3 April 2014

bj

The Evolution of Darkness

I. Introduction

One of the singular features of the Macdowell-Mansouri description of gravitation is that the metric tensor is directly constructed from the O(4,1) gauge potential A_μ. In addition, deSitter space is defined as the spacetime associated with a flat connection. In other words, the condition that the field strength $F_{\mu\nu}$(constructed from the gauge potential A_μ in the usual way) vanishes leads to deSitter space as a solution.

The fact that deSitter space is "pure gauge" implies that the gauge potential can be written as follows:

$$A_\mu \equiv \frac{\gamma_A \gamma_B}{2} A_\mu^{AB} = U^{-1} \partial_\mu U$$

Here we have used Clifford-algebra notation by contracting the potential into gamma matrices. I have constructed the matrix U for a variety of deSitter cartographies. They are typically of a product form as shown below, or a permutation of the 4 factors thereof:

$$U = U_1(x)\, U_2(y)\, U_3(z)\, U_0(t)$$

A generic submatrix typically has the form

$$U_i = e^{\Gamma_i x_i}$$

In this expression, Γ_i is the product of two gamma-matrices. Therefore it would seem that one could replace the exponent x_i with a monotonic function $X_I(x_i)$ which approaches a positive constant as the coordinate parameter x_i tends to positive infinity, and which approaches zero as x_i tends to negative infinity. If there is no obstacle to this procedure, it would seem that we could, at least asymptotically, "turn off" deSitter spacetime itself. We might also be able to create a number of isolated patches in the (Minkowskian) substrate within which the gauge potential is nontrivial and the metric tensor is nondegenerate. This is perhaps too strong a situation. The premise in this note is that, for the real-world, dark-energy-dominated spacetime in our universe, it is topological constraints that prevent one from gauging deSitter spacetime into a nothingness where the metric tensor identically vanishes.

This note is devoted to exploring this curious situation in more detail. The hope is that we might gain insight in understanding the fundamental conundrum of the dark energy problem, which I state as follows:

"After 10 billion years or so, a liter of dark energy will have evolved into two liters, containing twice as much energy and twice as much darkness. How does this happen at the microscopic level?"

II. Coordinate Patches

We are interested in is deSitter spacetime. A small amount, e.g. a cubic meter, will suffice for our purposes. So we can consider a quasi-Galilean situation, where all relevant velocities are small compared to c. Furthermore, we can, if it is convenient, compactify the space onto a torus, i.e. impose periodic boundary conditions.

If we put a proton into this volume, the region within its sphere of influence (radius of order 20 cm.) will be dominated by the spacetime curvature created by the proton mass. The most convenient description, using the Painleve-Gullstrand metric, utilizes a radial stationary-flow parameter v ("shift"). It is the same as the velocity of a test particle released from rest just inside the sphere of influence. On the other hand, the velocity parameter outside the sphere of influence becomes proportional to the distance from the proton. It describes the motion of a similar test particle released just outside the sphere of influence and accelerating outward under the influence of the dark energy.

The equations accompanying the above description are as follows. The Painleve-Gullstrand metric is

$$ds^2 = dt^2 - (d\vec{x} - \vec{v}\,dt)^2 \Rightarrow dt^2 - (dr - v(r)dt)^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

The expression for the velocity $\vec{v}$ is Galilean, even though we are doing general relativity:

$$\frac{v^2}{2} = \frac{GM}{r} + \frac{\Lambda r^2}{6} = \frac{M}{M_{pl}^2\, r} + \frac{H^2 r^2}{2}$$

The precise definition of the radius R of the sphere of influence of the proton (which I also call the rift zone) is where the radial gradient of the velocity vanishes:

$$v\frac{\partial v}{\partial r}\Big|_{r=R} = 0 \qquad \Rightarrow \qquad R^3 = \frac{M}{M_{pl}^2 H^2}$$

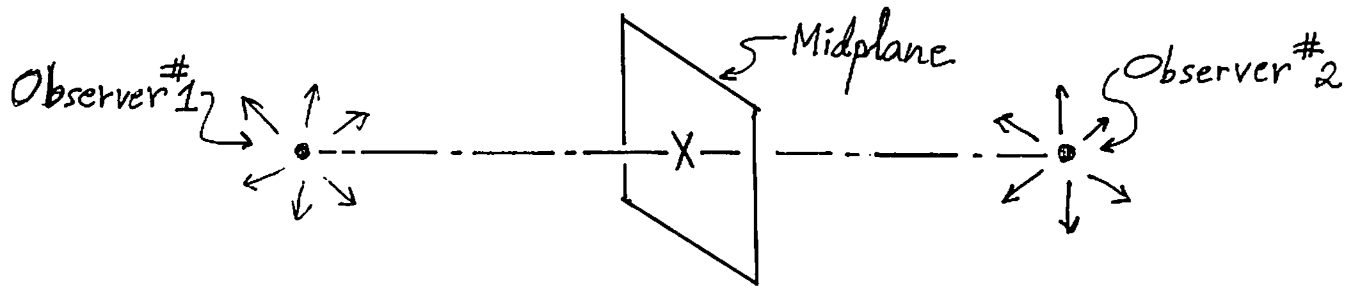
The change of sign of the velocity vector $\vec{v}$, which occurs at the surface of the sphere of influence, can be handled by a coordinate patch. While the metric tensor changes discontinuously across the boundary, the Riemann curvature tensor does not. It is totally smooth, because all components are even under a sign change of the velocity parameter. However, we are interested in the density of darkness n(r) . For this application the formula reads as follows:

$$n(r) = \frac{M_{pl}^2}{16\pi H^2} \cdot \frac{v^2}{r^2}\frac{\partial v}{\partial r}$$

It changes sign at the rift zone, unlike the Riemann tensor.

Our main reason for reviewing this material is the presence and apparent nontriviality of the coordinate patch. It stimulated consideration of pure deSitter space itself. What happens if we apply coordinate patches within pure deSitter space? In considering this, we first stay with the Painleve-Gullstrand form of the metric. In that description, there is a "priviliged observer", located at the origin of coordinates. (Note that such "observers" are found to be conceptually useful. For example they are needed to define the well-studied properties of horizon structure in deSitter space.)

In the Painleve-Gullstrand description, the privileged observer remains at rest. Can we add a second privileged observer? The answer appears to be affirmative. The construction is shown below.



The divider between the coordinate patches is a plane normal to the vector connecting the two "observers", and located midway between them. The squared velocity is continuous across the dividing plane, and the Riemann tensor (a multiple of the identity matrix when expressed in Petrov form) is trivially continuous across the boundary.

Once one starts doing this, it is hard to stop. The most extravagant option, as well as the most cogent one, is to assume that each unit of darkness is associated with one of these preferred observers. Put them into space at the darkness density characterized by the distance scale of 10^-13 cm. Construct the cells around these sites. They will be polyhedra, whose walls have the property that the mirror image of the privileged observer within the polyhedron is also the location of a privileged observer. (These are known as Voronoi cells.) It is fun to construct such geometries to get a feel of how this works. If one asks for high symmetry, the answer is a tightly packed hexagonal lattice. If one compactifies the space and scatters observers randomly within the unit cell, then a less symmetrical lattice structure will appear in the universal covering space. Without any compactification and any high degree of symmetry, the geometry is even more intricate. We will return to that case later.

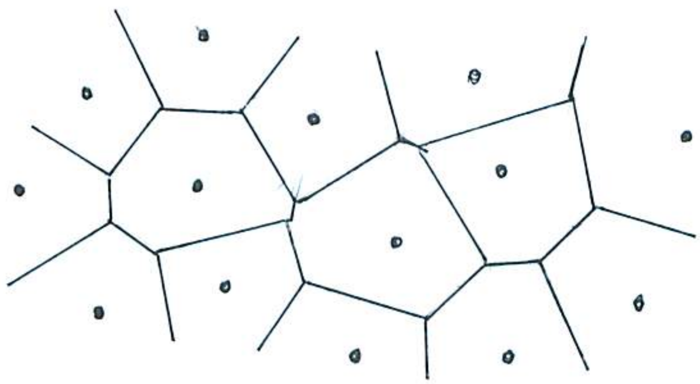
The individual cells, constructed to contain one unit of darkness per cell, also contain a fundamental unit of energy, namely the energy associated with the Hubble parameter H, of order 10^-33 eV.

$$\rho_{Dark} = \frac{3H^2 M_{pl}^2}{8\pi} \quad \Rightarrow \quad \langle E \rangle_{cell} = \frac{\rho_{Dark}}{n} = 6H$$

It is in a sense the smallest unit of energy worth talking about. In order to determine a value of energy more accurate than this, the measurement must take place over a time longer than the doubling time of the universe. In addition to be utterly impractical, there may be fundamental reasons why this is in practice impossible to accomplish.

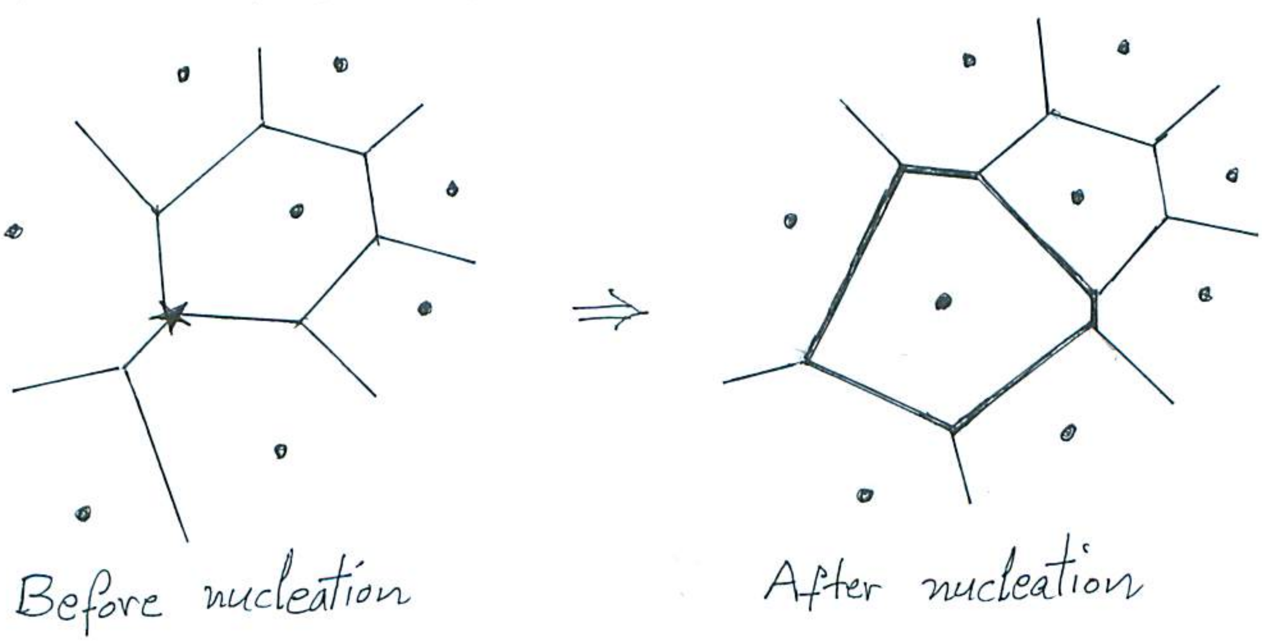
III. How DeSitter Space Inflates

In this section we will create a coarse visualization of how deSitter space inflates. The central issue will be to describe how the individual units of darkness appear as the universe expands. We first do this in FRW comoving coordinates. Suppose at $t = 0$ there exist a gas-like distribution of darkness elements, with density approximately equal to the desired darkness density. As outlined in the previous section, we now divide the space into Voronoi cells. In two space dimensions, these cells will be polygons, as shown below.



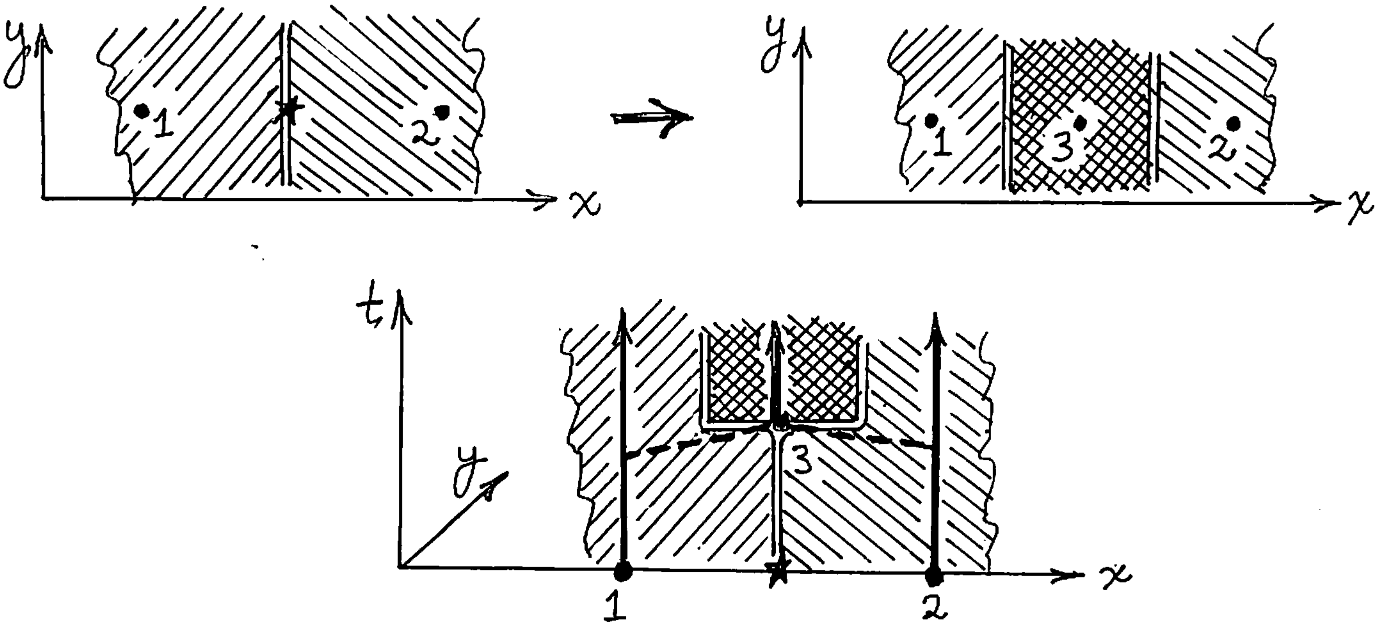
As the universe expands, the individual cell volumes increase exponentially, and we expect new cells to be nucleated, with each new cell containing a unit of darkness. Where is the most natural place for the next new unit to appear? It is simply at the corner of one of the polygons (or, in three dimensions, a corner of the polyhedron). Arguably, the first to go will be that corner which is furthest away from the neighboring sources of darkness. (Note that, by construction, the distances from the corner to the adjacent darkness sources are all the same.)

Once the darkness source is created, the Voronoi construction needs to be redone. The reconstruction is quasi-local; an example, again in two space dimensions is shown below.



Before nucleation — After nucleation

Evidently this construction generalizes easily to the three space dimension description of interest. We can think of the procedure in terms of a branching process. Each element of darkness then possesses a kind of genealogical history as shown below for an example where only one space dimension is involved:



In this case, we see that the piece of spacetime originally "owned" by an element of darkness, and which grows exponentially, is divided into pieces as time progresses. Most of the pieces are ceded to the progeny of the parent darkness element, with the parent retaining one darkness-unit of space in the long run.

Again, the process illustrated in the figure above for the one-space-dimension example generalizes easily to higher dimensions. But the mathematical description is a bit clumsier, albeit doable. In the sections which follow, we will set aside this gas-like description of darkness in favor of a crystal-like, Cartesian, description. Our motivation is simply to keep the equations as simple as possible.

IV. DeSitter Inflation in FRW Geometry

In this section, we endeavor to express this description in the MacDowell-Mansouri formalism which motivated it. The mathematics is simplest in FRW comoving coordinates, while the motivation for the description appears to be simplest in the Painleve-Gullstrand choice of coordinates. It turns out that there exists a metric which interpolates between the two:

$$ds^2 = dt^2 - e^{2(1-\lambda)t}\left(d\vec{x} - \lambda \vec{x}\, dt\right)^2$$

$$= dt^2 - e^{2(1-\lambda)t}\left[(dr - \lambda r\, dt)^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

(In what follows, we will often set the parameter H = 1 .)

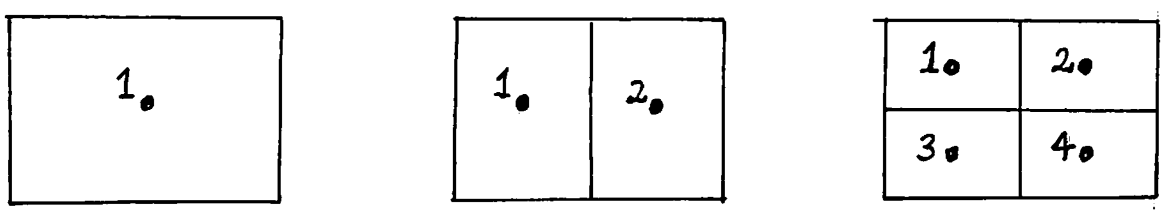
In the MacDowell-Mansouri language, the corresponding gauge potential is

$$A_t^{05} = 1 \qquad\qquad A_x^{05} = 0$$
$$A_t^{15} = -\lambda x e^{(1-\lambda)t} \quad \text{etc.} \qquad A_x^{15} = e^{(1-\lambda)t} \quad \text{etc.}$$
$$A_t^{01} = \lambda x e^{(1-\lambda)t} \quad \text{etc.} \qquad A_x^{01} = -e^{(1-\lambda)t} \quad \text{etc.}$$

It is easy to check that indeed $F = 0$ for this choice.

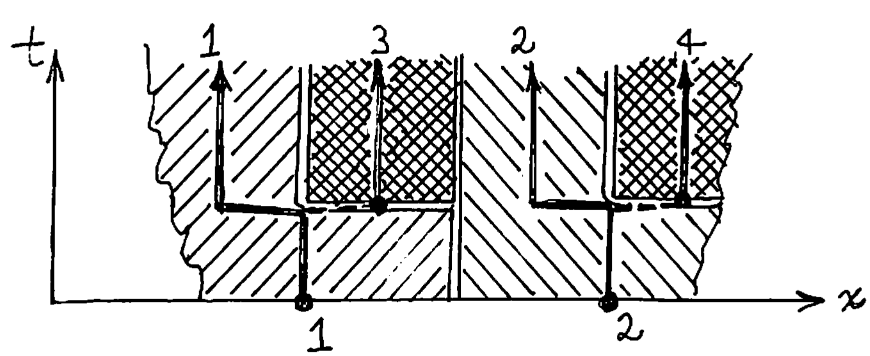
We are now ready to introduce the analog of the Voronoi cells. They will in this case be boxes, with distinct edge lengths. We choose boxes rather than cubes in order to be just a little closer to the case described in the previous section.

We choose to stay with the FRW language for a while, and set $\lambda = 0$. Then, as the universe expands, the boxes will divide. How this works is illustrated below for the case of two space dimensions.



While the genealogical details are slightly different, we believe that the idea expressed here is essentially the same as the idea expressed in the previous section. But here the rule for nucleating a new unit of darkness is as follows. When the area (or volume in the case of three space dimensions) exceeds a critical value, the cell divides in half with respect to its maximal dimension. After the cell division, the darkness units are placed at the centers of the new cells. (In three dimensions, we assume that at any time the cell dimensions L_a, L_b, L_c are such that $L_a < L_b < L_c < 2L_a$. Then there will be a self-similar sequence of cell divisions with period three.)

The genealogical tree is a little different than what was shown for the Voronoi case. It is depicted below, again for the case of one space dimension:



V. Connecting the Cells

Thus far we have decomposed deSitter space into cells with dimensions of order the Zeldovich, or darkness, scale of 10^-13 cm. We have also decomposed time into segments, separated by tne nucleation of new darkness elements. In addition, whenever new darkness elements appear, we have restructured some of the cells as well.

Each cell can be described in terms ot local coordinates, centered about the privileged observer. The most intuitive and attractive choice for these local coordinates is Painleve-Gullstrand. On the other hand, we have found the FRW comoving coordinates the easiest to deal with in describing the global properties of the space, in particular regarding the relationships of the cells with their neighbors—both in space and in time.

What we need is a language flexible enough to encompass both descriptions. In this section, we will construct such a language, using the ideas expressed in the introductory section of this note. The main idea will be that each cell can be embedded as an island within a surrounding ocean of nothingness (which we denote as the substrate), where all components of the gauge potential vanish and where the only guideposts are the substrate coordinates t, x, y, and z. In this ocean of total nothingness, the metric tensor vanishes, along with the usual notion of spacetime. In order to reconstruct the global spacetime from which we started, we must provide instructions on how to identify boundary points of the sundry spacetime islands with corresponding boundary points on neighboring spacetime islands.

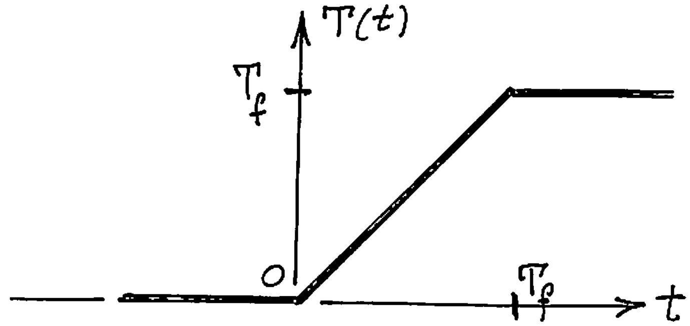
All of the above constructions must, for the deSitter space of interest, be pure gauge transformations—the MacDowell-Mansouri field strength F must vanish everywhere. So we begin by considering one of our FRW Cartesian boxes, and remove—via gauge transformations--all of its neighbor cells, as well as removing its prehistory and posthistory. We will eventually use the hybrid FRW/PG metric introduced in Section IV. The associated gauge potentials, contracted into gamma matrices, can be expressed in terms of a 4 x 4 matrix U as follows:

$$\frac{A_\mu}{2} = U^{-1}\partial_\mu U \qquad U(t,x) = e^{\frac{\lambda t \gamma_5\gamma_0}{2}}\left[1+\frac{\vec{\gamma}\cdot\vec{x}}{2}(\gamma_5-\gamma_0)\right]e^{\frac{(1-\lambda)t\,\gamma_5\gamma_0}{2}}$$

However, we will first stay with the FRW description, and set $\lambda = 0$. Now, as suggested in the introduction, replace the substrate coordinates t, x, y, z with functions of the substrate variables in the following way:

$$t \longrightarrow T(t)$$
$$x \longrightarrow X(x)$$
$$y \longrightarrow Y(y)$$
$$z \longrightarrow Z(z)$$

Inside the cell, the new variables T(t), X(x), Y(y), and Z(z) will be directly identified with t, x, y, and z. But outside the cell we will demand that the new variables become constants. The easiest case to deal with is time. We can easily "stop the FRW clock" in this way by choosing T(t) as shown below.

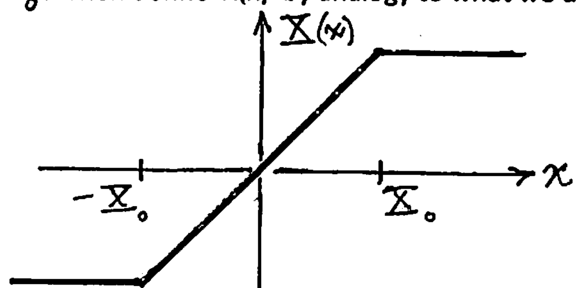


The field strength F still vanishes under these conditions. However, not all of the gauge potentials vanish under these circumstances. When T is a constant, either 0 or T_f, A_t vanishes trivially. But A_x does not vanish:

$$\frac{A_x}{2} = U^{-1}(T,x)\,\partial_x U(T,x) = \frac{\gamma_1}{2}(\gamma_5-\gamma_0)\,e^{\gamma_5\gamma_0 T} = \frac{\gamma_1(\gamma_5-\gamma_0)}{2}\,e^{T}$$

These nonvanishing components clearly match smoothly onto A_x in the spacetime region.

The same kind of construction also works in space. We temporarily retreat to the case of one space dimension. For a given time t in the active spacetime region, $0 < t < T_f$, let the boundaries of the spacetime box be $|x| < X_0$. Then define X(x) by analogy to what we did for time:

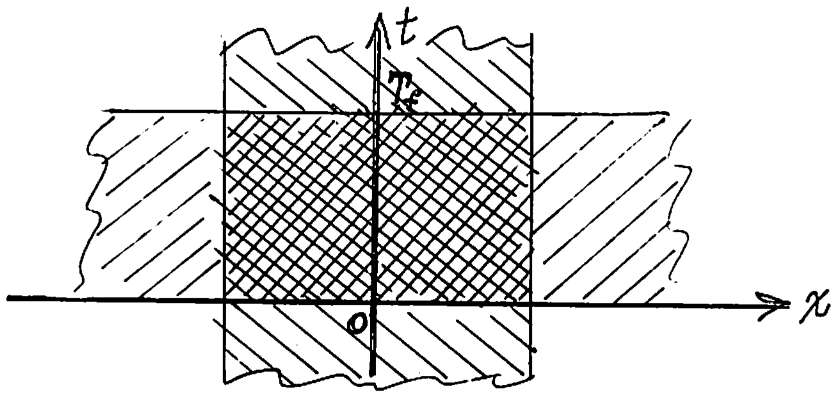


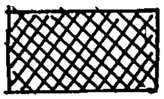
Evidently $A_x = 0$ outside the box, while A_t is nonvanishing:

$$\frac{A_t}{2} = e^{-\frac{\gamma_5\gamma_0 t}{2}}\left[1-\frac{\vec{\gamma}\cdot\vec{x}}{2}(\gamma_5-\gamma_0)\right]\left[1+\frac{\vec{\gamma}\cdot\vec{x}}{2}(\gamma_5-\gamma_0)\right]\frac{\gamma_5\gamma_0}{2}\,e^{\frac{\gamma_5\gamma_0 t}{2}} = \frac{\gamma_5\gamma_0}{2}$$

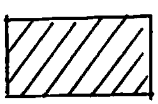
Again, this matches continuously to the value of A_t inside the spacetime box.

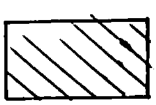
To summarize, the overall picture, as viewed in terms of the substrate variables, is shown below:



 Spacetime
$$A_\mu = \begin{array}{c} 05 \\ 15 \\ 01 \\ \\ \end{array}\!\!\begin{array}{c}\begin{pmatrix} 1 & 0 \\ 0 & e^t \\ 0 & -e^t \end{pmatrix} \\ \;\; t \qquad x \end{array}$$

 Pure substrate
$$A_\mu = \begin{array}{c} 05 \\ 15 \\ 01 \\ \\ \end{array}\!\!\begin{array}{c}\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \;\; t \qquad x \end{array}$$

 Substrate
$$A_\mu = \begin{array}{c} 05 \\ 15 \\ 01 \\ \\ \end{array}\!\!\begin{array}{c}\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \;\; t \qquad x \end{array}$$

 Substrate
$$A_\mu = \begin{cases} \begin{array}{c} 05 \\ 15 \\ 01 \\ \\ \end{array}\!\!\begin{array}{c}\begin{pmatrix} 0 & 0 \\ 0 & e^\tau \\ 0 & -e^\tau \end{pmatrix} \\ \;\; t \qquad x \end{array} \text{later} \\ \begin{array}{c} 05 \\ 15 \\ 01 \\ \\ \end{array}\!\!\begin{array}{c}\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \\ \;\; t \qquad x \end{array} \text{earlier} \end{cases}$$

This can be succinctly summarized as follows:

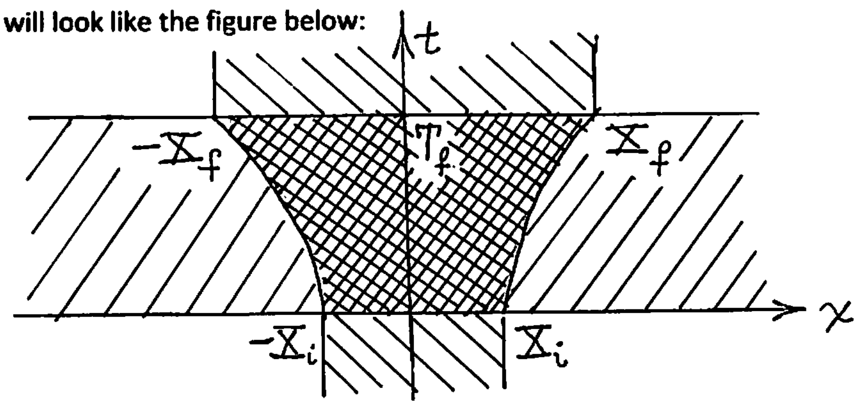
$$
A_\mu = \begin{array}{c} 05 \\ 15 \\ 01 \\ \end{array}
\begin{pmatrix} \dot{T}(t) & 0 \\ 0 & a(t)X' \\ 0 & -a(t)X' \end{pmatrix}
$$
$$
\qquad\qquad\quad t \qquad\quad x
$$

$$a(t) = e^{T(t)}$$

$$X'(x) = \frac{\partial X(x)}{\partial x} = \begin{cases} 1 & |x| < X_0 \\ 0 & |x| > X_0 \end{cases}$$

$$\dot{T}(t) = \frac{\partial T(t)}{\partial t} = \begin{cases} 1 & 0 < t < T_f \\ 0 & \text{otherwise} \end{cases}$$

We expect that, for the hybrid PG/FRW metric, the corresponding embedding of the physical spacetime into the substrate will look like the figure below:



Therefore the coordinate parameter X now can be expected to depend on the substrate time t as well as upon x. We therefore write the following form for the gauge potential:

$$
A_\mu = \begin{array}{c} 05 \\ 15 \\ 01 \\ \end{array}
\begin{pmatrix} \dot{T}(t) & 0 \\ -V(x,t) & X'(x,t) \\ +V(x,t) & -X'(x,t) \end{pmatrix}
$$
$$
\qquad\qquad\quad t \qquad\qquad x
$$

It is straightforward to compute the components of the field strength F. Demanding that they vanish leads to the following constraint:

$$\frac{\partial}{\partial x}\left[V + \dot{X} - X\dot{T}\right] = 0$$

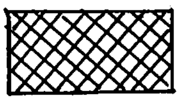
Consequently we may write in general for the gauge potential

$$
A_\mu = \begin{array}{c} 05 \\ 15 \\ 01 \\ \end{array}
\begin{pmatrix} \dot{T} & 0 \\ \dot{X} - X\dot{T} & X' \\ -\dot{X} + X\dot{T} & -X' \end{pmatrix}
$$
$$
\qquad\qquad\quad t \qquad\qquad x
$$

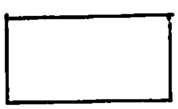
We may now easily determine the values taken by T(t) and by X(x, t) in the sundry substrate regions depicted on the previous page. First of all, the curved boundary satisfies the equation

$$\tilde{X}(x,t) = X_i\, e^{\lambda t} \quad \Rightarrow \quad X_f = X_i\, e^{\lambda T_f}$$

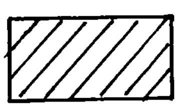
Within the various regions, we find



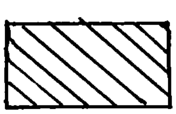
$$\begin{cases} T(t) = t \\ X(x,t) = x\,a(t) = x e^{(1-\lambda)t} \end{cases} \qquad A_\mu = \begin{pmatrix} 1 & 0 \\ -\lambda x a(t) & a(t) \\ \lambda x a(t) & -a(t) \end{pmatrix}$$



$$T(t) = \begin{cases} T_f & \text{later} \\ 0 & \text{earlier} \end{cases} \qquad A_\mu = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$



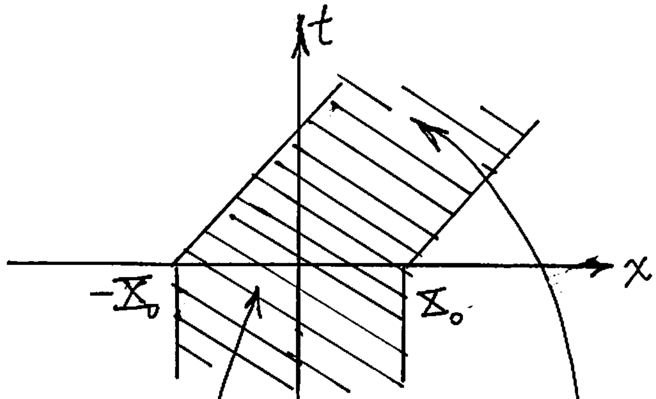
$$\begin{cases} T(t) = t \\ X(x,t) = \pm X_i e^{\lambda t} \cdot e^{(1-\lambda)t} = \pm X_i e^{t} \end{cases} \qquad A_\mu = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$



$$\begin{cases} T(t) = \begin{cases} T_f & \text{later} \\ 0 & \text{earlier} \end{cases} \\ X(x,t) = \begin{cases} x e^{(1-\lambda)T} & \text{later} \\ x & \text{earlier} \end{cases} \end{cases} \qquad A_\mu = \begin{cases} e^{(1-\lambda)t}\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} & \text{later} \\ \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} & \text{earlier} \end{cases}$$

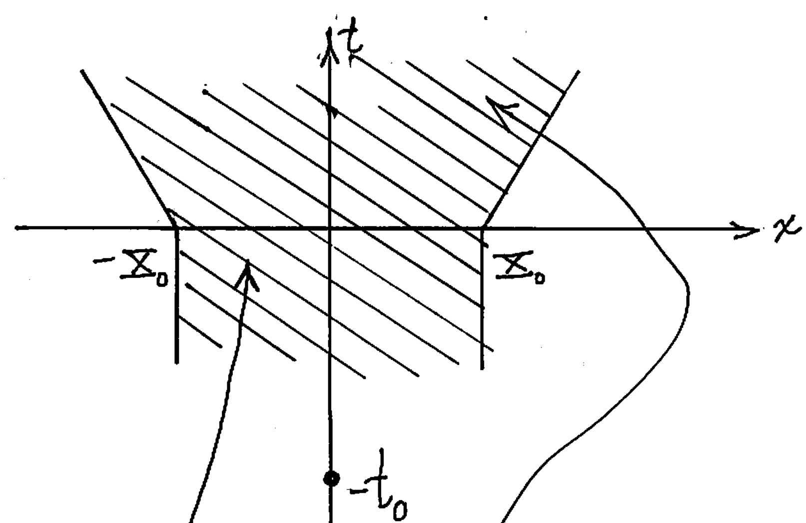
We see that there is a smooth generalization from the simpler FRW case.

Given these examples, we can now better see how to manipulate a single island of spacetime. For example, if we need to move it rigidly from one location in space to another, we may do it by turning off the spacetime clock and then sending it off in uniform motion in the desired direction.



$$\left.\begin{aligned} \dot{T} &= 0 \\ \dot{X} &= 0 \\ X' &= 0 \\ X(x) &= x \end{aligned}\right\}
\qquad\qquad
\left\{\begin{aligned} \dot{T} &= 0 \\ \dot{X} &= -v_0 \\ X' &= 1 \\ X(x,t) &= x - v_0 t \end{aligned}\right.$$

Likewise, if we wish to expand or contract the box with respect to substrate variables, while leaving the spacetime volume fixed, this can also be done while the spacetime clock is turned off:



$$\left.\begin{aligned} \dot{T} &= 0 \\ \dot{X} &= 0 \\ X' &\neq 0 \\ X &= x \end{aligned}\right\}
\qquad\qquad
\left\{\begin{aligned} \dot{T} &= 0 \\ \dot{X} &= \frac{x}{t_0} \\ X' &= \left(1 + \frac{t}{t_0}\right) \\ X(x,t) &= x\left(1 + \frac{t}{t_0}\right) \end{aligned}\right.$$

These Cartesian constructions can be expanded to three space dimensions. In that case, the gauge potential for the hybrid PG/FRW form will be as follows:

$$A_\mu = \begin{array}{cc} \begin{array}{c} 05 \\ 15 \\ 25 \\ 35 \\ 01 \\ 02 \\ 03 \\ 23 \\ 31 \\ 12 \end{array} & \begin{pmatrix} \dot{T} & 0 & 0 & 0 \\ \dot{X} - X\dot{T} & X' & 0 & 0 \\ \dot{Y} - Y\dot{T} & 0 & Y' & 0 \\ \dot{Z} - Z\dot{T} & 0 & 0 & Z' \\ -\dot{X} + X\dot{T} & -X' & 0 & 0 \\ -\dot{Y} + Y\dot{T} & 0 & -Y' & 0 \\ -\dot{Z} + Z\dot{T} & 0 & 0 & -Z' \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ & \begin{array}{cccc} t & x & y & z \end{array} \end{array}$$

$$T = T(t)$$
$$X = X(x,t)$$
$$Y = Y(y,t)$$
$$Z = Z(z,t)$$

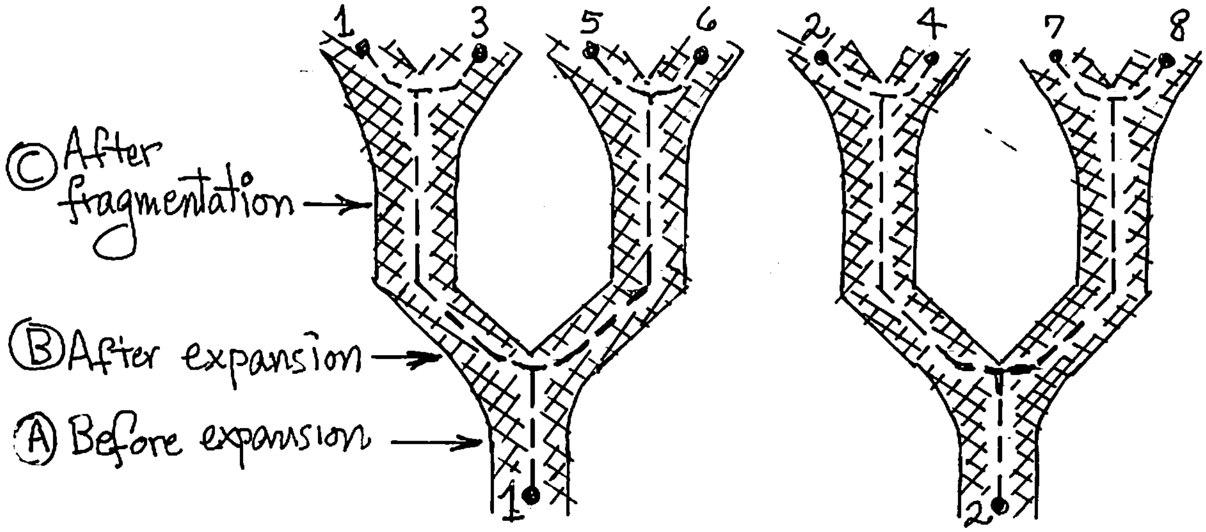
A more suggestive rewrite of this is as follows:

$$A^{AB}_\mu dx^\mu = \begin{cases} 05: & dT \\ 15: & dX - X\,dT \\ 25: & dY - Y\,dT \\ 35: & dZ - Z\,dT \\ 01: & -dX + X\,dT \\ 02: & -dY + Y\,dT \\ 03: & -dZ + Z\,dT \end{cases}$$

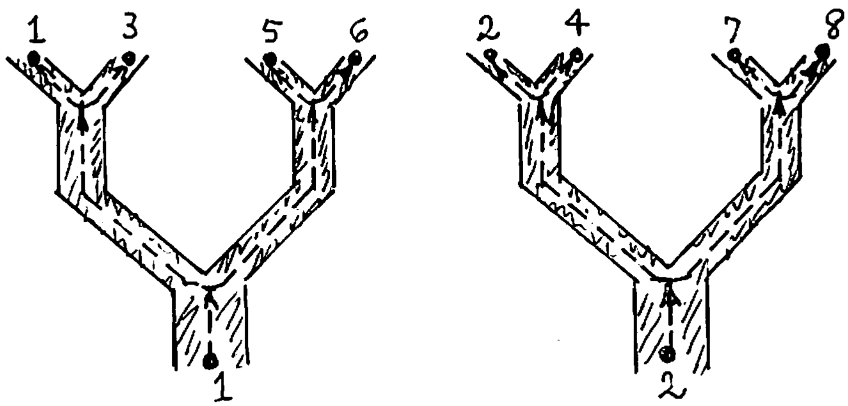
$$dT = \dot{T}\,dt$$
$$dX = \dot{X}\,dt - X'\,dx$$
$$dY = \dot{Y}\,dt - Y'\,dy$$
$$dZ = \dot{Z}\,dt - Z'\,dz$$

We see that the quantities X (x, t), Y (y, t), Z (z, t) and T (t) are the central, "gauge invariant" degrees of freedom in this description.

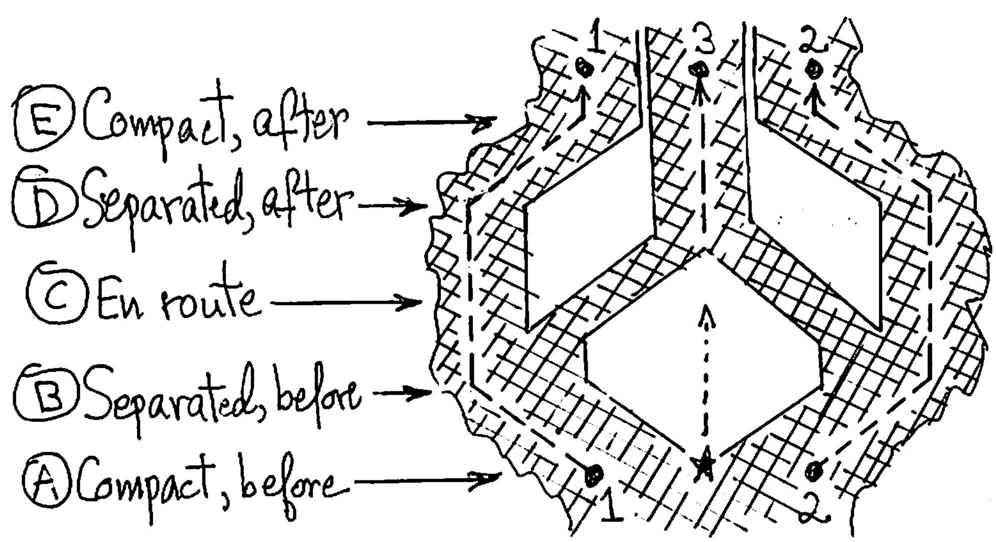
With the above examples in hand, it should be clear how to manipulate the definitions of X, Y, Z. and T in terms of the substrate variables in order to describe in more deconstructed terms the time evolution of inflation and the creation of new elements of darkness. Some examples are shown on the next pages, first for one space dimension and then for two. Some of these examples include the Voronoi-cell description. We have not done the explicit mathematics for such cases. It, hopefully, should be clear that it is doable. We invite the interested reader to do it for himself/herself.



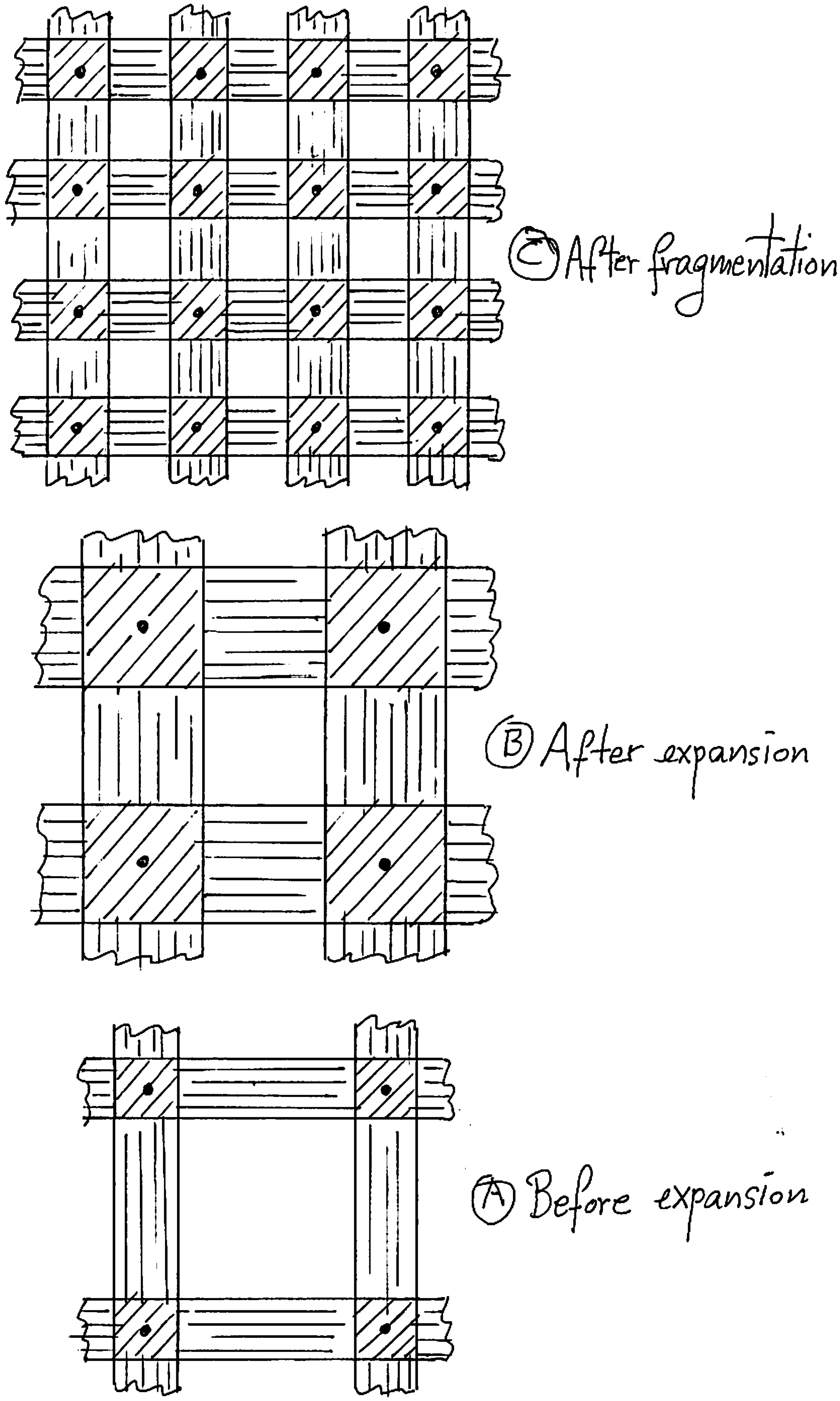
Painleve-Gullstrand "Cartesian" evolution in one space dimension



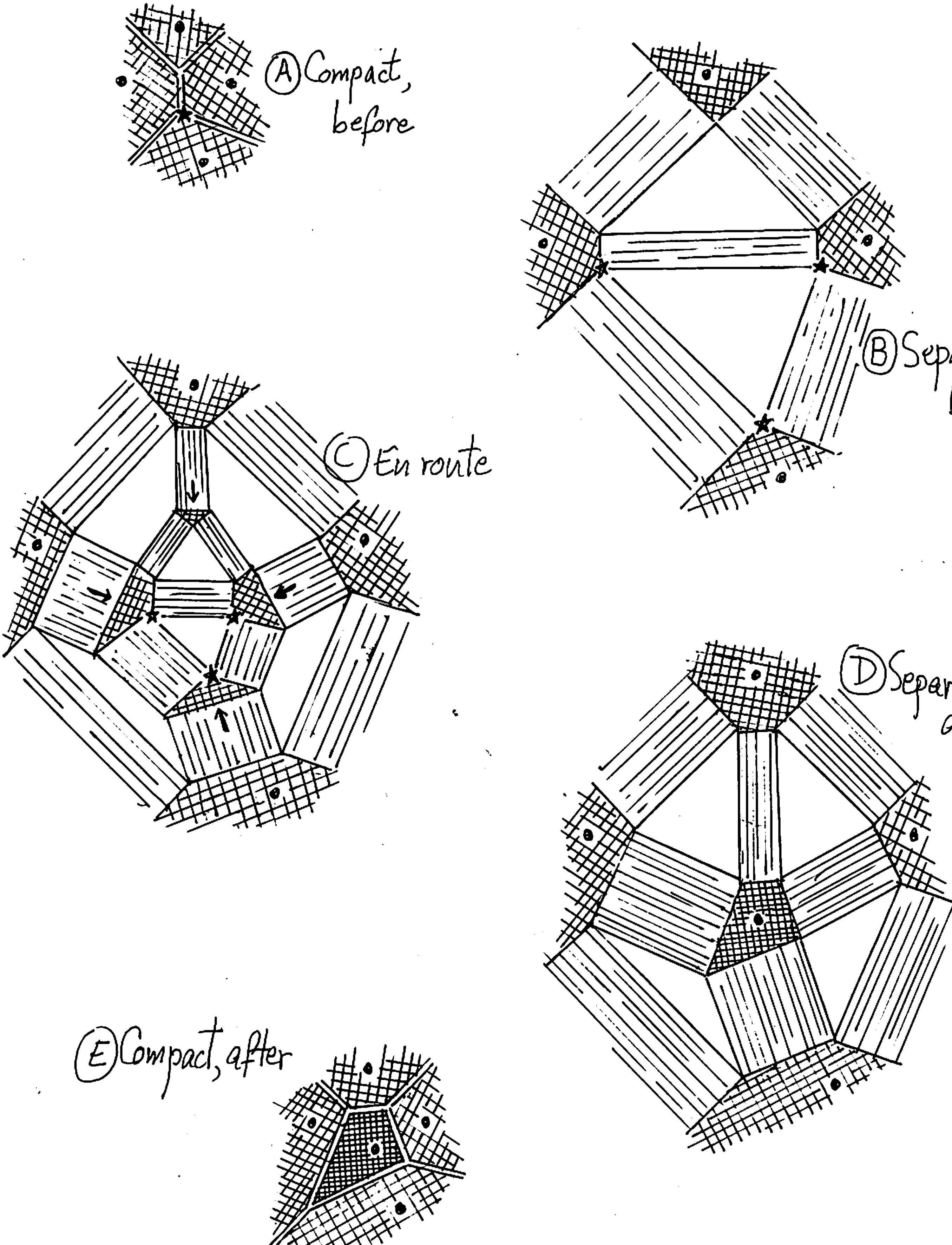
FRW "Cartesian" evolution in one space dimension



FRW "Voronoi" evolution in one space dimension



Painleve-Gullstrand "Cartesian" evolution in two space dimensions



FRW "Voronoi" evolution in two space dimensions

There appears to be no obstacle in going to three space dimensions. We have seen that, in the Cartesian description, the spatial structure of the substrate consists of a lattice containing cells, rods, plates, and voids. The physical, metric space lies only within the time intervals for which $N = 1$, and only within the cells, where the gauge potentials are hybrid FRW/PG. In the voids, all components of the gauge potential vanish except for A_t. The lapse component

$$A_t^{05} = N(t) = \dot{T}(t)$$

controls the clock. The "shift" components

$$A_t^{15} = -A_t^{01} = \dot{X}(x,t) - X\dot{T}(t) \qquad etc,$$

control the expansion. In a rod extending in the x-direction, A_y and A_z are allowed to be nonvanishing, while A_x vanishes. Their values are

$$A_y^{25} = A_z^{35} = -A_y^{02} = -A_z^{03} = a(t) = Y' = Z'$$

On a yz plate (which extends in y and z but not in x), only A_x is allowed to be nonvanishing. Its value is

$$A_x^{15} = -A_x^{01} = X'$$

The properties of these regions will be best described by the gauge-invariant quantities. Simple examples for the three cases are depicted below:

CELL VOLUME (provided the integration volume encloses it):

$$\text{Volume} = \int dx\,dy\,dz\, A_x^{15} A_y^{25} A_z^{35} = \int dx\,dy\,dz\, X'(x,t)\, Y'(y,t)\, Z'(z,t) = \Delta X\, \Delta Y\, \Delta Z$$

ROD CROSS-SECTIONAL AREA (provided the rod fully punctures the integration surface):

$$\text{Area} = \int dy\,dz\, A_y^{25} A_z^{35} = \Delta Y\, \Delta Z$$

PLATE THICKNESS (provided the curve C fully punctures the plate):

$$\text{Thickness} = \int dx\, A_x^{15} = \Delta X$$

VI. Summary

This note has gotten into a lot of dirty details. What are the Big Issues behind all of this?

1. The MacDowell-Mansouri version of deSitter space contrasts the role of the metric spacetime, described by the vierbein constructed from the gauge potentials, with the role of the rigid substrate, described by the "flat-space" coordinates t,x,y,z in terms of which the gauge potentials are defined. Darkness, here defined by hypothesis in terms of "privileged observers", is the tool used to try to understand the relationship better.

2. The very large freedom of description associated with the choice of gauge is another tool for obtaining a better intuitive understanding of this relation of substrate to spacetime.

3. We are tempted by the above description to regard deSitter spacetime, at the present time, as an occupant of a small fraction of the rigid substrate, but a fraction which grows exponentially with time. This in turn leads to an intuitive picture of the "reheating" event, which with near certainty can be expected to terminate the inflationary epoch which is now being initiated. "Reheating" most likely occurs---given the Painleve-Gullstrand description--- when the fraction of the substrate occupied by spacetime approaches a number of order unity.

4. With the experience gained from the exercises performed in this note, we anticipate moving away from the specific micromechanisms of interest here, and moving toward a coarse-grained version of rectangular arrays of rods, plates, and cells. These hopefully can be chosen, thanks to the gauge freedom, to have a simple Cartesian geometry. And if the darkness per macroscopic cell is a very large number, then statistical averages may be available for dealing with issues such as the description of the inflationary growth of darkness and the associated dark energy.

5. We emphasize again that the intuition being acquired is essentially nonAbelian, and appears to lead us toward a language close to, but not identical with, the spin networks and spin foams used in loop quantum gravity.

6. The next steps will be to apply this intuition to FRW cosmology and to the gravitational environment of a proton (or heavier counterpart such as a nucleus or neutron star). The necessary theoretical tools seem to be in place.

7. Again, it is hoped that the Newtonian limit is sufficient for gaining important insights into this problem of the nature of darkness, dark energy, and the role of the MacDowell-Mansouri rigid substrate. And it should be noted in this regard that the presence of large numbers in the description can be expected to be an advantage, not a disadvantage. The spacetimes of interest might best be described as a dilute component of a very large substrate. If that is a defensible notion, we may be dealing with a system whose role is analogous to the role of ideal gases in the development of thermodynamics and of statistical mechanics.