C_5(3y)^5$

$= 1 \times 32x^5 + 5 \times 48x^4y + 10 \times 72x^3y^2 + 10 \times 108x^2y^3 + 5 \times 162xy^4 + 1 \times 243y^5$
$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

(h) $(x + 2)^6$
$= {}^6C_0(x)^6 + {}^6C_1(x)^5 2^1 + {}^6C_2(x)^4 2^2 + {}^6C_3(x)^3 2^3 + {}^6C_4(x)^2 2^4 + {}^6C_5(x)^1 2^5 + {}^6C_6 2^6$
$= 1 \times x^6 + 6 \times 2x^5 + 15 \times 4x^4 + 20 \times 8x^3 + 15 \times 16x^2 + 6 \times 32x + 1 \times 64$
$= x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$

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The binomial expansion
Exercise C, Question 2

Question:

Find the term in x^3 of the following expansions:

(a) $(3 + x)^5$

(b) $(2x + y)^5$

(c) $(1 - x)^6$

(d) $(3 + 2x)^5$

(e) $(1 + x)^{10}$

(f) $(3 - 2x)^6$

(g) $(1 + x)^{20}$

(h) $(4 - 3x)^7$

Solution:

(a) $(3 + x)^5$
Term in x^3 is ${}^5C_3 (3)^2 (x)^3 = 10 \times 9x^3 = 90x^3$

(b) $(2x + y)^5$
Term in x^3 is ${}^5C_2 (2x)^3 (y)^2 = 10 \times 8x^3y^2 = 80x^3y^2$

(c) $(1 - x)^6$
Term in x^3 is ${}^6C_3 (1)^3 (- x)^3 = 20 \times (- 1x^3) = - 20x^3$

(d) $(3 + 2x)^5$
Term in x^3 is ${}^5C_3 (3)^2 (2x)^3 = 10 \times 72x^3 = 720x^3$

(e) $(1 + x)^{10}$
Term in x^3 is ${}^{10}C_3(1)^7 (x)^3 = 120 \times 1x^3 = 120x^3$

(f) $(3 - 2x)^6$
Term in x^3 is ${}^6C_3 (3)^3 (- 2x)^3 = 20 \times (- 216x^3) = - 4320x^3$

(g) $(1 + x)^{20}$
Term in x^3 is ${}^{20}C_3 (1)^{17} (x)^3 = 1140 \times 1x^3 = 1140x^3$

(h) $(4 - 3x)^7$
Term in x^3 is ${}^7C_3 (4)^4 (- 3x)^3 = 35 \times (- 6912x^3) = - 241 \quad 920x^3$

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The binomial expansion
Exercise C, Question 3

Question:

Use the binomial theorem to find the first four terms in the expansion of:

(a) $(1+x)^{10}$

(b) $(1-2x)^{5}$

(c) $(1+3x)^{6}$

(d) $(2-x)^{8}$

(e) $\left(2-\frac{1}{2}x\right)^{10}$

(f) $(3-x)^{7}$

(g) $(x+2y)^{8}$

(h) $(2x-3y)^{9}$

Solution:

(a) $(1+x)^{10}$
$= {}^{10}C_0 1^{10} + {}^{10}C_1 1^9 x^1 + {}^{10}C_2 1^8 x^2 + {}^{10}C_3 1^7 x^3 + \ldots$
$= 1 + 10 \times 1x + 45 \times 1x^2 + 120 \times 1x^3 + \ldots$
$= 1 + 10x + 45x^2 + 120x^3 + \ldots$

(b) $(1-2x)^{5}$
$= {}^{5}C_0 1^5 + {}^{5}C_1 1^4 (-2x)^1 + {}^{5}C_2 1^3 (-2x)^2 + {}^{5}C_3 1^2 (-2x)^3 + \ldots$
$= 1 \times 1 + 5 \times (-2x) + 10 \times 4x^2 + 10 \times (-8x^3) + \ldots$
$= 1 - 10x + 40x^2 - 80x^3 + \ldots$

(c) $(1+3x)^{6}$
$= {}^{6}C_0 1^6 + {}^{6}C_1 1^5 (3x)^1 + {}^{6}C_2 1^4 (3x)^2 + {}^{6}C_3 1^3 (3x)^3 + \ldots$
$= 1 \times 1 + 6 \times 3x + 15 \times 9x^2 + 20 \times 27x^3 + \ldots$
$= 1 + 18x + 135x^2 + 540x^3 + \ldots$

(d) $(2-x)^{8}$
$= {}^{8}C_0 2^8 + {}^{8}C_1 2^7 (-x)^1 + {}^{8}C_2 2^6 (-x)^2 + {}^{8}C_3 2^5 (-x)^3 + \ldots$
$= 1 \times 256 + 8 \times (-128x) + 28 \times 64x^2 + 56 \times (-32x^3) + \ldots$
$= 256 - 1024x + 1792x^2 - 1792x^3 + \ldots$

(e) $\left(2-\frac{1}{2}x\right)^{10}$

$= {}^{10}C_0 2^{10} + {}^{10}C_1 2^9 \left(-\frac{1}{2}x \right)^1 + {}^{10}C_2 2^8 \left(-\frac{1}{2}x \right)^2 + {}^{10}C_3 2^7 \left(-\frac{1}{2}x \right)^3 + \ldots$

$= 1 \times 1024 + 10 \times (-256x) + 45 \times 64x^2 + 120 \times (-16x^3) + \ldots$

$= 1024 - 2560x + 2880x^2 - 1920x^3 + \ldots$

(f) $(3 - x)^7$

$= {}^7C_0 3^7 + {}^7C_1 3^6 (-x)^1 + {}^7C_2 3^5 (-x)^2 + {}^7C_3 3^4 (-x)^3 + \ldots$

$= 1 \times 2187 + 7 \times (-729x) + 21 \times 243x^2 + 35 \times (-81x^3) + \ldots$

$= 2187 - 5103x + 5103x^2 - 2835x^3 + \ldots$

(g) $(x + 2y)^8$

$= {}^8C_0 x^8 + {}^8C_1 x^7 (2y)^1 + {}^8C_2 x^6 (2y)^2 + {}^8C_3 x^5 (2y)^3 + \ldots$

$= 1 \times x^8 + 8 \times 2x^7y + 28 \times 4x^6y^2 + 56 \times 8x^5y^3 + \ldots$

$= x^8 + 16x^7y + 112x^6y^2 + 448x^5y^3 + \ldots$

(h) $(2x - 3y)^9$

$= {}^9C_0 (2x)^9 + {}^9C_1 (2x)^8 (-3y)^1 + {}^9C_2 (2x)^7 (-3y)^2 + {}^9C_3 (2x)^6 (-3y)^3 + \ldots$

$= 1 \times 512x^9 + 9 \times (-768x^8y) + 36 \times 1152x^7y^2 + 84 \times (-1728x^6y^3) + \ldots$

$= 512x^9 - 6912x^8y + 41472x^7y^2 - 145152x^6y^3 + \ldots$

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The binomial expansion
Exercise C, Question 4

Question:

The coefficient of x^2 in the expansion of $(2 + ax)^6$ is 60.
Find possible values of the constant a.

Solution:

$(2 + ax)^6$
Term in x^2 is ${}^6C_2 2^4 (ax)^2 = 15 \times 16a^2x^2 = 240a^2x^2$
Coefficient of x^2 is $240a^2$.
If this is equal to 60 then
$240a^2 = 60 \quad (\div 240)$

$a^2 = \frac{1}{4} \quad \left(\sqrt{} \right)$

$a = \pm \frac{1}{2}$

Therefore $a = \pm \frac{1}{2}$.

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The binomial expansion
Exercise C, Question 5

Question:

The coefficient of x^3 in the expansion of $(3 + bx)^5$ is -720.
Find the value of the constant b.

Solution:

$(3 + bx)^5$
Term in x^3 is ${}^5C_3 3^2 (bx)^3 = 10 \times 9b^3x^3 = 90b^3x^3$
Coefficient of x^3 is $90b^3$.
If this is equal to -720 then
$90b^3 = -720 \quad (\div 90)$
$b^3 = -8 \quad (\sqrt{\ })$
$b = -2$
Hence $b = -2$.

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The binomial expansion
Exercise C, Question 6

Question:

The coefficient of x^3 in the expansion of $(2+x)(3-ax)^4$ is 30.
Find the values of the constant a.

Solution:

$(3-ax)^4$
$= {}^4C_0 3^4 + {}^4C_1 3^3(-ax) + {}^4C_2 3^2(-ax)^2 + {}^4C_3 3^1(-ax)^3 + {}^4C_4(-ax)^4$
$= 1 \times 81 + 4 \times (-27ax) + 6 \times 9a^2x^2 + 4 \times (-3a^3x^3) + 1 \times a^4x^4$
$= 81 - 108ax + 54a^2x^2 - 12a^3x^3 + a^4x^4$

$(2+x)(3-ax)^4 =$

$$(2+x)(81-108ax+54a^2x^2-12a^3x^3+a^4x^4)$$

Term in x^3 is $2 \times (-12a^3x^3) + x \times 54a^2x^2 = -24a^3x^3 + 54a^2x^3$
Hence
$-24a^3 + 54a^2 = 30 \quad (\div 6)$
$-4a^3 + 9a^2 = 5$
$0 = 4a^3 - 9a^2 + 5 \quad (4 \times 1^3 - 9 \times 1^2 + 5 = 0 \quad \Rightarrow \quad a = 1 \text{ is a root})$
$0 = (a-1)(4a^2 - 5a - 5)$
So $a = 1$ and
$4a^2 - 5a - 5 = 0$
Using the formula for roots,
$a = \frac{5 \pm \sqrt{25+80}}{8} = \frac{5 \pm \sqrt{105}}{8}$

Possible values of a are 1, $\frac{5+\sqrt{105}}{8}$ and $\frac{5-\sqrt{105}}{8}$

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The binomial expansion
Exercise C, Question 7

Question:

Write down the first four terms in the expansion of $\left(1-\frac{x}{10}\right)^6$.

By substituting an appropriate value for x, find an approximate value to $(0.99)^6$. Use your calculator to find the degree of accuracy of your approximation.

Solution:

$\left(1-\frac{x}{10}\right)^6$

$= {}^6C_0 1^6 + {}^6C_1 1^5\left(-\frac{x}{10}\right) + {}^6C_2 1^4\left(-\frac{x}{10}\right)^2 + {}^6C_3 1^3\left(-\frac{x}{10}\right)^3 + \ldots$

$= 1\times 1 + 6\times\left(-\frac{x}{10}\right) + 15\times\frac{x^2}{100} + 20\times\left(-\frac{x^3}{1000}\right) + \ldots$

$= 1 - 0.6x + 0.15x^2 - 0.02x^3 + \ldots$

We need to find $(0.99)^6$

So $1-\frac{x}{10} = 0.99$

$\Rightarrow \frac{x}{10} = 0.01$

$\Rightarrow x = 0.1$

Substitute $x = 0.1$ into our expansion for $\left(1-\frac{x}{10}\right)^6$

$\Rightarrow \left(1-\frac{0.1}{10}\right)^6 = 1 - 0.6\times 0.1 + 0.15\times(0.1)^2 - 0.02\times(0.1)^3 + \ldots$

$\Rightarrow (0.99)^6 = 0.94148$

From a calculator $(0.99)^6 = 0.941480149$
Accurate to 5 decimal places.

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The binomial expansion
Exercise C, Question 8

Question:

Write down the first four terms in the expansion of $\left(2 + \frac{x}{5} \right)^{10}$.

By substituting an appropriate value for x, find an approximate value to $(2.1)^{10}$. Use your calculator to find the degree of accuracy of your approximation.

Solution:

$\left(2 + \frac{x}{5} \right)^{10}$

${}^{10}C_0 2^{10} + {}^{10}C_1 2^9 \left(\frac{x}{5} \right)^1 + {}^{10}C_2 2^8 \left(\frac{x}{5} \right)^2 + {}^{10}C_3 2^7 \left(\frac{x}{5} \right)^3 + \ldots$

$= 1 \times 1024 + 10 \times \frac{512x}{5} + 45 \times \frac{256x^2}{25} + 120 \times \frac{128x^3}{125} + \ldots$

$= 1024 + 1024x + 460.8x^2 + 122.88x^3 + \ldots$

If we want to find $(2.1)^{10}$ we need

$2 + \frac{x}{5} = 2.1$

$\Rightarrow \frac{x}{5} = 0.1$

$\Rightarrow x = 0.5$

Substitute $x = 0.5$ into the expansion for $\left(2 + \frac{x}{5} \right)^{10}$

$(2.1)^{10} = 1024 + 1024 \times 0.5 + 460.8 \times (0.5)^2 + 122.88 \times (0.5)^3 + \ldots$
$(2.1)^{10} = 1024 + 512 + 115.2 + 15.36 + \ldots$
$(2.1)^{10} = 1666.56$
From a calculator
$(2.1)^{10} = 1667.988 \ldots$
Approximation is correct to 3 s.f. (both 1670).

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The binomial expansion
Exercise D, Question 1

Question:

Use the binomial expansion to find the first four terms of

(a) $(1+x)^8$

(b) $(1-2x)^6$

(c) $\left(1+\frac{x}{2}\right)^{10}$

(d) $(1-3x)^5$

(e) $(2+x)^7$

(f) $(3-2x)^3$

(g) $(2-3x)^6$

(h) $(4+x)^4$

(i) $(2+5x)^7$

Solution:

(a) Here $n=8$ and $x=x$

$(1+x)^8 = 1 + 8x + \frac{8\times 7}{2!}x^2 + \frac{8\times 7\times 6}{3!}x^3 + \ldots$

$(1+x)^8 = 1 + 8x + 28x^2 + 56x^3 + \ldots$

(b) Here $n=6$ and $x=-2x$

$(1-2x)^6 = 1 + 6\left(-2x\right) + \frac{6\times 5}{2!}(-2x)^2 + \frac{6\times 5\times 4}{3!}(-2x)^3 + \ldots$

$(1-2x)^6 = 1 - 12x + 60x^2 - 160x^3 + \ldots$

(c) Here $n=10$ and $x=\frac{x}{2}$

$\left(1+\frac{x}{2}\right)^{10} = 1 + 10\left(\frac{x}{2}\right) + \frac{10\times 9}{2!}\left(\frac{x}{2}\right)^2 + \frac{10\times 9\times 8}{3!}\left(\frac{x}{2}\right)^3 + \ldots$

$\left(1+\frac{x}{2}\right)^{10} = 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \ldots$

(d) Here $n=5$ and $x=-3x$

$(1-3x)^5 = 1 + 5\left(-3x\right) + \frac{5\times 4}{2!}(-3x)^2 + \frac{5\times 4\times 3}{3!}(-3x)^3 + \ldots$

$(1-3x)^5 = 1 - 15x + 90x^2 - 270x^3 + \ldots$

(e) $(2+x)^7 = \left[2\left(1+\frac{x}{2}\right)\right]^7 = 2^7\left(1+\frac{x}{2}\right)^7$

Here $n = 7$ and $x = \frac{x}{2}$, so

$(2+x)^7 = 128\left[1 + 7\left(\frac{x}{2}\right) + \frac{7\times6}{2!}\left(\frac{x}{2}\right)^2 + \frac{7\times6\times5}{3!}\left(\frac{x}{2}\right)^3 + \ldots\right]$

$(2+x)^7 = 128\left(1 + \frac{7}{2}x + \frac{21}{4}x^2 + \frac{35}{8}x^3 + \ldots\right)$

$(2+x)^7 = 128 + 448x + 672x^2 + 560x^3 + \ldots$

(f) $(3-2x)^3 = \left[3\left(1-\frac{2x}{3}\right)\right]^3 = 3^3\left(1-\frac{2x}{3}\right)^3$

Here $n = 3$ and $x = \frac{-2x}{3}$, so

$(3-2x)^3 = 27\left[1 + 3\left(-\frac{2x}{3}\right) + \frac{3\times2}{2!}\left(-\frac{2x}{3}\right)^2 + \frac{3\times2\times1}{3!}\left(-\frac{2x}{3}\right)^3\right]$

$(3-2x)^3 = 27\left(1 - 2x + \frac{4}{3}x^2 - \frac{8}{27}x^3\right)$

$(3-2x)^3 = 27 - 54x + 36x^2 - 8x^3$

(g) $(2-3x)^6 = \left[2\left(1-\frac{3x}{2}\right)\right]^6 = 2^6\left(1-\frac{3x}{2}\right)^6$

Here $n = 6$ and $x = -\frac{3x}{2}$, so

$(2-3x)^6 = 64\left[1 + 6\left(-\frac{3x}{2}\right) + \frac{6\times5}{2!}\left(\frac{-3x}{2}\right)^2 + \frac{6\times5\times4}{3!}\left(\frac{-3x}{2}\right)^3 + \ldots\right]$

$(2-3x)^6 = 64\left(1 - 9x + \frac{135}{4}x^2 - \frac{135}{2}x^3 + \ldots\right)$

$(2-3x)^6 = 64 - 576x + 2160x^2 - 4320x^3 + \ldots$

(h) $(4+x)^4 = \left[4\left(1+\frac{x}{4}\right)\right]^4 = 4^4\left(1+\frac{x}{4}\right)^4$

Here $n = 4$ and $x = \frac{x}{4}$, so

$(4+x)^4 = 256\left[1 + 4\left(\frac{x}{4}\right) + \frac{4\times3}{2!}\left(\frac{x}{4}\right)^2 + \frac{4\times3\times2}{3!}\left(\frac{x}{4}\right)^3 + \ldots\right)$

$(4+x)^4 = 256\left(1 + x + \frac{3}{8}x^2 + \frac{1}{16}x^3 + \ldots\right)$

$(4+x)^4 = 256 + 256x + 96x^2 + 16x^3 + \ldots$

(i) $(2+5x)^7 = \left[2\left(1+\frac{5x}{2}\right)\right]^7 = 2^7\left(1+\frac{5x}{2}\right)^7$

Here $n = 7$ and $x = \frac{5x}{2}$, so

$$(2 + 5x)^7 = 128\left[1 + 7\left(\frac{5x}{2}\right) + \frac{7 \times 6}{2!}\left(\frac{5x}{2}\right)^2 + \frac{7 \times 6 \times 5}{3!}\left(\frac{5x}{2}\right)^3 + \ldots\right]$$

$$(2 + 5x)^7 = 128\left(1 + \frac{35}{2}x + \frac{525}{4}x^2 + \frac{4375}{8}x^3 + \ldots\right)$$

$$(2 + 5x)^7 = 128 + 2240x + 16800x^2 + 70000x^3 + \ldots$$

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The binomial expansion
Exercise D, Question 2

Question:

If x is so small that terms of x^3 and higher can be ignored, show that:
$(2+x)(1-3x)^5 \approx 2-29x+165x^2$

Solution:

$$(1-3x)^5 = 1 + 5\left(-3x\right) + \frac{5\times 4}{2!}(-3x)^2 + \dots$$

$$(1-3x)^5 = 1 - 15x + 90x^2 + \dots$$

$$(2+x)(1-3x)^5$$
$$= (2+x)(1-15x+90x^2+\dots)$$
$$= 2 - 30x + 180x^2 + \dots$$
$$+ \quad x - 15x^2 + \dots$$
$$= 2 - 29x + 165x^2$$

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The binomial expansion
Exercise D, Question 3

Question:

If x is so small that terms of x^3 and higher can be ignored, and
$(2 - x) (3 + x)^4 \approx a + bx + cx^2$
find the values of the constants a, b and c.

Solution:

$(3 + x)^4$
$= \left[3 \left(1 + \frac{x}{3} \right) \right]^4$
$= 3^4 \left(1 + \frac{x}{3} \right)^4$
$= 81 \left[1 + 4 \left(\frac{x}{3} \right) + \frac{4 \times 3}{2!} \left(\frac{x}{3} \right)^2 + \frac{4 \times 3 \times 2}{3!} \left(\frac{x}{3} \right)^3 + \ldots \right]$
$= 81 \left(1 + \frac{4}{3}x + \frac{2}{3}x^2 + \frac{4}{27}x^3 + \ldots \right)$
$= 81 + 108x + 54x^2 + 12x^3 + \ldots$

$(2 - x) (3 + x)^4$
$= (2 - x) (81 + 108x + 54x^2 + 12x^3 + \ldots)$
$= 162 + 216x + 108x^2 + \ldots$
$\quad - 81x - 108x^2 + \ldots$
$= 162 + 135x + 0x^2 + \ldots$
Therefore $a = 162$, $b = 135$, $c = 0$

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The binomial expansion
Exercise D, Question 4

Question:

When $(1-2x)^p$ is expanded, the coefficient of x^2 is 40. Given that $p>0$, use this information to find:

(a) The value of the constant p.

(b) The coefficient of x.

(c) The coefficient of x^3.

Solution:

$(1-2x)^p$

$= 1 + p\left(-2x\right) + \frac{p(p-1)}{2!}(-2x)^2 + \ldots$

$= 1 - 2px + 2p(p-1)x^2 + \ldots$

Coefficient of x^2 is $2p(p-1) = 40$

$\Rightarrow \quad p(p-1) = 20$

$\Rightarrow \quad p^2 - p - 20 = 0$

$\Rightarrow \quad (p-5)(p+4) = 0$

$\Rightarrow \quad p = 5$

(a) Value of p is 5.

(b) Coefficient of x is $-2p = -10$.

(c) Term in $x^3 = \frac{p(p-1)(p-2)}{3!}(-2x)^3 = \frac{5\times4\times3}{3!}\left(-8x^3\right) = -80x^3$

Coefficient of x^3 is -80.

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The binomial expansion
Exercise D, Question 5

Question:

Write down the first four terms in the expansion of $(1 + 2x)^{8}$. By substituting an appropriate value of x (which should be stated), find an approximate value of 1.02^8. State the degree of accuracy of your answer.

Solution:

$(1 + 2x)^{8}$

$= 1 + 8 \times 2x + \frac{8 \times 7}{2\,!} (2x)^{2} + \frac{8 \times 7 \times 6}{3\,!} (2x)^{3} + \ldots$

$= 1 + 16x + 112x^2 + 448x^3 + \ldots$

If we want an approximate value to $(1.02)^{8}$ we require
$1 + 2x = 1.02$
$2x = 0.02$
$x = 0.01$
Substitute $x = 0.01$ into our approximation for $(1 + 2x)^{8}$
$(1.02)^{8}$
$= 1 + 16 \times 0.01 + 112 \times (0.01)^{2} + 448 \times (0.01)^{3}$
$= 1 + 0.16 + 0.0112 + 0.000448$
$= 1.171648$
By using a calculator
$(1.02)^{8} = 1.171659$
Approximation is correct to 4 s.f. (1.172 for both solutions)

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The binomial expansion
Exercise E, Question 1

Question:

When $\left(1-\frac{3}{2}x\right)^p$ is expanded in ascending powers of x, the coefficient of x is -24.

(a) Find the value of p.
(b) Find the coefficient of x^2 in the expansion.
(c) Find the coefficient of x^3 in the expansion.

[E]

Solution:

$$\left(1-\frac{3x}{2}\right)^p = 1 + p\left(-\frac{3x}{2}\right) + \frac{p(p-1)}{2!}\left(-\frac{3x}{2}\right)^2 + \frac{p(p-1)(p-2)}{3!}\left(-\frac{3x}{2}\right)^3 + \ldots$$

(a) Coefficient of x is $-\frac{3p}{2}$

We are given its value is -24

$\Rightarrow \quad -\frac{3p}{2} = -24$

$\Rightarrow \quad p = 16$

(b) Coefficient of x^2 is $\frac{p(p-1)}{2} \times \frac{9}{4} = \frac{16 \times 15}{2} \times \frac{9}{4} = 270$

(c) Coefficient of x^3 is $-\frac{p(p-1)(p-2)}{3!} \times \frac{27}{8} = -\frac{16 \times 15 \times 14}{3!} \times \frac{27}{8} = -1890$

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The binomial expansion
Exercise E, Question 2

Question:

Given that:
$(2 - x)^{13} \equiv A + \text{B}x + Cx^2 + \ldots$
Find the values of the integers A, B and C.

[E]

Solution:

$(2 - x)^{13}$
$= 2^{13} + {}^{13}C_1 2^{12} (- x) + {}^{13}C_2 2^{11} (- x)^2 + \ldots$
$= 8192 + 13 \times (- 4096x) + 78 \times 2048x^2 + \ldots$
$= 8192 - 53248x + 159744x^2 + \ldots$
$\equiv A + \text{B}x + Cx^2 + \ldots$
So $A = 8192$, $B = - 53248$, $C = 159744$

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The binomial expansion

Exercise E, Question 3

Question:

(a) Expand $(1-2x)^{10}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient in the expansion.

(b) Use your expansion to find an approximation to $(0.98)^{10}$, stating clearly the substitution which you have used for x.

[E]

Solution:

(a) $(1-2x)^{10}$

$= 1 + 10\left(-2x\right) + \frac{10 \times 9}{2!}(-2x)^2 + \frac{10 \times 9 \times 8}{3!}(-2x)^3 + \ldots$

$= 1 + 10 \times (-2x) + 45 \times 4x^2 + 120 \times (-8x^3) + \ldots$

$= 1 - 20x + 180x^2 - 960x^3 + \ldots$

(b) We need $(1-2x) = 0.98$

$\Rightarrow \quad 2x = 0.02$

$\Rightarrow \quad x = 0.01$

Substitute $x = 0.01$ into our expansion for $(1-2x)^{10}$

$(1 - 2 \times 0.01)^{10} = 1 - 20 \times 0.01 + 180 \times 0.01^2 - 960 \times 0.01^3 + \ldots$

$(0.98)^{10} = 1 - 0.2 + 0.018 - 0.00096 + \ldots$

$(0.98)^{10} = 0.81704 + \ldots$

So $(0.98)^{10} \approx 0.81704$

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The binomial expansion

Exercise E, Question 4

Question:

(a) Use the binomial series to expand $(2-3x)^{10}$ in ascending powers of x up to and including the term in x^3, giving each coefficient as an integer.

(b) Use your series expansion, with a suitable value for x, to obtain an estimate for 1.97^{10}, giving your answer to 2 decimal places.

[E]

Solution:

(a) $(2-3x)^{10}$

$= 2^{10} + {}^{10}C_1 2^9 (-3x)^1 + {}^{10}C_2 2^8 (-3x)^2 + {}^{10}C_3 2^7 (-3x)^3 + \ldots$

$= 1024 + 10 \times (-1536x) + 45 \times 2304x^2 + 120 \times (-3456x^3) + \ldots$

$= 1024 - 15360x + 103680x^2 - 414720x^3 + \ldots$

(b) We require $2 - 3x = 1.97$

$\Rightarrow \quad 3x = 0.03$

$\Rightarrow \quad x = 0.01$

Substitute $x = 0.01$ in both sides of our expansion of $(2-3x)^{10}$

$(2 - 3 \times 0.01)^{10} = 1024 - 15360 \times 0.01 + 103680 \times 0.01^2 - 414720 \times 0.01^3 + \ldots$

$(1.97)^{10} \approx 1024 - 153.6 + 10.368 - 0.41472 = 880.35328 = 880.35$ (2 d.p.)

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The binomial expansion

Exercise E, Question 5

Question:

(a) Expand $(3 + 2x)^4$ in ascending powers of x, giving each coefficient as an integer.

(b) Hence, or otherwise, write down the expansion of $(3 - 2x)^4$ in ascending powers of x.

(c) Hence by choosing a suitable value for x show that $(3 + 2\sqrt{2})^4 + (3 - 2\sqrt{2})^4$ is an integer and state its value.

[E]

Solution:

(a) $(3 + 2x)^4$ has coefficients and terms

1	4	6	4	1
3^4	$3^3(2x)$	$3^2(2x)^2$	$3(2x)^3$	$(2x)^4$

Putting these together gives

$(3 + 2x)^4 = 1 \times 3^4 + 4 \times 3^3 \times 2x + 6 \times 3^2 \times (2x)^2 + 4 \times 3 \times (2x)^3 + 1 \times (2x)^4$

$(3 + 2x)^4 = 81 + 216x + 216x^2 + 96x^3 + 16x^4$

(b) $(3 - 2x)^4 = 1 \times 3^4 + 4 \times 3^3 \times (-2x) + 6 \times 3^2 \times (-2x)^2 + 4 \times 3 \times (-2x)^3 + 1 \times (-2x)^4$

$(3 - 2x)^4 = 81 - 216x + 216x^2 - 96x^3 + 16x^4$

(c) Using parts (a) and (b)

$(3 + 2x)^4 + (3 - 2x)^4 =$

$$\begin{array}{r} 81 + 216x + 216x^2 + 96x^3 + 16x^4 \\ + 81 - 216x + 216x^2 - 96x^3 + 16x^4 \\ \hline 162 \qquad\qquad + 432x^2 \qquad\qquad + 32x^4 \end{array}$$

Substituting $x = \sqrt{2}$ into both sides of this expansion gives

$(3 + 2\sqrt{2})^4 + (3 - 2\sqrt{2})^4 = 162 + 432(\sqrt{2})^2 + 32(\sqrt{2})^4 = 162 + 432 \times 2 + 32 \times 4 = 162 + 864 + 128 = 1154$

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The binomial expansion
Exercise E, Question 6

Question:

The coefficient of x^2 in the binomial expansion of $\left(1+\frac{x}{2}\right)^n$, where n is a positive integer, is 7.

(a) Find the value of n.

(b) Using the value of n found in part (a), find the coefficient of x^4.

[E]

Solution:

$$\left(1+\frac{x}{2}\right)^n = 1+n\left(\frac{x}{2}\right)+\frac{n(n-1)}{2!}\left(\frac{x}{2}\right)^2+\frac{n(n-1)(n-2)}{3!}\left(\frac{x}{2}\right)^3+\frac{n(n-1)(n-2)(n-3)}{4!}\left(\frac{x}{2}\right)^4+\ \ldots$$

(a) We are told the coefficient of x^2 is 7

$\Rightarrow \quad \frac{n(n-1)}{2}\times\frac{1}{4}=7$

$\Rightarrow \quad n(n-1)=56$

$\Rightarrow \quad n^2-n-56=0$

$\Rightarrow \quad (n-8)(n+7)=0$

$\Rightarrow \quad n=8$

(b) Coefficient of x^4 is

$$\frac{n(n-1)(n-2)(n-3)}{4!}\times\frac{1}{2^4}=\frac{\cancel{8}\times 7\times \overset{2}{\cancel{6}}\times 5}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1}\times\frac{1}{\underset{8}{\cancel{16}}}=\frac{35}{8}$$

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The binomial expansion
Exercise E, Question 7

Question:

(a) Use the binomial theorem to expand $(3 + 10x)^4$ giving each coefficient as an integer.

(b) Use your expansion, with an appropriate value for x, to find the exact value of $(1003)^4$. State the value of x which you have used.

[E]

Solution:

(a) $(3 + 10x)^4$
$= 3^4 + {}^4C_1 3^3 (10x) + {}^4C_2 (3)^2 (10x)^2 + {}^4C_3 (3)^1 (10x)^3 + (10x)^4$
$= 3^4 + 4 \times 270x + 6 \times 900x^2 + 4 \times 3000x^3 + 10000x^4$
$= 81 + 1080x + 5400x^2 + 12000x^3 + 10000x^4$

(b) We require $1003 = 3 + 10x$
$\Rightarrow \quad 1000 = 10x$
$\Rightarrow \quad 100 = x$
Substitute $x = 100$ in both sides of our expansion
$(3 + 10 \times 100)^4 = 81 + 1080 \times 100 + 5400 \times 100^2 + 12000 \times 100^3 + 10000 \times 100^4$
$(1003)^4 = 81 + 108\,000 + 54\,000\,000 + 12\,000\,000\,000 + 1\,000\,000\,000\,000$
$(1003)^4 =$

```
1 000 000 000 000
   12 000 000 000
       54 000 000
          108 000
               81
-----------------
1 012 054 108 081
```

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The binomial expansion
Exercise E, Question 8

Question:

(a) Expand $(1+2x)^{12}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

(b) By substituting a suitable value for x, which must be stated, into your answer to part (a), calculate an approximate value of $(1.02)^{12}$.

(c) Use your calculator, writing down all the digits in your display, to find a more exact value of $(1.02)^{12}$.

(d) Calculate, to 3 significant figures, the percentage error of the approximation found in part (b).

[E]

Solution:

(a) $(1+2x)^{12}$

$= 1 + 12\left(2x\right) + \frac{12 \times 11}{2\,!}(2x)^2 + \frac{12 \times 11 \times 10}{3\,!}(2x)^3 + \ldots$

$= 1 + 24x + 264x^2 + 1760x^3 + \ldots$

(b) We require $1 + 2x = 1.02$

$\Rightarrow \quad 2x = 0.02$

$\Rightarrow \quad x = 0.01$

Substitute $x = 0.01$ in both sides of expansion

$(1 + 2 \times 0.01)^{12} = 1 + 24 \times 0.01 + 264 \times 0.01^2 + 1760 \times 0.01^3$

$(1.02)^{12} = 1 + 0.24 + 0.0264 + 0.00176$

$(1.02)^{12} = 1.26816$

(c) Using a calculator

$(1.02)^{12} = 1.268241795$

(d) % error $= \frac{|\text{Answer b} - \text{Answer c}|}{\text{Answer c}} \times 100$

% error $= 0.006449479$

% error $= 0.00645$ %

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The binomial expansion
Exercise E, Question 9

Question:

Expand $\left(x - \frac{1}{x} \right)^5$, simplifying the coefficients.

[E]

Solution:

$\left(x - \frac{1}{x} \right)^5$ has coefficients and terms

1	5	10	10	5	1
x^5	$x^4 \left(- \frac{1}{x} \right)$	$x^3 \left(- \frac{1}{x} \right)^2$	$x^2 \left(- \frac{1}{x} \right)^3$	$x \left(- \frac{1}{x} \right)^4$	$\left(- \frac{1}{x} \right)^5$

Putting these together gives

$$\left(x - \frac{1}{x} \right)^5 = 1x^5 + 5x^4 \left(- \frac{1}{x} \right) + 10x^3 \left(- \frac{1}{x} \right)^2 + 10x^2 \left(- \frac{1}{x} \right)^3 + 5x \left(- \frac{1}{x} \right)^4 + 1 \left(- \frac{1}{x} \right)^5$$

$$\left(x - \frac{1}{x} \right)^5 = x^5 - 5x^3 + 10x - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}$$

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The binomial expansion
Exercise E, Question 10

Question:

In the binomial expansion of $(2k+x)^n$, where k is a constant and n is a positive integer, the coefficient of x^2 is equal to the coefficient of x^3.

(a) Prove that $n = 6k + 2$.

(b) Given also that $k = \frac{2}{3}$, expand $(2k+x)^n$ in ascending powers of x up to and including the term in x^3, giving each coefficient as an exact fraction in its simplest form.

[E]

Solution:

(a) $(2k+x)^n = (2k)^n + {}^nC_1(2k)^{n-1}x + {}^nC_2(2k)^{n-2}x^2 + {}^nC_3(2k)^{n-3}x^3 + \ldots$
Coefficient of x^2 = coefficient of x^3
${}^nC_2(2k)^{n-2} = {}^nC_3(2k)^{n-3}$

$$\frac{\cancel{n!}}{(n-2)!2!}\left(2k\right)^{n-2} = \frac{\cancel{n!}}{(n-3)!3!}(2k)^{n-3}$$

$$\frac{(2k)^{n-2}}{(2k)^{n-3}} = \frac{(n-2)!2!}{(n-3)!3!} \quad \text{(Use laws of indices)}$$

$$(2k)^1 = \frac{(n-2)!2!}{(n-3)!3!} \quad \left[\left(n-2\right)! = \left(n-2\right)\times\left(n-3\right)!\right]$$

$$2k = \frac{(n-2)\times\cancel{2}}{\cancel{6}\ 3}$$

$3 \times 2k = n - 2$
$6k = n - 2$
$n = 6k + 2$

(b) If $k = \frac{2}{3}$ then $n = 6 \times \frac{2}{3} + 2 = 6$

Expression is

$$\left(2\times\frac{2}{3}+x\right)^6$$

$$= \left(\frac{4}{3}+x\right)^6$$

$$= \left(\frac{4}{3}\right)^6 + {}^6C_1\left(\frac{4}{3}\right)^5x^1 + {}^6C_2\left(\frac{4}{3}\right)^4x^2 + {}^6C_3\left(\frac{4}{3}\right)^3x^3 + \ldots$$

$$= \frac{4096}{729} + \frac{2048}{81}x + \frac{1280}{27}x^2 + \frac{1280}{27}x^3 + \ldots$$

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The binomial expansion

Exercise E, Question 11

Question:

(a) Expand $(2+x)^6$ as a binomial series in ascending powers of x, giving each coefficient as an integer.

(b) By making suitable substitutions for x in your answer to part (a), show that $(2+\sqrt{3})^6-(2-\sqrt{3})^6$ can be simplified to the form $k\sqrt{3}$, stating the value of the integer k.

[E]

Solution:

(a) $(2+x)^6$
$= 2^6 + {}^6C_1 2^5 x + {}^6C_2 2^4 x^2 + {}^6C_3 2^3 x^3 + {}^6C_4 2^2 x^4 + {}^6C_5 2x^5 + x^6$
$= 64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$

(b) With $x = \sqrt{3}$
$(2+\sqrt{3})^6 = 64 + 192\sqrt{3} + 240(\sqrt{3})^2 + 160(\sqrt{3})^3 + 60(\sqrt{3})^4 + 12(\sqrt{3})^5 + (\sqrt{3})^6$ ①
with $x = -\sqrt{3}$
$(2-\sqrt{3})^6 = 64 + 192(-\sqrt{3}) + 240(-\sqrt{3})^2 + 160(-\sqrt{3})^3 + 60(-\sqrt{3})^4 + 12(-\sqrt{3})^5 + (-\sqrt{3})^6$
$(2-\sqrt{3})^6 = 64 - 192\sqrt{3} + 240(\sqrt{3})^2 - 160(\sqrt{3})^3 + 60(\sqrt{3})^4 - 12(\sqrt{3})^5 + (\sqrt{3})^6$ ②
① − ② gives
$(2+\sqrt{3})^6 - (2-\sqrt{3})^6$
$= 384\sqrt{3} + 320(\sqrt{3})^3 + 24(\sqrt{3})^5$
$= 384\sqrt{3} + 320 \times 3\sqrt{3} + 24 \times 3 \times 3\sqrt{3}$
$= 384\sqrt{3} + 960\sqrt{3} + 216\sqrt{3}$
$= 1560\sqrt{3}$
Hence $k = 1560$

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The binomial expansion

Exercise E, Question 12

Question:

The coefficient of x^2 in the binomial expansion of $(2 + kx)^8$, where k is a positive constant, is 2800.

(a) Use algebra to calculate the value of k.

(b) Use your value of k to find the coefficient of x^3 in the expansion.

[E]

Solution:

(a) The term in x^2 of $(2 + kx)^8$ is
${}^8C_2 2^6 (kx)^2 = 28 \times 64k^2x^2 = 1792k^2x^2$
Hence $1792k^2 = 2800$
$k^2 = 1.5625$
$k = \pm 1.25$
Since k is positive $k = 1.25$.

(b) Term in x^3 of $(2 + kx)^8$ is
${}^8C_3 2^5 (kx)^3 = 56 \times 32k^3x^3$
Coefficient of x^3 term is $1792k^3 = 1792 \times 1.25^3 = 3500$

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The binomial expansion

Exercise E, Question 13

Question:

(a) Given that
$(2+x)^5+(2-x)^5 \equiv A+Bx^2+Cx^4$,
find the value of the constants A, B and C.

(b) Using the substitution $y=x^2$ and your answers to part (a), solve
$(2+x)^5+(2-x)^5=349$.

[E]

Solution:

(a) $(2+x)^5$ will have coefficients and terms

1	5	10	10	5	1
2^5	2^4x	2^3x^2	2^2x^3	$2x^4$	x^5

Putting these together we get
$(2+x)^5=1\times2^5+5\times2^4x+10\times2^3x^2+10\times2^2x^3+5\times2x^4+1\times x^5$
$(2+x)^5=32+80x+80x^2+40x^3+10x^4+x^5$
Therefore
$(2-x)^5=32-80x+80x^2-40x^3+10x^4-x^5$
Adding $(2+x)^5+(2-x)^5=64+160x^2+20x^4$
So $A=64$, $B=160$, $C=20$

(b) $(2+x)^5+(2-x)^5=349$
$64+160x^2+20x^4=349$
$20x^4+160x^2-285=0 \quad (\div 5)$
$4x^4+32x^2-57=0$
Substitute $y=x^2$
$4y^2+32y-57=0$
$(2y-3)(2y+19)=0$
$y=\frac{3}{2}, -\frac{19}{2}$

But $y=x^2$, so $x^2=\frac{3}{2} \Rightarrow x=\pm\sqrt{\frac{3}{2}}$

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The binomial expansion
Exercise E, Question 14

Question:

In the binomial expansion of $(2 + px)^5$, where p is a constant, the coefficient of x^3 is 135. Calculate:

(a) The value of p,

(b) The value of the coefficient of x^4 in the expansion.

[E]

Solution:

(a) The term in x^3 in the expansion of $(2 + px)^5$ is
$^5C_3 2^2 (px)^3 = 10 \times 4p^3x^3 = 40p^3x^3$
We are given the coefficient is 135 so
$40p^3 = 135 \quad (\div 40)$
$p^3 = 3.375 \quad \left(\sqrt[3]{\quad} \right)$
$p = 1.5$

(b) The term in x^4 in the expansion of $(2 + px)^5$ is
$^5C_4 2^1 (px)^4 = 5 \times 2p^4x^4 = 5 \times 2(1.5)^4 x^4 = 50.625x^4$
Coefficient of x^4 is 50.625