Write your name here

Surname	Other names

Core Mathematics C12

SWANASH

A* Practice Paper

Time: 2 hours 30 minutes	Paper - E	Year: 2017-2018

The formulae that you may need to answer some questions are found at the end of this A star practice paper.

A student may use any basic scientific calculator except: facility for symbolic algebra manipulation, differentiation, integration, retrievable mathematical formulae.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more/less space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answer if you have time at the end.
- Practicing many **Swanash A-star** papers will enhance your final exam grades.

1. (a) Given that ${}^{14}C_r = 1001$, find the values of r

(2)

(b) Find the first 3 terms in *descending* powers of x of

$$\left(2+\frac{x}{3}\right)^8$$

giving each term in its simplest form.

(4)

Question 1 continued

(Total 6 marks)

2.

$$f(x) = -x^3 + \frac{4x - 3}{3x^5} + x, \qquad x \neq 0$$

Giving your answers in their simplest form, find

(a) $f'(x) - f'(1)$

(5)

(b) $\int f(x)\,dx$

(3)

Question 2 continued

(Total 8 marks)

3. Given that $f(x) = 6x^3 - 13x^2 - 46x + 8$

(a) Find the remainder when $f(x)$ is divided by

(i) x

(ii) $x + 2$

(iii) $2x - 3$

(5)

(b) Hence factorise $f(x)$ completely.

(4)

Question 3 continued

(Total 9 marks)

4. Using *only* the left-hand side (LHS) show, without the use of a calculator, that

$$\frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

(4)

Question 4 continued

(Total 4 marks)

5. A sequence is defined by

$$u_1 = 1$$

$$u_n = n + u_{n-1}, \quad n \geqslant 2$$

Find the values of

(a) u_2, u_3 and u_4

(3)

(b) u_{72}

(1)

(c) $\sum_{i=72}^{74} u_i$

(3)

Question 5 continued

(Total 7 marks)

6. Given that , $a = \log_6 12$ and $b = \log_{16} 81$

Show,without the use of a calculator, that

$$a(1 + b) = 2 + b$$

(6)

Question 6 continued

(Total 6 marks)

7. (a) Show that

$$\cos^2(45° + x) + \cos^2(45° - x) \equiv 1$$

(2)

(b) Solve, for $0 \leqslant \theta < 300°$, the equation

$$\sin\theta + \cos\theta = 1$$

showing each stage of your working.

(5)

Question 7 continued

(Total 7 marks)

8. (a) Given that the x-axis is a tangent to the curve with the equation

$$kx^2 + 8x + 2(k + 7) = 0$$

find the two possible values of the constant k

(5)

(b) Solve,

$$6^x + 72 = 9(2^x) + 8(3^x)$$

(5)

Question 8 continued

(Total 10 marks)

9. In the first month after opening, a car shop sold 100 cars. A model for future sales assumes that the number of cars sold will increase by 6% per month, so that 100×1.06 will be sold in the second month, 100×1.06^2 in the third month, and so on.

Using this model, calculate

(a) the number of cars sold in the 25th month,

(2)

(b) the total number of cars sold over the whole 25 months.

(2)

This model predicts that, in the Nth month, the number of cars sold in that month exceeds 2000 for the first time.

(c) Find the value of N.

(3)

Question 9 continued

(Total 7 marks)

10. The curve C has equation $y = \sin\left(x - \frac{\pi}{2}\right)$, $0 \leqslant x \leqslant 2\pi$

(a) In the space below, sketch the curve C.

(2)

(b) Write down the exact coordinates of the points at which C meets the coordinate axes.

(3)

(c) Solve, for x in the interval $0 \leqslant x \leqslant 2\pi$,

$$\sin\left(x - \frac{\pi}{2}\right) = \frac{1}{\sqrt{2}}$$

giving your answers in the form $k\pi$, where k is a rational number.

(4)

Question 10 continued

(Total 9 marks)

11. Three consecutive terms of an arithmetic series are $(8k - 2)$, $(6k + 4)$ and $(5k + 2)$ respectively.

(a) Find the value of the constant k.

(3)

(b) Write down the smallest positive term in the series.

(1)

Given also that the series has r positive terms,

(c) show that the sum of the positive terms of the series is given by $r(5r - 3)$.

(3)

Question 11 continued

(Total 7 marks)

12.

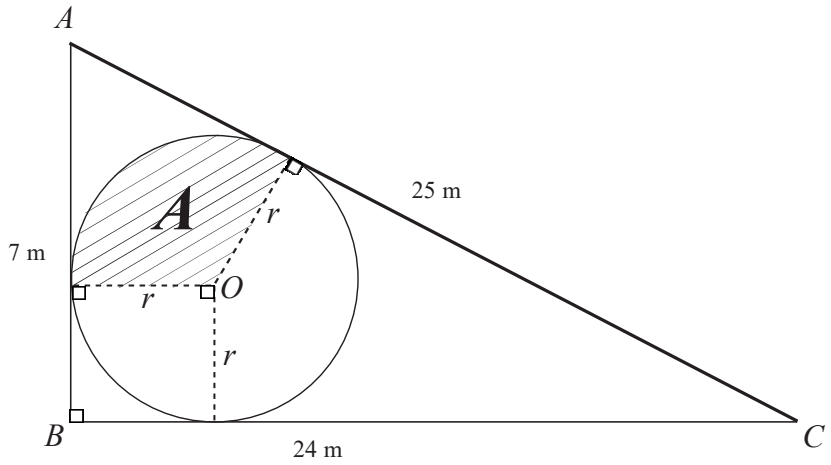

Diagram **NOT** drawn to scale

(□ = 90°)

Figure 1

Figure 1 shows a inscribed circle, of radius r with centre O, inside a right angle triangle ABC. The straight line AB is vertical and has length 7 m. The straight line BC is horizontal and has length 24 m. The straight line AC has length 25 m.

Find

(a) the radius, r, of the circle with centre O.

(2)

(b) the area of the shaded region, $\boldsymbol{A}$, to 3 significant figures,

(4)

Question 12 continued

(Total 6 marks)

13.

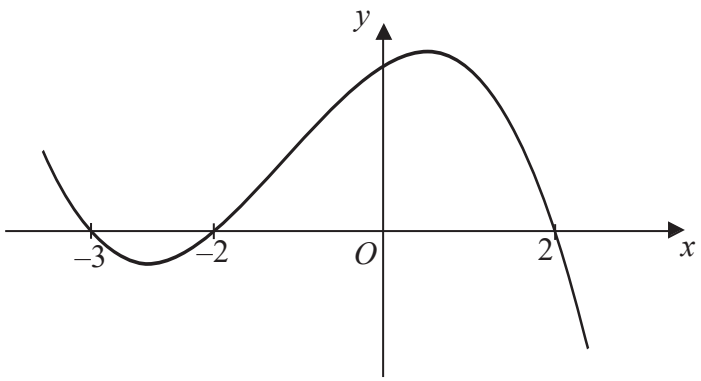

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x + 2)$ where the curve crosses the x-axis at $(–3, 0)$, $(–2, 0)$ and $(2, 0)$

(a) In the space below, sketch the curve C with equation $y = f(x – 1)$ and state the coordinates of the points where the curve C meets the x-axis.

(3)

(b) Given that the original curve $y = f(x)$ passes through the point $(1, 12)$. Find $f(x)$

(3)

(c) Find the x-coordinates of the stationary points on the curve $y = f(x)$, giving your answers to 2 decimal places.

(6)

(d) Find the coordinates of the maximum and minimum points.

(3)

(e) State the set of values of k, to 1 decimal place, for which the equation $f(x) = k$ has **three** solutions.

(2)

Question 13 continued

(Total 17 marks)

14.

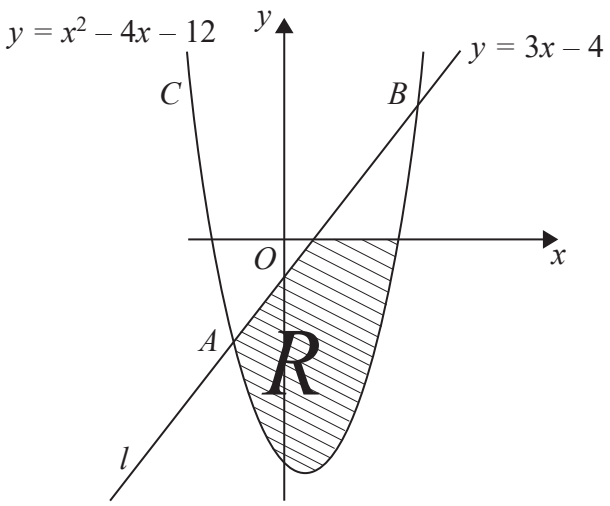

Diagram **NOT** drawn to scale

Figure 2

Figure 2 shows part of the line l with equation $y = 3x - 4$ and part of the curve C with equation $y = x^2 - 4x - 12$

The line l and the curve C intersect at the points A and B as shown.

(a) Use algebra to find the coordinates of A and the coordinates of B.

(5)

In Figure 2, the shaded region, R, is bounded by the line l, the curve C and the positive x-axis.

(b) Use integration to calculate an exact value for the shaded area of R.

(5)

Question 14 continued

(Total 10 marks)

15.

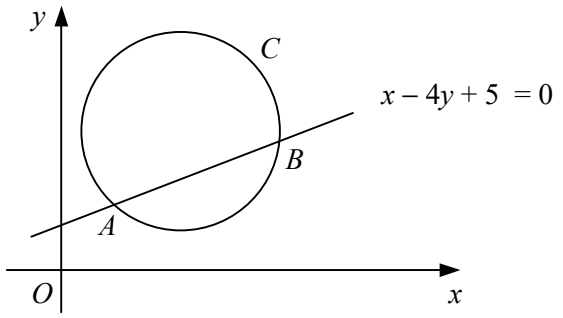

Diagram **NOT** drawn to scale

Figure 3

Figure 3 shows the line with equation $x - 4y + 5 = 0$ intersects the circle C at the points A and B.

Given that the centre of C has coordinates (6, 7),

(a) Find the coordinates of the mid-point of the chord AB.

(6)

Given also that the x-coordinate of the point A is 3,

(b) Find the coordinates of the point B,

(3)

(c) Find an equation for C.

(3)

Question 15 continued

(Total 12 marks)

TOTAL FOR PAPER: 125 MARKS

Formulae for Core Mathematics C12

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a + (n-1)d]$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots (|x| < 1,\ n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

The trapezium rule

$$\int_a^b y\ \mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}, \text{ where } h = \frac{b-a}{n}$$