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THE COLLAPSING CHOICE THEORY: DISSOCIATING CHOICE AND JUDGMENT IN DECISION MAKING

ABSTRACT. Decision making theory in general, and mental models in particular, associate judgment and choice. Decision choice follows probability estimates and errors in choice derive mainly from errors in judgment. In the studies reported here we use the Monty Hall dilemma to illustrate that judgment and choice do not always go together, and that such a dissociation can lead to better decision-making. Specifically, we demonstrate that in certain decision problems, exceeding working memory limitations can actually improve decision choice. We show across four experiments that increasing the number of choice alternatives forces people to collapse choices together, resulting in better decision-making. While choice performance improves, probability judgments do not change, thus demonstrating an important dissociation between choice and probability judgments. We propose the Collapsing Choice Theory (CCT) which explains how working memory capacity, probability estimation, choice alternatives, judgment, and regret all interact and effect decision quality.

KEY WORDS: choice, judgment, working memory, mental models, decision making, monty hall dilemma

JEL CLASSIFICATIONS: D70 · D80 · D81 · D84

1. INTRODUCTION

Decision-making plays an important and inseparable role in almost every domain of human cognition. For the most part, the same general mechanisms are used when decisions are made regardless of the specific domain, including categorization (Nosofsky and Palmeri, 1997; Stibel, 2006a,b), memory recognition (Ratcliff, 1978) and perceptual processes (Link, 1992). For most decision-making tasks, alternatives are considered and their benefits are measured until a decision threshold is reached (e.g., Busemeyer and Townsend, 1993), at which stage a decision is determined. A large body of experimental studies has examined a variety of factors that affect how decisions are reached. These studies focus on how differing data are computed, probability estimated, and how such factors are mediated by time pressure and other parameters (e.g., Dror, 2007; Dror et al., 1999; Ariely and Zakay, 2001).

One phenomenon of decision-making relates to how problems are framed and represented. Different representations of the same problem often result in different decisions (Kahneman and Tversky, 1973). This is mainly due to how framing and representations affect probability judgment and subsequent evaluations of alternative choices (Stanovich and West, 2000; Stibel, 2005a,b). Typically, frames are developed to improve performance by leveraging the strengths of the human mind, such as taking advantage of cognitive heuristics. But can some of our limitations also increase performance on decision-making tasks?

In the studies reported here we manipulate working memory load so as to force people to collapse together different choices in a classic decision-making task. Such memory load has been previously shown to decrease decision-making performance. The common reasons for such decrement in decision quality is the inability to properly consider, examine, and compare different alternative choices. As information load increases and cognitive resources are depleted, the decision maker is forced to adapt and take 'short cut' strategies (see for example, Ariely, 2008; Biggs et al., 1985; Dror, 2007; Shields, 1983; Stibel, 2007). Thus, memory load results in a drop in decision-making performance.

However, short cut strategies have also been shown to create cognitive advantages. With difficult problems, certain short cuts, heuristics, or frames often provide valuable tools to lead people toward the correct decision (see for example, Gigerenzer and Goldstein, 1996; Gigerenzer et al., 1999; Sloman et al., 2003; Hogarth and Karelaia, 2007). Our results are consistent with the latter explanation; namely that working memory limitations can in fact improve decision-making.

Across four experiments, we demonstrate that collapsing alternative choices selectively changes decisions but does not affect probability judgments. This provides the underpinnings for a new theory that dissociates choice and judgment in decision-making. The *collapsing choice theory* argues that, when multiple choices exceed working memory, the choices are collapsed. People assign increased weight to the collapsed set without changing the underlying probabilities. This can cause people to change their decisions without changing their understanding of the problem and thereby creates a disassociation between choice and judgment.

We examine these issues and develop our theory using a classic decision-making task, the Monty Hall dilemma. These cognitive illusions provide interesting insights and a good platform to examine decision-making. Just like visual and other illusions, the Monty Hall dilemma is a cognitive task that people consistently get wrong. The Monty Hall dilemma is so powerful that even after explaining the proper probabilities and correct decision, people continue to make incorrect choices, and do so with high levels of confidence. By examining errors, when the decision-making system fails, we can get a good view of the underlining mechanisms. This is particularly true in cases where the illusion disappears under certain conditions. In this article, we demonstrate such conditions whereby the Monty Hall dilemma is much less powerful.

2. THE MONTY HALL DILEMMA

Few problems defy common sense as much as the Monty Hall dilemma. Not only does it almost always lead people to give the wrong answer, but also when the correct answer is presented and explained, people continue to answer incorrectly. This problem first stirred controversy when it appeared in Marilyn vos Savant's column in Parade magazine (vos Savant, 1990a):

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the other doors, opens another door, say No. 3, which has a goat. He then says to you, 'Do you want to pick door No. 2?' Is it to your advantage to take the switch?

The solution to this problem is not intuitive. Contrary to common sense, people should switch to the remaining door, as it will increase their probability of winning. Yet people consistently choose to stay with their first choice and report with high levels of certainty that the odds are 50:50 that the prize is behind either of the remaining doors (Falk, 1992; Gilovich et al., 1995; Shimojo and Ichikawa, 1989).

A thought experiment helps most people understand the true probabilities of this problem. Imagine that you originally chose door 1. If the host now asked you to choose between your current choice or switching to both doors 2 and 3, you would switch. The Monty Hall dilemma is no different except that the host is giving you additional information (that the prize is specifically not behind one of the other doors). The key to the problem is to realize that, because the host's choice is not random and is constrained (he will never open the door with the prize), the probability of the prize being behind the chosen door remains one out of three. A formal Bayesian proof of the problem is presented in Appendix A.

Why is the wrong answer to the Monty Hall dilemma so compelling? One possibility is that under conditions of ignorance, people assign equal probabilities to unknown events. For example, Shimojo and Ichikawa (1989) showed that people report erroneous beliefs concerning a formally equivalent problem called the three-prisoners dilemma (see Appendix B). Of particular interest are two subjective theorems they proposed: *number of cases* and *constant ratio.* The number of cases theorem states that when the number of possible alternatives is N, the probability of each alternative is 1/N. The constant ratio theorem states that when one alternative is eliminated, the ratio of probabilities for the other alternatives remains the same as their prior probabilities. Taken together, the default probability attached to each choice remains the same across all conditions regardless of whether choices have been eliminated.

2.1. *Can Increasing Working Memory Demands Facilitate Probabilistic Choice?*

Johnson-Laird (1983) posited a mental model theory of naïve probabilistic reasoning that incorporates many of the ideas later proposed by Shimojo and Ichikawa (1989). Mental model theories of mind date back to Wittgenstein's (1922) representational theory of language and Craik's (1943) theory that "small-scale models" of reality are used to predict events. Johnson-Laird (1983) and Johnson-Laird et al. (1999) provided a mental model account of decision-making based on three principles: (a) the "truth principle," that people represent what they know to be true of the different possibilities that a problem affords; (b) the equiprobabilty principle, which corresponds to the "number of cases" theorem; and (c) the proportionality principle, which argues that the probability of event A depends on the proportion of the models in which the event occurs, such that $p(A) = \frac{n_A}{n}$, where n_A stands for the number of models containing A and n.

In the context of the Monty Hall dilemma, people create three models, one for each possible state (the prize is behind door 1, door 2, or door 3, respectively). They then ascribe equal probabilities across the choices prior to making an initial decision. People fail, however, to correct those probabilities when given new information, such as a choice being eliminated by a knowledgeable host. Rather than increasing the complexity of the model after the elimination of the door, people use an incorrect and simplified model of ascribing 50:50 to the remaining unopened doors.

Johnson-Laird (1983) and Johnson-Laird et al. (1999) argue that failures in these types of problems occur as a result of working memory overload, which cause people to create an incomplete set of models. According to Johnson-Laird and Byrne (2000), "The greater the number of models that a task elicits, and the greater the complexity of individual models, the poorer performance is. Reasoners focus on a subset of the possible models of multiple-model problems – often just a single model – and are led to erroneous conclusions and irrational decisions."

In contrast to Johnson-Laird (1983) and Johnson-Laird et al. (1999), we propose that working memory limitations can actually be advantageous by inducing people to make a normative correct choice. According to mental model theory, people create unique equiprobable models for each of the three choices in the Monty Hall dilemma. Presenting people with more choices would require the use of more models. We hypothesize that increasing the number of choices will increase working memory demands yet improve performance. Performance will grow with the number of models and asymptote once working memory is fully exhausted.

The collapsing choice theory argues that by increasing memory load, people will be forced to create two models. As the number of choices increase, the need for additional models will tax working memory. When demand exceeds capacity, working memory will be forced to create and deal with only two models: one for the initial choice and one for the remaining options. If initially, with little memory load of three choices, people choose option A, then their decision model can easily consist of "choosing A, VS. alternative choices B or C." However, with increased memory demand, say 7 choices, rather than a decision model of "choosing A, VS alternative choices B, C, D, E, F, or G," they will collapse together the alternative choices to create the more cognitively economical two model of "choosing A, VS. alternative choices." Thus minimizing the load by adopting the two model when memory load demand surpasses cognitive resources.

2.2. *Naïve Probability Versus Correct Choice*

If exhausting working memory can facilitate correct responses, can it also facilitate an understanding of the problem? Mental model theory does not distinguish between correct responses and an understanding thereof. Johnson-Laird et al. (1999) assumed that exhausting working memory could only have negative consequences, both in terms of performance and understanding. This in turn, led to a unified theory of probability judgment and choice. We believe that the two can be dissociated; performance (choice) and understanding (probability judgments) do not necessarily go together.

Consider the facilitation that people experience when they are asked to estimate frequencies (in contrast to probabilities) on problems such as Linda the bank teller, which tend to elicit conjunction errors. Studies have manipulated the number of constituents in the problem to exhaust working memory and eliminate the difference in performance between frequency and probability problems (Sloman et al., 2003). When Tversky and Kahneman (1983) manipulated 'Linda the bank teller problem' to be about "100 Lindas," the conjunction fallacy was reduced substantially. A number of authors have argued that people do not actually understand probabilities and this creates the cognitive illusion (Gigerenzer, 2004; Cosmides and Tooby, 1996). Kahneman and Tversky (1983) argued that asking people to estimate frequencies (vs. probabilities) induced people to appreciate the *inclusion rule*, "If A includes B, then the $p(A) \geq p(B)$." These manipulations increase correct choice but do not necessarily affect subjects' probability judgments.

The collapsing choice theory argues that a large number of alternatives in decision-making problems may force people to create a reduced choice representation, often to a simplistic 2-choice set. We claim that this is not a rare occurrence. Often the cognitive system tries to optimize its processes so as to free resources for other tasks. Rarely does it engage in complex processing when it is not needed. Thus, both as a general principle of economy in use of resources, and as a result of constant demands, the cognitive system often adopts the less computationally demanding processes; in this case the more simplistic 2-choice set. By moving to a reduced choice model, people change their underlying mental representations and conceptualization without altering the probabilities. Similar to support theory (Tversky and Koehler, 1994), this leads to better performance without necessarily leading to a better understanding of the problem or its underlying probabilities. Our proposal is that increasing the number of options improves performance, not by improving probability judgment, but by increasing the likelihood that different weighted mental models are used to represent the chosen and non-chosen sets.

The study reported here uses the Monty Hall dilemma to empirically examine this possibility. In the Monty Hall problem, the sets are mentally partitioned before any doors are opened and people's probability assumptions rarely change from this initial model. To examine the relation between choice and probability, we asked participants to evaluate the probability of winning after their final choice was made. We demonstrated that choice was independent of probability judgment, thus revealing a dissociation. Since our proposal suggests that choice is not necessarily mediated by probability judgment, it is consistent with the possibility of a dissociation between choice and judgment. Such a result is not consistent with mental model theory, which assumes that choice is governed directly by probability judgment. Whereas we do not argue that choice is never governed by probability judgments, we propose that there are other factors that can affect choice. This enables choice and judgment to be dissociated.

The current study is also designed to examine whether increasing memory load can contribute to making correct probabilistic choices. According to the mental model theory, people create different and yet equiprobable models for each of the choices in this problem. The collapsing choice theory argues that increasing memory load (by increasing the number of choices and the corresponding mental models) will force people to reduce those choices to a more simplistic model. This is achieved by collapsing different choices together, resulting in a model based on non-probabilistic evidence. Thus, rather than considering "stay with current choice, or switching to alternative A or B," increasing the number of alternatives induced a different model of choice (but not necessarily different in the probabilistic estimations). With the new model, the decision makers now consider whether to "stay with current choice, or switch to an alternative" (as the number of alternatives is increased, these are collapsed together). To test this theory, we increased the number of choices in order to create increasing degrees of memory load. The resulting function shows that the number of people who decide to switch increases with the number of choices, until an asymptote is reached at the level of working memory capacity.

We further investigate other factors that play a role in decision-making that are not related to probability judgments. Particularly, we examine the role of regret in deciding whether to reconsider a previous decision or not. The possible regret about choosing the wrong alternative choice makes some decisions psychologically less preferable than others (Hoelzl and Loewenstein, 2005). This is particularly notable in situations where a decision has already been taken and no other action is necessary, (but is still possible). In these situations, an active step can be taken, and if that can lead to a mistake, the regret of taking this step (in contrast to sticking to the previous existing decision) is particularly strong.

Such regret is applicable in the Monty Hall dilemma where a choice has been taken and an option to change it is presented, in games like black-jack where an option to take an additional card is presented (see, Dror et al., 1999), and in a variety of police-related decisions (see, Dror, 2007). The awareness and focus on such regret can play a significant role in the choice alternative that is selected, regardless of the probabilistic judgments (Wright and Ayton, 2005). This type of influence relates to internal psychological feeling of regret, in contrast to externally generated fear of having to justify and be accountable for decision choices (Dror 2007; Tetlock and Boettger, 1994). In a final experiment, we review to what extent regret played a role in people's choice and judgment.

3. EXPERIMENT 1

According to the collapsing choice theory, in problems such as the Monty Hall dilemma, a larger number of choice alternatives make people more likely to make a correct decision. When all options but one are eliminated from one model, the remaining alternative inherits the weighted evidence associated with its parent set without changing the underlying probabilities. One way to do this is by increasing the number of alternatives to exhaust working memory. Indeed, after headlines pronounced, "Marilyn is wrong," Marilyn vos Savant (1990b) argued for switching on the Monty Hall dilemma as follows:

> Yes, you should switch. The first door has a 1/3 chance of winning, but the second door has 2/3 chance. Here's a good way to visualize what happened. Suppose there are a million doors, and you pick door No. 1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except for door No. 777,777. You'd switch to that door pretty fast, wouldn't you?

Despite facilitating correct responses, a larger number of choices do not necessarily cause people to drop the incorrect equiprobability assumption even though it may be transparently violated. The collapsing choice theory predicts that people will retain their equiprobability assumption yet assign additional weight to the correct response. We predict that people will assign additional weight to the collapsed set without changing their underlying probability beliefs.

3.1. *Method*

Participants. Participants consisted of 32 Williams College undergraduates in statistics and psychology courses.

Procedure. Participants were randomly assigned to either the 3- or 100-choice condition of the Monty Hall dilemma. In both conditions, they were given a variation of the dilemma, which depicted boxes and a cash prize, instead of doors and a car. The structure of the dilemma was left intact. The reason for changing the content was to provide participants with a more feasible scenario (a room with 100 boxes is easier to imagine than one with 100 doors). After participants solved the dilemma, they were asked to state the probability that the prize was contained in their final choice. All participants were asked whether they have heard of the Monty Hall dilemma or a similar problem and anyone who said yes was eliminated from the results.

3.2. *Materials*

3.2.1. *The 3-box condition*

Imagine that you are participating in a game show. Your host shows you a room with one table and 3 boxes (Box #1, Box #2, and Box #3) on top of it. He tells you that one-dollar bills have been placed in two of the boxes and that $20 has been placed in a third box. He also tells you that the $20 is equally likely to be in any one of the three boxes and was placed in one of the boxes randomly prior to the start of the game show. He also reminds you that only he, the fair and unbiased game show host, knows the exact location of the $20. He then asks you to select a box, and you randomly choose Box #3. Your host then tells you that out of the two boxes you did not choose, at least one of them (and possibly both) does not contain the $20. Your host then proceeds to tell you that he will make your decision even easier by eliminating one of the two boxes that you did not choose that does not have the $20. He then opens Box #2 and reveals a one-dollar bill. Your host then gives you the option to stay with your initial choice (Box #3) or switch to the remaining box (Box #1). What would you do?

3.2.2. *The 100-box condition*

Imagine that you are participating in a game show. Your host shows you a room with one table and 100 boxes (Box #1, Box #2, Box #3, etc.) on top of it. He tells you that one-dollar bills have been placed in 99 of the boxes and that $20 has been placed in one box. He also tells you that the $20 is equally likely to be in any one of the 100 boxes and was placed in one of the boxes randomly prior to the start of the game show. He also reminds you that only he, the fair and unbiased game show host, knows the exact location of the \$20. He then asks you to select a box, and you randomly choose Box #3. Your host then tells you that out of the 99 boxes you did not choose, at least 98 of them (and possibly all 99) do not contain the \$20. Your host then proceeds to tell you that he will make your decision even easier by eliminating 98 of the 99 boxes that you did not choose that do not have the \$20. He then opens Box #1 and Box #2, Boxes #4–7, and Boxes #9–100 and reveals the \$1.00. Your host then gives you the option to stay with your initial choice (Box #3) or switch to the remaining box (Box #8). What would you do?

3.3. *Results and Discussion*

Very few participants chose to switch in the 3-box condition (1 out of 16). In contrast, significantly more people switched in the 100-box condition (8 out of 16). A chi-square analysis was conducted comparing the two conditions of number of choice alternatives with the correct decisions as the dependent measure. As can be seen in Table I, there was a significant effect due to increasing the number of choices (boxes) and exceeding working memory capacity, χ^2 $(1, N = 32) = 7.57$, $p < .001$. A corrected test, using Yates' correction, was also performed and did not change the results, indicating that the findings are justified and robust.

Furthermore, the probability judgments were dissociated from participant's choices. In response to the question of:

TABLE I

Frequencies of staying and switching in Experiment 1

	Decision	
	Stay	Switch
3 Boxes	15	1
100 Boxes	8	8

"What is the probability that the $20 is in the box you just chose?" the median responses in the 3- and 100- choice conditions were .50. The only person who switched in the 3-choice condition did not report an estimate, and 4 out of 6 participants (the remaining subjects did not provide probability estimates) who switched in the 100-choice condition reported a .50. In both conditions, the majority of participants who decided to stay reported a judgment of .50 (12 out of 15 in the 3-choice condition and 6 out of 8 in the 100-choice condition).

The collapsing choice theory states that more people will switch their choice to the degree that the number of options in the problem exhausts working memory. This implies that the likelihood of switching should increase until a value that is clearly beyond all participants' working memory capacity is reached. An alternative hypothesis may be that the difference between the 3- and 100-box condition is related to the way that numbers are mentally represented. For example, one view which emerges from the study of linear-distance effects, is that numbers are represented in an analog fashion such that differences between numbers are salient in proportion to their size (Moyer and Landauer, 1967).

If people represent the number of boxes on an analog scale and this representation is responsible for the effect, then the likelihood of switching should be monotonically related to the number of boxes. Unlike the collapsing choice theory, which relies on working memory limitations to produce a shift in problem representation, no asymptote should be observed when working memory capacity is exceeded in an analog scaling model. Experiment 2 was designed to compare these views by plotting the effect of the number of boxes across multiple values.

4. EXPERIMENT 2

If people collapse choices in their representation of the problem due to taxation of working memory capacity, then we should see a leap in correct responses as the number of

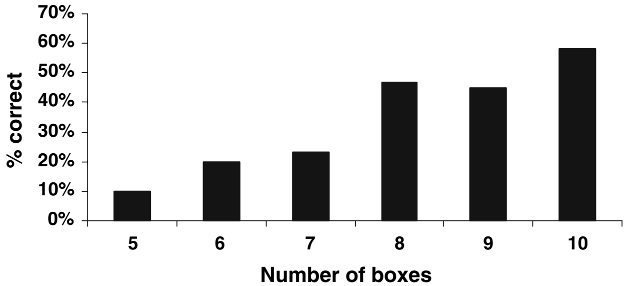

Figure 1. Percentages of switching across conditions in Experiment 2.

options overwhelms capacity. This number is roughly seven choices or models (Miller, 1956). The current experiment was designed to increase the number of models until they exceeded working memory. Although we expected individual differences in capacity (e.g., some participants have a limit of six models, some of eight) to obscure a clean break, we predicted an overall increase in correct responses across participants as the number of alternatives increased beyond the limits of working memory.

4.1. *Method*

Participants. The participants consisted of 152 Brown University undergraduates in statistics and psychology courses.

Procedure and Materials. The procedure and materials were identical to the ones described in Experiment 1, with the exception that participants were randomly assigned to 5-, 6-, 7-, 8-, 9-, and 10-choice conditions.

4.2. *Results and Discussion*

As Figure 1 illustrates, increasing the number of boxes facilitated correct responses. A tabulation of the frequency of people switching shows a significant change across experimental conditions, χ^2 $(5, N=132)=17.64$, $p=.003$.

In addition to the effect of increasing levels of switching with increased number of choices, an important aspect of our hypothesis concerns people's behavior at the point at which short-term memory capacity is exhausted. To test for discontinuities in the number of people switching in each condition, the seven- and eight-box conditions were compared to each other. A comparison of the two conditions reveals only a marginal effect, χ^2 $(1, N = 41) = 2.75$, $p = .10$. This result should not be surprising, given that differences exist in individuals' capacities, and thus obscure jumps in the number of correct responses (Miller, 1956, argued that short-term memory was limited to 7, plus or minus 2 discreet "bits of information").

Another way to approach the data is to ask with which other conditions the seven-box and 8-box conditions belong. A logistic regression, using staying versus switching as the criterion, contrasted the 7-box condition with the 8-, 9-, and 10-box conditions. It also contrasted the eight-box condition with the nine-box and 10-box conditions. The model, including the number of boxes as predictors (-2 Log Likelihood $= 150.84$), led to significant improvement over a model that contained just a constant (-2 Log Likelihood $= 169.39$), χ^2 $(5) = 18.55$, $p = .002$. As shown in Table II, the people in the 7-box condition exhibited a significantly lower probability of switching than people in the higher conditions, whereas those in the 8-box condition did not. In a separate model, the contrasts were reversed, so that the 8-box condition was contrasted with the 5-, 6-, and 7-box conditions, and the 7-box condition was contrasted with the two below it. Again, there was a significantly higher probability of switching in the 8-box condition than in the lower conditions. As predicted, there was no difference between the 7-box and the 5- and 6-box conditions. These results show a "leap" in the number of correct responses with eight options, which is where one can expect most people's memory capacity to become overloaded. This data provides evidence to support the influence of working memory exhaustion on correct response.

TABLE II

Regression model and contrast results for Experiment 2

Source	B	SE	df	Wald
General model				
Model			5	15.67**
Constant	–0.80	0.21	1	14.11***
Contrasts with greater number of boxes				
5 vs 6,7,8,9,10	–1.68[a]	0.77	1	4.69*
6 vs. 7,8,9,10	–1.08	0.55	1	3.87*
7 vs. 8,9,10	–1.23	0.57	1	4.66*
8 vs. 9,10	–0.16	0.55	1	0.09
9 vs. 10	–0.51	0.60	1	0.73
Contrasts with lower number of boxes				
10 vs. 5,6,7,8,9	1.33	0.47	1	8.20**
9 vs. 5,6,7,8	1.03	0.53	1	3.75*
8 vs. 5,6,7	1.50	0.57	1	6.80**
7 vs. 5,6	0.57	0.68	1	0.70
6 vs. 5	0.81	0.90	1	0.82

[a]The sign of the coefficient for each contrast indicates the direction of change in the odds of switching in the contrast category compared to the others. Thus, it is less likely that a person will switch in the 5-box condition, compared to the odds of switching in the conditions with greater number of choices.

*$p < .05$. **$p < .01$. ***$p < .001$.

5. EXPERIMENT 3

Experiment 2 demonstrated a step performance curve suggesting that working memory overload is the underlying cause of the increase in correct responses. However, a number of other factors could be underlying this effect. If increasing the number of choices and thereby exhausting working memory capacity is the underlying cause of the increase in correct responses, then we should see increased performance under other conditions where working memory is exhausted. Experiment 3 was designed to directly test the effects of working memory on decision-making in the Monty Hall dilemma. We introduced a classic memory task designed to exhaust working memory without changing the underlying problem or increasing the level of complexity. We predicted that as working memory was exhausted, correct choice, but not judgment, would be facilitated.

5.1. *Method*

Participants. 82 participants were recruited for an online survey using traditional marketing methods. Participants were randomly assigned to each condition, were roughly split between male and female, and had a mean age of approximately 34. This pool of participants was different than the participants used in the other reported studies that used undergraduate students. Using different participant pools allows us to draw conclusions that generalize across different populations, and thus strengthens our findings.

Procedure and Materials. The procedure and materials were identical to the ones described in Experiment 1, with the following exceptions. The study was performed over the Internet using online survey technology. Participants were placed into one of two conditions that utilized materials from the 3-box condition of Experiment 1. One condition was identical in all respects to its counterpart in Experiment 1. A second condition was designed to overload working memory by introducing a classic memory task. In this condition, subjects were presented with a set of 7 shapes and were asked to remember them directly after reading the dilemma. They were then instructed to answer the choice and judgment questions followed by a recall task. No remuneration was given for completing the tasks.

5.2. *Results and Discussion*

As predicted by the collapsing choice theory, the memory test increased performance on the Monty Hall dilemma without

TABLE III
Frequencies of staying and switching in Experiment 3

	Decision	
	Stay	Switch
Classic dilemma	31	5
Memory overload	23	23

changing the underlying probabilities, χ^2 $(1, N = 82) = 11.71$, $p < .001$. A post hoc analysis did not find significant differences between the 100-box condition of Experiment 1 and the memory overload condition in this experiment. Taxing working memory provided roughly the same increase in performance as increasing the number of choices. As can be seen in Table III, the results provide strong evidence that working memory overload can increase performance.

Consistent with Experiment 1, there was a dissociation between choice and judgment across conditions. In both conditions, the median judgment was .50. Among those subjects that answered correctly, the vast majority had probability judgments that were inconsistent with their responses. Of the correct choices, 60% responded with a .50 probability in the 3-box condition and 75% in the working memory load condition. This study (in contrast to Experiment 1) has a larger number of participants and also uses a different method of recruitment. This allowed us to recruit a variety of participants from a variety of environments, and thus our conclusions are more applicable and robust.

While the three previous experiments provide strong evidence to support the collapsing choice theory, it is important to note that increased facilitation was never generated across all participants in the memory overload condition. Experiment 4 was designed to remove additional confounds that could prevent subjects from selecting the correct choice.

6. EXPERIMENT 4

Despite the evidence supporting the collapsing choice theory across the previous experiments, the majority of subjects still did not choose the correct response. At best, we were only able to improve performance for roughly half of the participants. We speculate that memory overload does not provide enough additional evidentiary weight for all subjects to overcome other psychological factors, such as regret and cognitive dissonance.

Gilovich et al. (1995) argued that people tend to stay most of the time on the Monty Hall dilemma because people give more psychological weight to errors of commission rather than to errors of omission. They demonstrated that people are more willing to risk that their initial choice was wrong than to switch, only later to find out that their original choice was right. Other related theories, such as Lichtenstein and Slovic's (1971, 1973) illusion of control, assign incremental weight to self-selected choices as opposed to options that are given to people. The implications of this work to the current study are that the initial subjective choice is yet another factor that may conspire to keep people from switching. Regret may therefore mitigate additional facilitation that would result from increasing the number of choices. The present experiment tests this by eliminating the subjective choice while keeping the underlying structure of the Monty Hall dilemma intact.

6.1. *Method*

Participants. Participants consisted of 61 Brown University undergraduates in statistics and psychology courses.

Procedure and Materials. The procedure and materials were identical to the ones described in Experiment 1, with the exception that participants were given a variation of the 3- and 100-box Monty Hall problem that removed the initial choice. The experiment was designed to create a partition between the different options that was not produced by the subject's initial choice. By removing the initial choice, we expected to eliminate regret and cognitive dissonance while keeping the underlying structure of the problem intact.

6.1.1. *The 3-box condition*

Imagine that you are participating in a game show. Your host shows you a room with two tables – one table (Table A) with one box (Box #1) and another table (Table B) with 2 boxes (Box #2 and Box #3). He tells you that one-dollar bills have been placed in two of the boxes and that $20 has been placed in a third box. He also tells you that the $20 is equally likely to be in any one of the three boxes and was placed in one of the boxes randomly prior to the start of the game show. He also reminds you that only he, the fair and unbiased game show host, knows the exact location of the $20. Your host then asks you to ignore Table A for now, and focus only on Table B. Your host then tells you that because Table B has 2 boxes, at least one of them (and possibly both) does not contain the $20. He proceeds to tell you that he will make your decision even easier by eliminating one of the two boxes on Table B that does not have the $20. He then opens Box #2 and reveals a one-dollar bill. Your host then asks you to select a remaining box – either Box #1 on Table A, or Box #3 on Table B. What would you do?

6.1.2. *The 100-box condition*

Imagine that you are participating in a game show. Your host shows you a room with two tables–one table (Table A) with one box (Box #1) and another table (Table B) with 99 boxes (Box #2, Box #3, Box #4, etc.). He tells you that one-dollar bills have been placed in 99 of the boxes and that $20 has been placed in one box. He also tells you that the $20 is equally likely to be in any one of the 100 boxes and was placed in one of the boxes randomly prior to the start of the game show. He also reminds you that only he, the fair and unbiased game show host, knows the exact location of the $20. Your host then asks you to ignore Table A for now, and focus only on Table B. Your host then tells you that because Table B has 99 boxes, at least 98 of them (and possibly all 99) do not contain the $20. He proceeds to tell you that he will make your decision even easier by eliminating 98 of the 99 boxes on Table B that do not have the $20. He then opens Boxes #2–7 and Boxes #9–100 and reveals one-dollar bills beneath each of them. Your host then asks you to select a remaining box—either Box #1 on Table A, or Box #8 on Table B. What would you do?

6.2. *Results and Discussion*

Unlike Experiments 1 and 3, the majority of participants in the 100-box condition selected the correct choice (22 out of 29). Additionally, by eliminating regret from the problem, fewer people made an incorrect choice in the 3-box condition (16 out of 32) than in the previous experiments. While performance improved across all conditions, participants in the 100-box condition performed significantly better than those in the 3-box condition, $\chi^2(1, N = 61) = 4.33$, $p = .037$. This lends support to the claim that, absent regret or other evidence to the contrary, people rely on the constant ratio theorem. When compared with Experiment 1, the effects of regret appear clearly. Figure 2 illustrates two findings: that regret and dissonance reduction lead people to make an incorrect choice; and that once regret is mitigated, the collapsed choices yield incremental correct responses.

As with the previous experiments, few people knew the correct probabilities, indicating a dissociation between choice and judgment. Of the participants who made the correct choice (and answered the probability question) in the 3-box condition, most believed the probability was .50 (9 out of 12), and only two got the correct probability rating. In the 100-box condition, 15 out of the 19 people who chose correctly (and answered the probability question) believed the probability was .50, and only 2 participants responded with an accurate probability rating.

These results, along with the work of Gilovich et al. (1995), help explain the effects of regret on the collapsing choice

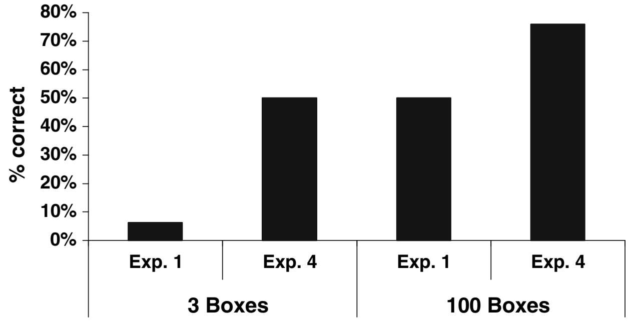

Figure 2. Percentages of switching across conditions in Experiment 1 as compared to those in Experiment 4.

theory. In the 3-box condition, participants were equally likely to choose the correct or incorrect option, conforming to the constant ratio hypothesis. This finding is markedly different than the pattern we observed previously, where few of the participants were willing to switch in the 3-box condition. By eliminating the subjects' initial choice, and thereby eliminating any potential for regret, subjects were no more likely to choose one over another.

A more striking finding was found in the 100-box condition, where most people made the normatively correct decision. In contrast to all the other conditions of this study, the majority of participants identified and made the correct choice. In sum, when regret is eliminated, the collapsing choices have an even greater impact on participants' decision-making. However, regret is only one possibility; other alternative (or additional) contributions, such as escalation of commitment and bias, can play a role in the decision-making process (see Dror and Charlton, 2006). In these cases, people select an alternative choice because of external contextual influences that bias their decisions (see Dror, 2007; Dror and Rosenthal, 2008).

7. GENERAL DISCUSSION

Two generalizations summarize the main findings of our experiments. First, correct responses increased as the number of alternative choices grew, up to an asymptote at a value exceeding the capacity of working memory. This finding supports the primary claim of the collapsing choice theory: when working memory capacity is exceeded, people tend to construct reduced choice models to represent multiple choice sets.

Second, despite the improvement in choice, probability judgments were unaffected by the number of alternatives. This finding is consistent with the second claim of the collapsing choice theory: that choice can be independent of probability judgments. Indeed, participants who decided to switch most often held the belief that the probability of winning the prize was .50. These results suggest a dissociation between choice and probability judgment; making a correct probabilistic choice does not necessarily entail an understanding of the underlying probabilities. In principle, the same effect of working memory limitations can be obtained using any hard problem, which has a variable number of possibilities, and probabilistically correct partitioning is used to encode possibilities whose number increase working memory capacity. It is interesting to compare 'probability estimation' and 'perception of risk' (see Dror, 2007).

The current results stand in contrast to the conventional view that problem solving is always enhanced by greater working memory capacity. In fact, taken to its logical extreme, one could argue that people with higher intelligence and therefore higher working memories (Stanovich and West, 2000), may actually be poor decision makers on tasks such as the Monty Hall dilemma. We have shown that working memory limitations can be advantageous in inducing people to make a correct probabilistic choice. The collapsing choice theory accounts for this seeming paradox: increasing memory demands force people to collapse the multiple-choice mental model, in this case into an equiprobable binary set with evidentiary weight assigned to the collapsed model. Thus, even though people may utilize a constant ratio theorem when they are confronted with few options (e.g., 3 choices), people are more likely to partition sets in a way consistent with the correct choice when more alternatives are imposed (e.g., 100 choices). Regardless of increased facilitation, probability judgments do not change. Instead of using probability estimates to make a decision, people focus on representations that can be affected by choice collapsing. Choice is affected by evidential strength generated by collapsing sets. This lends weight to the correct choice without changing the underlying probabilities. In the case of the Monty Hall dilemma, the collapsing choices generated additional evidence to support the correct choice.

The dissociation observed between judgment and choice is captured both by Shafer's belief functions (reviewed in Shafer, 1990) and by Support Theory (Tversky and Koehler, 1994; Hadjichristidis et al., 1999). Both of these theories explicitly distinguish evidential strength – the impact of evidence on a belief or the support provided by evidence for a belief – from judged probability. One difference is that judgments of evidential strength are sensitive to the support evidence that provides for alternative hypotheses; judgments of probability are not. In terms of the Monty Hall dilemma, the number of boxes may affect the perceived strength of evidence for the non-chosen boxes without affecting the judged probability that they contain the prize.

The tendency to focus on evidential strength is consistent with people's reliance on subjective theorems, such as the constant ratio theorem. People's reasoning on the traditional version of the Monty Hall dilemma is an example of the belief in the equiprobabilities of the remaining alternatives. In a similar set of experiments, Fox and Rottenstreich (2003) demonstrated that people not only judge the likelihood of an event, but also consider the number of alternatives available. In the current study, we showed that even though most people do tend to stay with their initial choice erroneously, there are conditions under which people are induced to give more weight to the correct probabilistic choice. One such condition is exceeding working memory capacity.

One question that pervades much of this research is why a large number of people continue to answer incorrectly, despite the role of the collapsed choices. Evans and Over (1996) argued that human rationality can be assessed by either a personal or impersonal theory. The *personal theory* takes people's goals into account and also whether people's reasoning helps to fulfill these goals. The *impersonal theory* is based on theories that make use of principles of logic and probability. For example, a researcher who positions her own hypothesis against one that she knows to be false (the null hypothesis) is using a *straw man* argument (because her results are bound to disconfirm the null hypothesis). From an impersonal viewpoint, she is committing a fallacy and being irrational. However, if her goal is to get people's attention and she is clever enough to conceal her faulty reasoning, then from a personal viewpoint, she is rational.

On the Monty Hall dilemma, people's reasoning may be affected by a personal theory of regret. That is, even though from a rational point of view a loss is still the same loss regardless of an error of commission or omission, people may find it psychologically more comforting to commit the latter (Gilovich et al., 1995). As the results from Experiment 4 have shown, when the effects of regret are removed, people are more likely to choose correctly.

An illusion of control on the part of subjects may also play a role in creating a personal theory favoring the initial choice. Illusions of control have been shown to influence individual's beliefs and judgments (Lichtenstein and Slovic, 1971, 1973). Thus, subjects perceive a chosen item as more valuable than one picked randomly or one assigned to them. For that reason, when subjects must rate two items, both equally valuable (probable), they will assign a higher weighting to the one they initially chose (Lichtenstein and Slovic, 1971, 1973). For that reason, subjects in the Monty Hall Dilemma, after having judged both doors as equally probable, may assign extra weight to their choice simply because they chose it. Due to this, they tend to stick with their initial choice, as opposed to switching to a quasi-random alternative. The results of Experiment 4 support this conclusion.

The results of this study have two implications for mental model theory. First, if the collapsing choice theory is correct, then people are more flexible in the way they encode possible states of the world than mental model theory affords. In the absence of any information to the contrary, people do tend to convert ignorance into a uniform distribution of probabilities (Falk, 1992; Johnson-Laird et al., 1999). However, people do not blindly obey the equiprobability principle. Instead, people ignore it and construct efficient representations when working memory is taxed. Second, the dissociation observed between choice and probability implies that the theory's assumption that behavior in uncertain domains is governed by the same set of mental models is not always viable. Either different mental models underlie choice and probability judgment or mental models mediate only one of the two tasks.

Does a dissociation between choice and judgment offer evidence for human irrationality? Not necessarily. If there is no obvious evidence for favoring one choice over another, it is logical for people to assume a uniform distribution of probabilities, particularly as an effective way for dealing with choices in every day life. Evans (1989) describes this idea:

> The view that I wish to argue here is that errors of thinking occur because of, rather than in spite of, the nature of our intelligence. In other words, they are an inevitable consequence of the way in which we think and a price to be paid for the extraordinary effectiveness with which we routinely deal with the massive information-processing requirements of everyday life.

Instead of trying to determine whether people are rational or irrational, it may be more useful to identify the circumstances under which people exhibit more or less effective reasoning.

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APPENDIX A

A Bayesian analysis of choices on the Monty Hall dilemma

The first step in the analysis is to compute the prior probabilities (i.e., the probabilities that the prize is behind each door, before the contestant chooses a door):

$$\mathrm{P(door1)} = \mathrm{P(door2)} = \mathrm{P(door3)} = .33.$$

As can be seen, the prior probabilities for the Monty Hall dilemma are all equal.

The next step is to calculate the conditional probabilities, namely, the probabilities that the prize is behind each one of the doors (door j), given that the host opens a particular door (door i), denoted as P(door i/door j). According to Bayes' theorem, the probability that the prize is in door j given that the host chooses to open door i is:

$$\mathrm{P}(door\ j/door\ i) = \frac{\mathrm{P(door}\,j) \bullet \mathrm{P(door}\,i/\mathrm{door}\,j)}{\mathrm{P(door\ 1)} \bullet \mathrm{P(door}\,i/\mathrm{door\ 1)} + \mathrm{P(door\ 2)} \bullet \mathrm{P(door}\,i/\mathrm{door\ 2)} + \mathrm{P(door\ 3)} \bullet \mathrm{P(door}\,i/\mathrm{door\ 3)}}$$

As an example, suppose the contestant chooses door3(i), and the host opens door2(j), the conditional probabilities become:

$$\mathrm{P(door2/door3)} = .5$$
$$\mathrm{P(door2/door2)} = 0$$
$$\mathrm{P(door2/door1)} = 1$$

Staying with door3 will result in the following probability that the prize is in door3 given that the host chooses to open door2:

$$P\mathrm{(door3/door2)} = \frac{(.33) \bullet (.5)}{(.33) \bullet (.5) + (.33) \bullet (0) + (.33) \bullet (1)} = .33$$

Switching to door1, on the other hand, results in an increased probability:

$$P(\text{door1}/\text{door2}) = \frac{(.33)\bullet(1)}{(.33)\bullet(1)+(.33)\bullet(0.5)+(.33)\bullet(0)} = .67$$

APPENDIX B

The three-prisoners dilemma

The three-prisoners dilemma is as follows: Tom, Dick, and Harry are awaiting execution in some remote country. The monarch of that country decides to pardon one of them by a fair draw. The warden is not allowed to say which one is to be pardoned, but Dick reasons that he already knows that someone else will be executed, so he asks the warden to tell him if it will be Tom or Harry. The warden names Harry. Immediately Dick becomes more optimistic saying: "before, I had a 1/3 chance of being pardoned, but now that only me and Tom are eligible for pardon, my chance has increased to 1/2." Is Dick's reasoning valid? (Falk, 1992).

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