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Research Article

Characterization of Insertion Property in Topological Spaces

Blasus Ogola, N. B. Okelo*

School of Mathematics and Actuarial Science,
Jaramogi Oginga Odinga University of Science and Technology,
P. O. Box 210-40601, Bondo-Kenya.

*Corresponding author's e-mail: bnyaare@yahoo.com

Abstract

In the present paper, for a topological space whose Λ-sets or kernel of sets are open, we give a sufficient condition for the weak cc-insertion property. Also for a space with the weak cc-insertion property, we give a sufficient conditions for the space to have the strong cc-insertion property.

Keywords: Characterization; Insertion Property; Topological Space.

Introduction

The concept of a preopen set in a topological space was introduced many decades back and many results have been obtained [1]. A subset A of a topological space (X, τ) is called preopen or locally dense or nearly open if A ⊆ I nt(C l(A)). A set A is called preclosed if its complement is preopen or equivalently if C l (I nt(A)) ⊆ A. The concept of a semi-open set in a topological space was introduced in [2]. A subset A of a topological space (X, τ) is called semi-open if A ⊆ C l (I nt(A)). A set A is called semi-closed if its complement is semi-open or equivalently if I nt (C l(A)) ⊆ A. In [3] they introduced a new class of generalized open sets in a topological space, so called b-open sets [4]. This type of sets discussed under the name of γ-open sets [5].

This class is closed under arbitrary union. The class of b-open sets contains all semi-open sets and preopen sets. The class of b-open sets generates the same topology as the class of preopen sets. Authors also studied several fundamental and interesting properties of b-open sets. A subset A of a topological space (X, τ) is called b-open if A ⊆ C l (I nt(A)) ∪ I nt(C l(A)) [6]. A set A is called b-closed if its complement is b-open or equivalently if C l (I nt(A)) ∩ I nt(C l(A)) ⊆ A. A generalized class of closed sets was considered in [7]. He investigated the sets that can be represented as union of closed sets and called them V-sets. Complements of V-sets, i.e., sets that are intersection of open sets are called Λ-sets [8, 9].

Research methodology

In this section we outline our research methodology. We begin by the following definition.

Definition 2.1

A real-valued function f defined on a topological space X is called A-continuous if the preimage of every open subset of R belongs to A, where A is a collection of subsets of X.

Remark 2.2

Most of the definitions of function used throughout this paper are consequences of the definition of A-continuity. However, for unknown concepts the reader may refer to [10]. In the recent literature many topologists had focused their research in the direction of investigating different types of generalized continuity. In [7] he introduced a new class of mappings called contra-continuity. A good number of researchers have also initiated different types of contra-continuous like mappings in the papers [1].

Definition 2.3

Hence, a real-valued function f defined on a topological space X is called contra-continuous (resp. contra-b-continuous) if the preimage of every open subset of R is closed (resp. b-closed) in X.

Results of Katˇetov [1,7] concerning binary relations and the concept of an indefinite lower cut set for a real-valued function, which is

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due to Brooks [3], are used in order to give a necessary and sufficient conditions for the insertion of a contra-continuous function between two comparable real- valued functions on such topological spaces that Λ-sets or kernel of sets are open [2]. If g and f are real-valued functions defined on a space X , we write $g \leq f$ in case $g(x) \leq f(x)$ for all x in X. The following definitions are modifications of conditions considered in [8].

Definition 2.4

A property P defined relative to a real-valued function on a topological space is a cc-property provided that any constant function has property P and provided that the sum of a function with property P and any contra-continuous function also has property P. If P_1 and P_2 are cc-properties, the following terminology is used:

(i) A space X has the weak cc-insertion property for (P_1, P_2) if and only if for any functions g and f on X such that $g \leq f$, g has property P_1 and f has property P2 , then there exists a contra- continuous function h such that $g \leq h \leq f$.

(ii) (ii) A space X has the strong cc−insertion property for (P_1, P_2) if and only if for any functions g and f on X such that $g \leq f$, g has property P_1 and f has property P_2, then there exists a contra-continuous function h such that $g \leq h \leq f$ and if $g(x) < f(x)$ for any x in X, then $g(x) < h(x) < f(x)$.

In this paper, for a topological space whose Λ-sets or kernel of sets are open, is given a sufficient condition for the weak cc-insertion property. Also for a space with the weak cc-insertion property, we give sufficient conditions for the space to have the strong cc-insertion property. Several insertion theorems are obtained as corollaries of these results. In addition, the weak insertion of a contra-b-function has also recently considered by the author in [3].

Results and discussion

Before giving a sufficient condition for insertability of a contra-continuous function, the necessary definitions and terminology are stated. The abbreviations cc and cbc are used for contra-continuous and contra-b-continuous, respectively.

Definition 3.1

Let A be a subset of a topological space (X, τ). We define the subsets A^{Λ} and A^{V} as follows:
$A^{\Lambda} = \cap \{O: O \supseteq A, O \in (X, \tau)\}$ and $A^{V} = \cup\{F : F \subseteq A, F c \in (X, \tau)\}$.

Remark 3.2

In [8], A^{Λ} is called the kernel of A. The family of all b-open and b-closed will be denoted by $bO(X, \tau)$ and $bC(X, \tau)$, respectively. We define the subsets $b(A^{\Lambda})$ and $b(A^{V})$ as follows:
$B(A^{\Lambda}) = \cap \{O : O \supseteq A, O \in bO(X, \tau)\}$ and $b(A^{V}) = \cup\{F : F \subseteq A, F \in bC(X, \tau)\}$. B (AΛ) is called the b-kernel of A.

Proposition 3.3

(i) The union of any family of b-open sets is a b-open set.
(ii) The intersection of an open and a b-open is a b-open set.

Definition 3.4.

If f is a real-valued function defined on a space X and if $\{x \in X : f(x) < i\} \subseteq A(f, i) \subseteq \{x \in X : f(x) \leq i\}$ for a real number i, then A(f, i) is called a lower indefinite cut set in the domain of f at the level i.

Theorem 3.5.

Let g and f be real-valued functions on the topological space X, in which kernel sets are open, with $g \leq f$. If there exists a strong binary relation ρ on the power set of X and if there exist lower indefinite cut sets A (f, t) and A (g, t) in the domain of f and g at the level t for each rational number t such that if $t_1 < t_2$ then $A(f, t_1)\ \rho\ A(g, t_2)$, then there exists a contra-continuous function h defined on X such that $g \leq h \leq f$.

Proof:

Let g and f be real-valued functions defined on the X such that $g \leq f$. By hypothesis there exists a strong binary relation ρ on the power set of X and there exist lower indefinite cut sets A(f, t) and A(g, t) in the domain of f and g at the level t for each rational number t such that if $t_1 < t_2$ then $A(f, t_1)\ \rho\ A(g, t_2)$. Define functions F and G mapping the rational numbers Q into the power set of X by F (t) = A (f, t) and G(t) = A(g, t). If t_1 and t_2 are any elements of Q with $t_1 < t_2$, then $F(t1)\ \bar{\rho}\ F(t_2)$, $G(t_1)\ \bar{\rho}\ G(t_2)$, and $F(t_1)\ \rho\ G(t_2)$.

By [9] it follows that there exists a function H mapping Q into the power set of X such that if t_1 and t_2 are any rational numbers with $t_1 < t_2$, then F (t_1) ρ H (t_2), H (t_1) ρ H (t_2) and H (t_1) ρ G(t_2). For any x in X, let h(x) = inf {t ∈ Q: x ∈ H (t)}. We first verify that g ≤ h ≤ f: If x is in H (t) then x is in G(t_0) for any $t_0 > t$; since x is in G(t_0) = A(g, t_0) implies that g(x) ≤ t_0, it follows that g(x) ≤ t. Hence g ≤ h. If x is not in H (t), then x is not in F (t_0) for any $t_0 < t$; since x is not in F (t_0) = A(f, t_0) implies that f (x) > t_0, it follows that f (x) ≥ t. Hence h ≤ f. Also, for any rational numbers t_1 and t_2 with $t_1 < t_2$, we have h−1(t_1, t_2) = H $(t_2)^V$ \H $(t_1)^\Lambda$. Hence h^{-1} (t_1, t_2) is closed in X, i.e., h is a contra-continuous function on X. The above proof used the technique of theorem 1 in [7]. Before stating the consequences of theorems 2.1, we suppose that X is a topological space whose kernel sets are open.

Corollary 3.6

If for each pair of disjoint b-open sets G_1, G_2 of X , there exist closed sets F_1 and F_2 of X such that $G_1 \subseteq F_1$, $G_2 \subseteq F_2$ and $F_1 \cap F_2 = \emptyset$ then X has the weak cc-insertion property for (cbc, cbc).

Proof:

Let g and f be real-valued functions defined on X, such that f and g are cbc, and g ≤ f. If a binary relation ρ is defined by A ρ B in case $b(A^\Lambda) \subseteq b(B^V)$, then by hypothesis ρ is a strong binary relation in the power set of X . If t_1 and t_2 are any elements of Q with $t_1 < t_2$, then we have that A(f, t_1) ⊆ {x ∈ X : f (x) ≤ t_1} ⊆ {x ∈ X : g(x) < t_2 } ⊆ A(g, t_2); since {x ∈ X : f (x) ≤ t1 } is a b-open set and since {x ∈ X : g(x) < t_2 } is a b-closed set, it follows that b(A(f, $t_1)^\Lambda$) ⊆ b(A (g, $t_2)^V$). Hence $t_1 < t_2$ implies that A (f, t_1) ρ A (g, t_2). By Theorem 3.5 the proof is complete.

Corollary 3.7.

If for each pair of disjoint b-open sets G_1, G_2 , there exist closed sets F_1 and F_2 such that $G_1 \subseteq F_1$, $G_2 \subseteq F_2$ and $F_1 \cap F_2 = \emptyset$ then every contra-b-continuous function is contra-continuous.

Proof:

Let f be a real-valued contra-b-continuous function defined on X. Set g = f, then by Corollary 3.6 there exists a contra-continuous function h such that g = h = f.

Corollary 3.8

If for each pair of disjoint b-open sets G_1, G_2 of X , there exist closed sets F_1 and F_2 of X such that $G_1 \subseteq F_1$, $G_2 \subseteq F_2$ and $F_1 \cap F_2 = \emptyset$ then X has the strong cc-insertion property for (cbc, cbc).

Proof:

Let g and f be real-valued functions defined on the X, such that f and g are cbc, and g ≤ f. Set h = (f + g)/2, thus g ≤ h ≤ f and if g(x) < f (x) for any x in X, then g(x) < h(x) < f (x). Also, by Corollary 3.2, since g and f are contra-continuous functions hence h is a contra-continuous function.

Conclusions

In the present paper, we have shown that for a topological space whose Λ-sets or kernel of sets are open, is given a sufficient condition for the weak cc-insertion property. Also for a space with the weak cc-insertion property, we have given sufficient conditions for the space to have the strong cc-insertion property.

Conflicts of interest

The authors declare no conflict of interest.

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