Mathematics Intervention

FERGUSON/FLORISSANT

DECEMBER 2017

Contact Information

Sarah Powell
- ◦ University of Texas at Austin

✉ srpowell@austin.utexas.edu

🐦 @sarahpowellphd

Make a Friend

Introduce yourself to someone – that person will be your partner today

Discuss your role in teaching mathematics

Explicit Instruction

MODELING

Clear explanation

Planned examples

PRACTICE

Guided and independent

Supporting Practices

Asking the right questions

Eliciting frequent responses

Providing immediate specific feedback

Maintaining a brisk pace

Explicit Instruction

I Do

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

We Do

You Do

Supporting Practices
- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Goal and importance

Modeling

Clear Explanation

Planned Examples

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

"Let's continue working with our three-dimensional shapes and volume. Understanding volume and calculating volume helps with measuring capacity."

Goal and importance

Modeling

Clear Explanation

Planned Examples

$$79$$
$$- 18$$

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

"To solve 26 plus 79, I first decide about the operation. Do I add, subtract, multiply or divide?"

"The plus sign tells me to add. So, I'll add 26 plus 79. I'll use the partial sums strategy. First, I add 20 plus 70. What's 20 plus 70?"

"20 plus 70 is 90. I write 90 right here."

"Then I add 6 plus 7. What's 6 plus 7?"

"6 plus 7 is 13. So, I write 13 here."

"Finally, we add the partial sums: 90 and 13. 90 plus 13 is 103. So, 26 plus 79 equals 103."

Modeling

Goal and importance

Model steps

Clear Explanation

Planned Examples

$$\begin{array}{r} 79 \\ -\ \underline{18} \end{array}$$

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

Concise language

"To solve 26 plus 79, I first decide about the operation. Do I add, subtract, multiply or divide?"

"The plus sign tells me to add. So, I'll add 26 plus 79. I'll use the partial sums strategy. First, I add 20 plus 70. What's 20 plus 70?"

"20 plus 70 is 90. I write 90 right here."

"Then I add 6 plus 7. What's 6 plus 7?"

"6 plus 7 is 13. So, I write 13 here."

"Finally, we add the partial sums: 90 and 13. 90 plus 13 is 103. So, 26 plus 79 equals 103."

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

Concise language

$$\begin{array}{r} 79 \\ -\ \underline{18} \end{array}$$

Examples

Modeling

Clear Explanation

Planned Examples

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

$$24 / 6 \qquad 28 \div 7 \qquad 35 \overline{)\, 5}$$

Modeling

Clear Explanation

➡ Planned Examples

Examples

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

$$24 / 6 \qquad 28 \div 7 \qquad 35\overline{)\,5}$$

Non-examples

$$32 \div 8 \qquad 42 \div 7 \qquad 25 - 5$$

Examples

Modeling

Clear
Explanation

Planned
Examples

Non-examples

$$\begin{array}{r} 79 \\ -\ \underline{18} \end{array}$$

Practice
Guided Practice
Independent Practice

We Do

You Do

Teacher and student practice together

Practice

Guided Practice

Independent Practice

Teacher and student practice together

Practice

→ Guided Practice

Independent Practice

$$
\begin{array}{r}
79 \\
-\ \underline{18} \\
\end{array}
$$

Practice

Guided Practice

Independent Practice

Teacher and student practice together

Student practices with teacher support

Practice

Guided Practice

Independent Practice

Teacher and student practice together

Student practices with teacher support

$$
\begin{array}{r}
79 \\
-\ 18 \\
\hline
\end{array}
$$

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Supporting Practices

Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

"What?"
"When?"

"Why?"
"How?"

Supporting Practices

Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Supporting Practices

Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

$$
\begin{array}{r}
79 \\
- 18 \\
\hline
\end{array}
$$

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Supporting Practices

- Asking the right questions
  Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

"Turn and discuss the formula for perimeter with your partner."

"Write the multiplication problem on your whiteboard."

"In your math journal, draw a picture to help you remember to term *parallelogram*."

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Supporting Practices

- Asking the right questions
  Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

$$
\begin{array}{r}
79 \\
- \ 18 \\
\hline
\end{array}
$$

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

"Good work using your word-problem attack strategy."

"Let's look at that again. Tell me how you added in the hundreds column."

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
Providing immediate specific feedback
- Maintaining a brisk pace

$$79 - 18$$

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

Planned and organized

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

Planned and organiz...

Supporting Practices
- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
Maintaining a brisk pace

$$79 - 18$$

Talk to a Friend

Why is explicit instruction important?

What are your strengths with implementing explicit instruction?

What are your areas for improvement with explicit instruction?
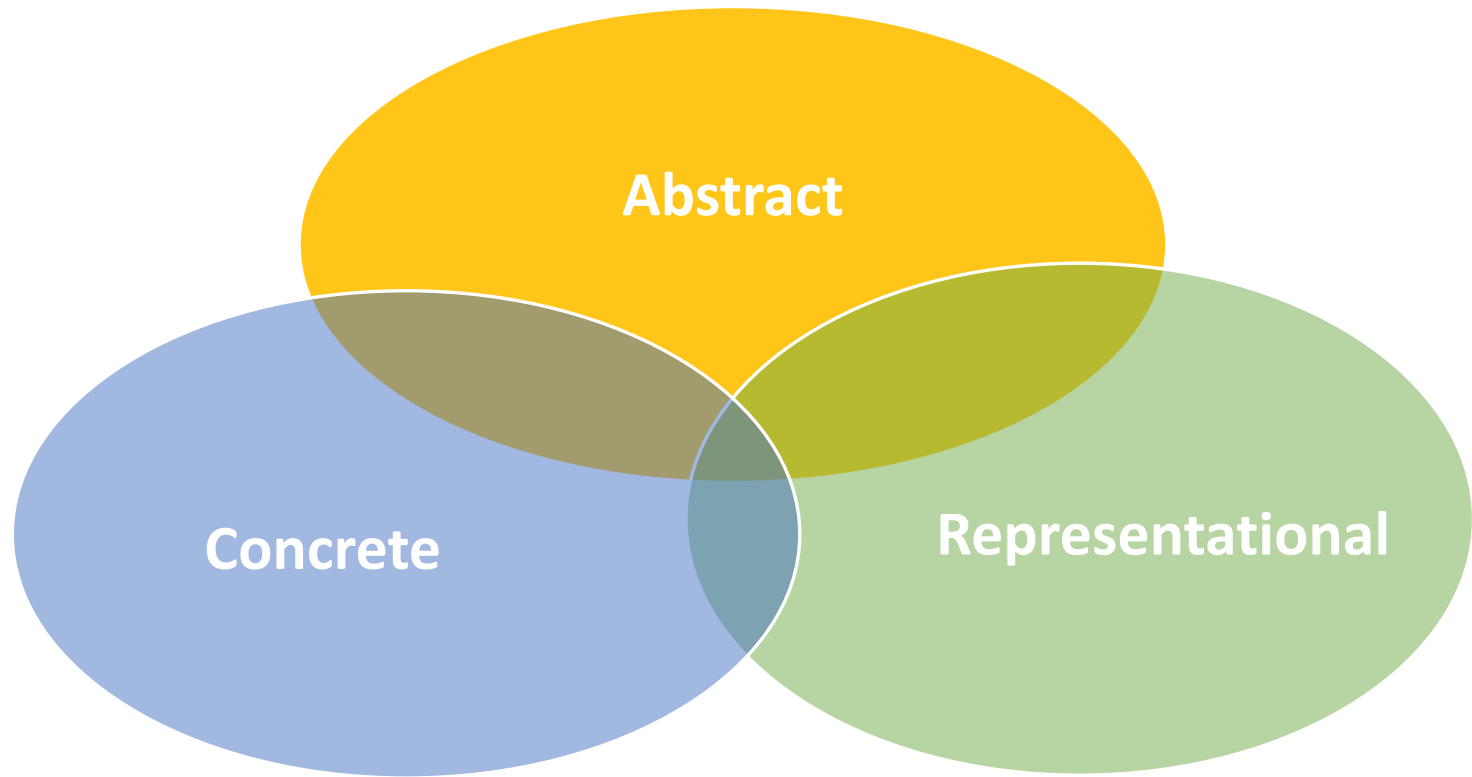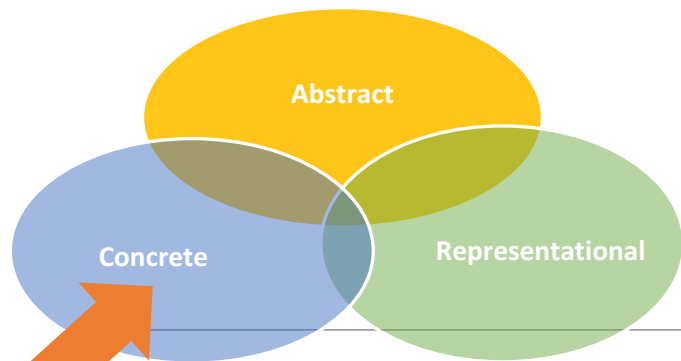
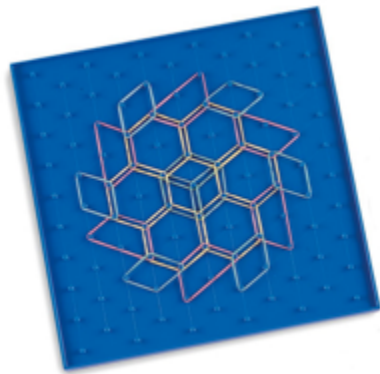# Multiple Representations

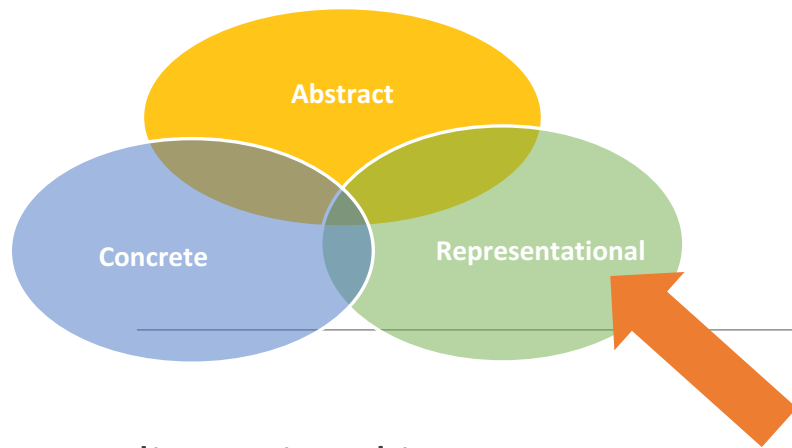# Concrete-Representational-Abstract

Multiple Representations

Abstract

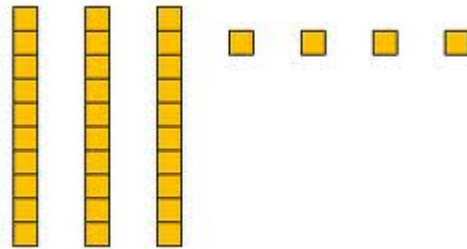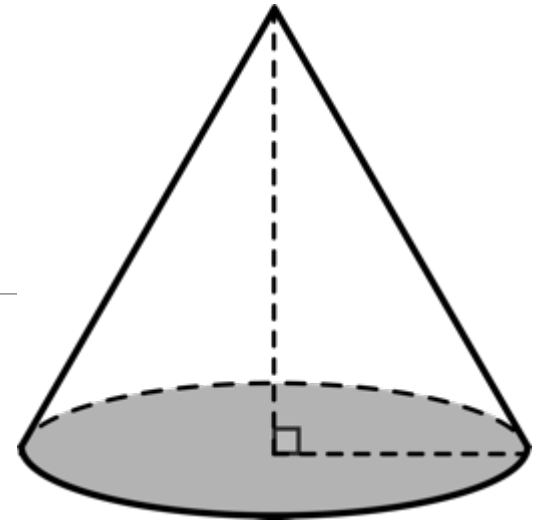Concrete
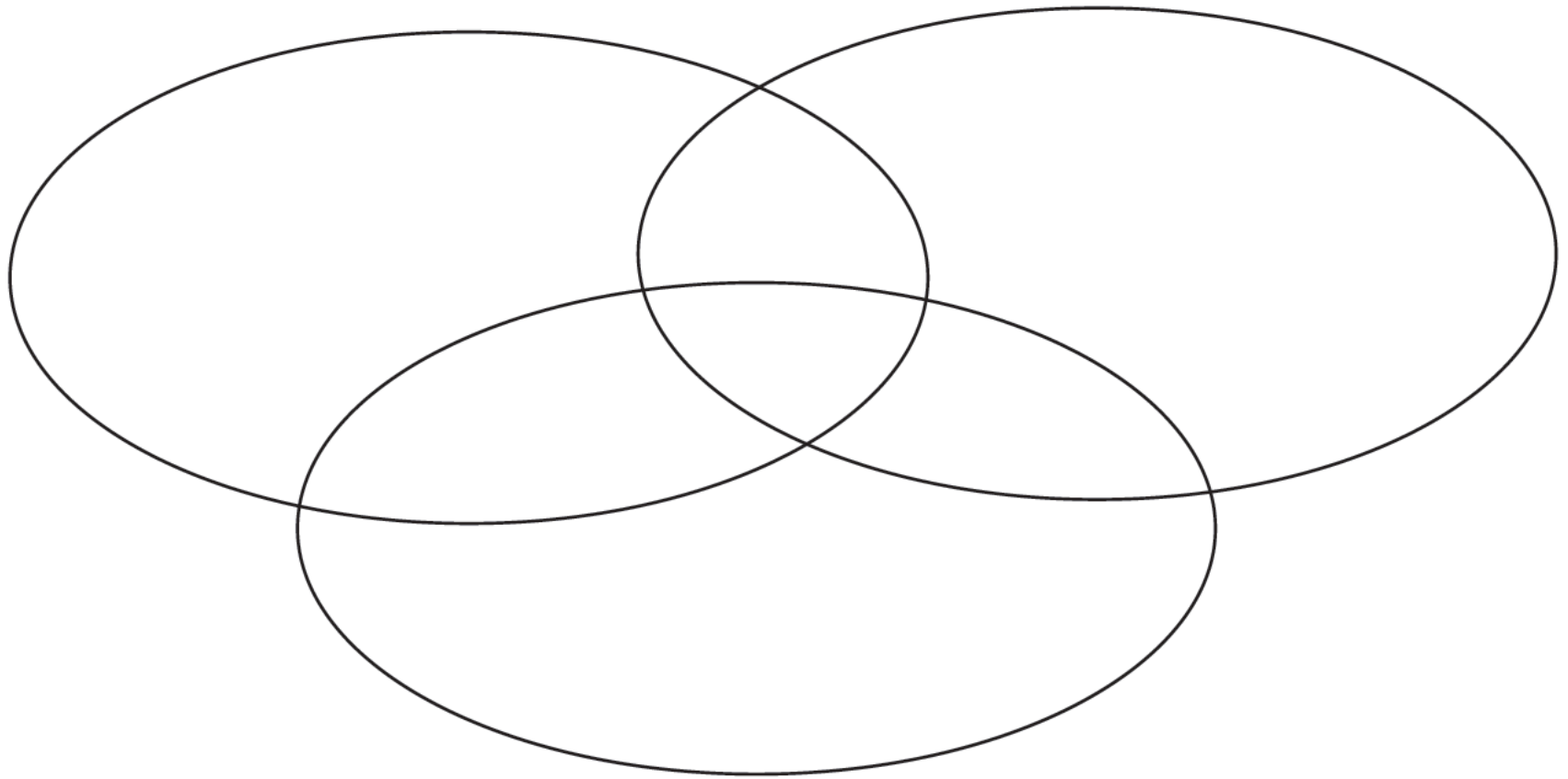
Representational

Three-dimensional objects

Abstract

Concrete

Representational

Two-dimensional images
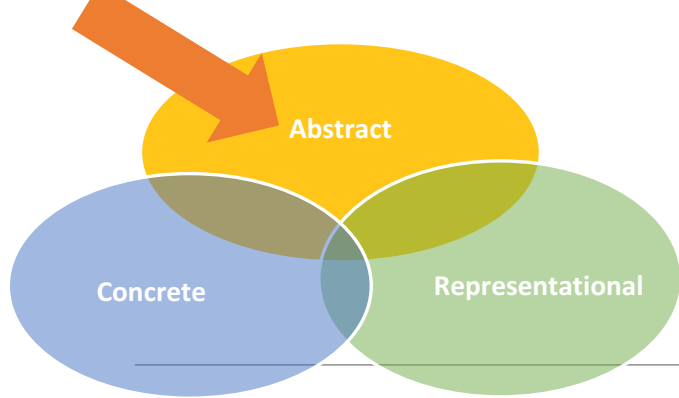
Numerals and symbols

$2 + 8 = 10$ $34 = 3$ tens and 4 ones

$x - 6 = 8$

$$4{,}179$$
$$+ \quad 569$$

Problem Solving

Word-Problem Solving

Three Takeaways About Word Problems

1	
2	
3	

Problem Solving Difficulties

Reading problems

Understanding vocabulary

Identifying relevant information

Ignoring irrelevant information

Interpreting charts and graphs

Identifying appropriate operation(s)

Performing the computation(s)

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

Package Delivery

6. **Part A**

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

Ⓐ 300

Ⓑ 340

Ⓒ 350

Ⓓ 360

Reading problems

Understanding vocabulary

Identifying relevant information

Ignoring irrelevant information

Interpreting charts and graphs

Identifying appropriate operation(s)

Performing the computation(s)

1 Don't tie **key words** to **operations**

2 Have an **attack strategy**

3 Teach word-problem **schemas**

Key Words

(Addition - orange plus)
combined
addition | in all | total
sum | together | plus
both | perimeter | add
more than

(Multiplication - pink X)
triple | twice
factor | product
multiply
each | per
in all | multiple | area
times | double

average

(Subtraction - green)
subtract | decrease | fewer
remain | take away | minus
less than | how many more...

(Division - yellow)
division | split | quotient
equal groups | divide
half | shared equally | each

distribute

©TheResourceRoomTeacher

(Anchor chart - Addition)
total · plus · in all · sum · join · together
Addition
What it looks like:
1 + 3 = 4

(Anchor chart - Subtraction)
minus · Part · left · take away · fewer
Subtraction
What it looks like:
4 - 1 = 3

Key Words Used in Math Word Problems

Addition Words
+ add
+ all together or altogether
+ and
+ both
+ combined
+ how many in all
+ how much
+ in all
+ increased by
+ plus
+ sum
+ together
+ total

Subtraction Words
- change
- decreased by
- difference
- fewer or fewer than
- how many are left (or have left)
- how many did not have
- how many (or much) more
- how much longer (shorter, taller, heavier, etc.)
- less or less than
- lost
- minus
- need to
- reduce
- remain
- subtract
- take away

Multiplication Words
x by (dimension)
x double
x each group
x every
x factor of
x increased by
x multiplied by
x of
x product
x times
x triple

Division Words
+ as much
+ cut up
+ each group has
+ equal sharing
+ half (or other fractions)
+ how many in each
+ parts
+ per
+ percent
+ quotient of
+ ratio of
+ separated
+ share something equally

Kasey made $42, and Mandy made $37. How much money did they make *in all*?

Kasey and Mandy made $79 *in all*. If Kasey made $42, how much money did Mandy make?

Kasey mowed 12 lawns on Monday. Then, she mowed 10 *more* on Tuesday. How many lawns has Kasey mowed?

Kasey mowed 22 lawns and Mandy mowed 7 lawns. How many *more* lawns did Kasey mow than Mandy?
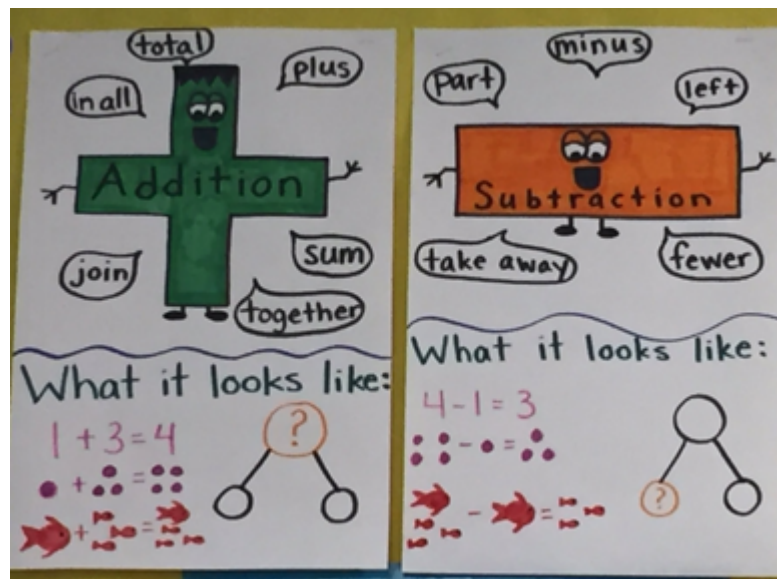
Becky has $70 *more than* Perla. If Becky has $120, how much money does Perla have?

Becky has $70 *more than* Perla. If Perla has $50, how much money does Becky have?

Becky had 9 dinosaurs and then her sister *took away* 4 of them. How many dinosaurs does Becky have now?

Becky had some dinosaurs and then her sister *took away* 4 of them. Now Becky has 5 dinosaurs. How many dinosaurs did she start with?

key words
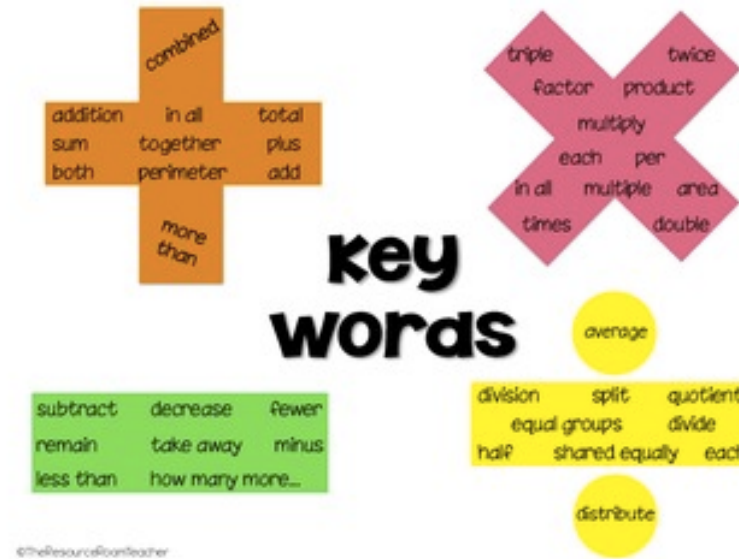
Matt baked 18 cookies. His brother baked *twice* as many. How many cookies did his brother bake?

Matt's brother baked *twice* as many cookies as Matt. If Matt's brother baked 36 cookies, how many did Matt bake?

Reece has 7 bags with 3 apples in *each* bag. How many apples does Reece have?

Reece had 21 apples and placed 3 apples *each* in several bags. How many bags does Reece need?
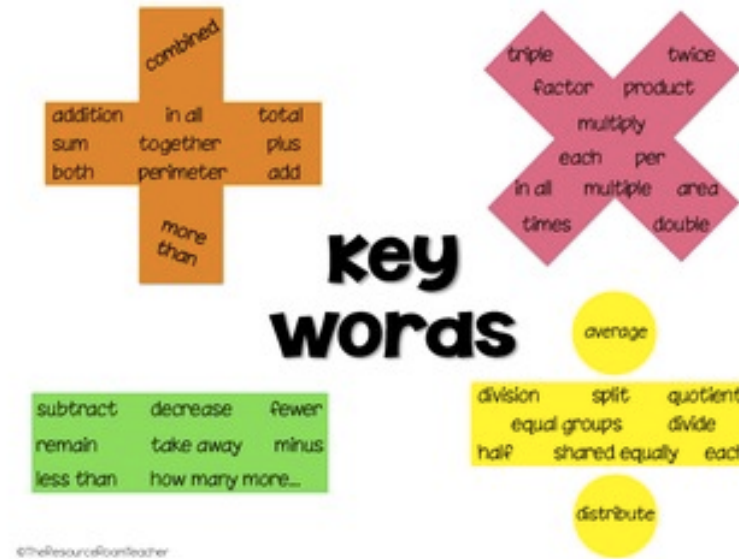
Rachel wants to *share* 36 brownies with 6 friends. How many cookies will *each* friend receive?

Rachel *shared* brownies with 6 friends. *Each* friend ate 6 brownies. How many brownies did Rachel have to start with?

Brent made 12 cupcakes. His brother made *half* as many cupcakes. How many cupcakes did Brent's brother bake?

Brent made 12 cupcakes. He cut each cupcake into *half*. How many pieces of cupcake does Brent have?

Michelle made 17 paper airplanes. Dante made 24 paper airplanes. How many airplanes did they make *altogether*?

Michelle and Dante made 41 paper airplanes *altogether*. If Dante made 24 paper airplanes, how many did Michelle make?

Michelle made 4 paper airplanes using 2 pieces of paper for each airplane. How much paper did Michelle use *altogether*?

Dante and Michelle made 40 paper airplanes *altogether*. Dante made 24 of the paper airplanes. If Michelle gave 7 of her paper airplanes to her friend Nicole, how many planes does Michelle have now?

Key Words Used in Math Word Problems

Addition Words	Subtraction Words
• add	• change
• all together or altogether	• decreased by
• and	• difference
• both	• fewer or fewer than
• combined	• how many are left (or have left)
• how many in all	• how many did not have
• how much	• how many (or much) more
• in all	• how much longer (shorter, taller, heavier, etc.)
• increased by	• less or less than
• plus	• lost
• sum	• minus
• together	• need to
• total	• reduce
	• remain
	• subtract
	• take away

Students need to understand *key words*. But, key words should not be directly tied to *operations*.

Name_____ Date_____

Read each problem. Write a number sentence and solve.

1. Mrs. Smith has 33 poodles and
    18 boxers. How many more
    poodles does Mrs. Smith have?

    _____

2. The kennel holds 91 dogs.
    Mr. Glass has 67 dogs in the
    kennel now. How many spaces
    does he have left?

    _____

3. Mr. Kelly has 44 beagles.
    26 of them are puppies.
    How many adult beagles
    does Mr. Kelly have?

    _____

4. Mrs. Green has 60 terriers.
    25 of them are boys. How many
    terriers are girls?

    _____

5. There were 58 kittens at the pet
    shop on Friday. 29 of them were
    sold on Saturday. How many
    kittens were left?

    _____

6. Pat counted 22 lizards in the tank
    at the pet shop. 8 were sold later
    that day. How many lizards were
    left in the tank?

    _____

1 Don't tie **key words** to **operations**

2 Have an **attack strategy**

3 Teach word-problem **schemas**

For every word problem

Regardless of problem type, students need an **attack** strategy for working through the problem

This strategy should work for any problem type

Routine Word Problems

A library has 126 books about trees.

24. Part A

The library has 48 fewer books about rivers than about trees.

What is the number of books the library has about rivers and what is the total number of books the library has about trees and rivers?

Ⓐ 78 and 126

Ⓑ 48 and 204

Ⓒ 48 and 126

Ⓓ 78 and 204

Instructional Word Problems

7. Which **three** shapes are quadrilaterals?

Ⓐ

Ⓑ

Ⓒ

Ⓓ

Ⓔ

Ⓕ

RIDGES

Read the problem.
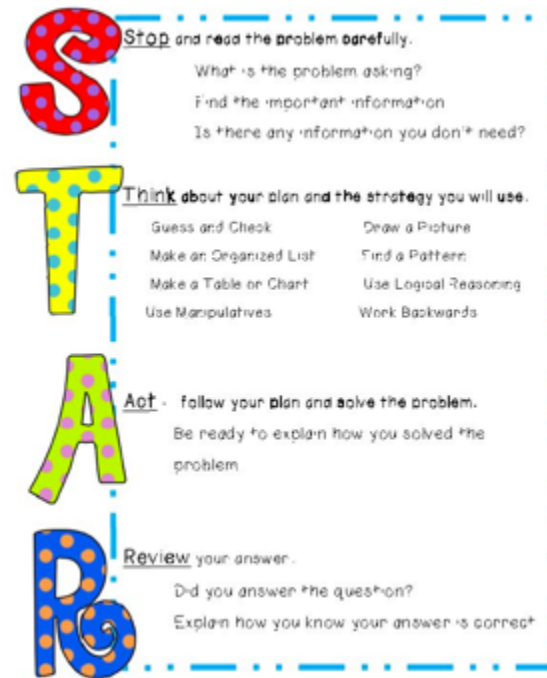**I** know statement.
Draw a picture.
Goal statement.
**E**quation development.
Solve the equation.

Stop and read the problem carefully.
What is the problem asking?
Find the important information
Is there any information you don't need?

Think about your plan and the strategy you will use.

Guess and Check   Draw a Picture
Make an Organized List   Find a Pattern
Make a Table or Chart   Use Logical Reasoning
Use Manipulatives   Work Backwards

Act - follow your plan and solve the problem.
Be ready to explain how you solved the problem

Review your answer.
Did you answer the question?
Explain how you know your answer is correct

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

Read and Record the problem.
Illustrate your thinking.
Compute.
Explain your thinking.
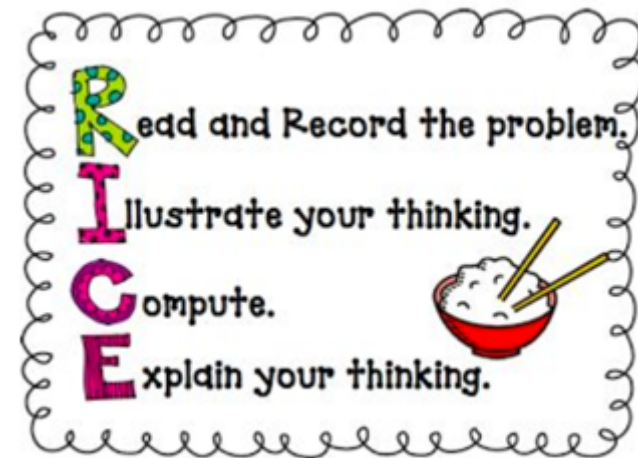
R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

Problem
Read your problem.

Analyze
Underline key words. Cross out information you do not need. Decide which operation you will need to use.

Work it out
Use the operation to work the problem out. Show your work.

Solve
Write your answer. Ask yourself, "Does my answer make sense?"

S lowly read the story problem twice.

U nderline the question and circle the numbers you need.

P icture it! Draw the scenario to show what is happening.

E xplain the problem with a number sentence.

R ewrite the answer in a sentence.

RUN!

1. _**R**ead_ the problem.

2. _**U**nderline_ the labels.

3. _**N**ame_ the problem type.

Total, **D**ifference, or **C**hange
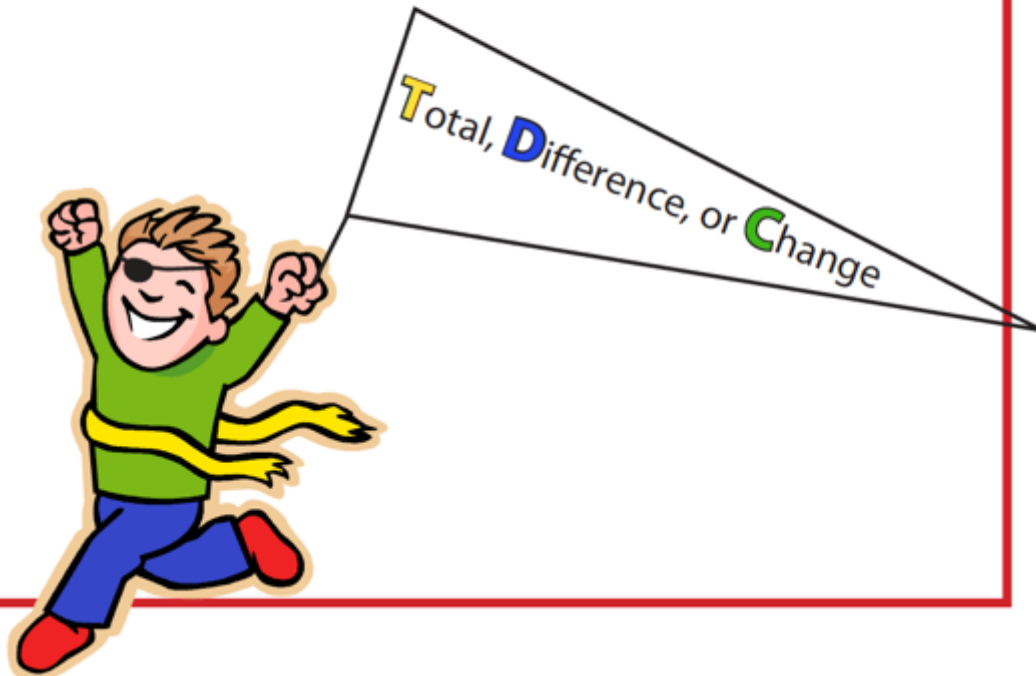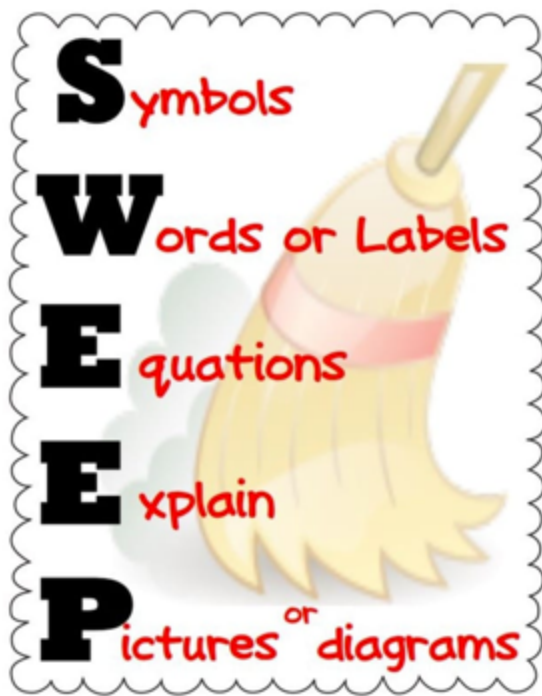
SIGNS

Survey questions

Identify key words

Graphically draw problem

Note operations

Solve and check

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

S	slowly and carefully **READ** the problem.	
H	highlight or underline **key information**.	
I	**identify the question** by drawing a circle around it.	
N	now **solve the problem** with numbers, pictures, and words. Show your work.	
E	**examine your work** for precision, accuracy, and clarity.	
S	share your answer by **writing a sentence.**	

SOLVING **LET'S S.K.A.T.E.**
Word Problems

S — Survey the word problem and identify the question.

"The skateboard store had a total of 11 skateboards on Monday. By Friday, there were only 4 skateboards left. How many skateboards were sold?"

K — Keep and highlight important information.

"The skateboard store had a total of 11 skateboards on Monday. By Friday, there were only 4 skateboards left. How many skateboards were sold?"

A — Attempt to estimate the answer. USE THE FACTS TO MAKE A REASONABLE GUESS!

T — Take time to solve the problem.

$11 - ? = 4$

By Friday, there were a total of 7 skateboards sold.

E — Examine your answer carefully.

7 is a reasonable answer.

GRASS

G — What is important in the question? What information is delivered to help you solve the question?

R — What is the problem? What are you being asked to do?

A — What problem solving strategy will you use? What approach can you take to find the solution?
☐ Use Manipulatives ☐ Act it out ☐ Make a model
☐ Make a table ☐ Draw a picture ☐ work backwards
☐ Look for patterns ☐ Use a Formula ☐ Process of Elimination
☐ make a graph ☐ Guess and Check

S — Time to do the math! Follow through on your strategy and find the answer!

S — A summary, in words, of what you did.
· I used _____ because _____
· I think _____ didn't work because _____
_____ is why I chose my second application
· I know that _____
· I found _____
· I have used _____ to solve something like this before
_____ worked better than _____

UNDERSTAND
- Read the problem carefully.
- Highlight or circle necessary data, key words, and labels.
- Identify the question.

PLAN
- Record the information needed to solve the problem.
- Draw a picture or diagram of the situation and label all parts.
- Choose the appropriate strategy, tool, or operation.

SOLVE
- Write an appropriate equation for the situation.
- Use your plan and data to solve.
- Write your solution with units if applicable.

CHECK
- Check your math (substitute the value(s) into your equation).
- Did you answer the question?
- Is your answer reasonable?

Steps to Solve a Problem: UPS Check!

Understand
- Read the problem.
- What am I looking for?
  [Bracket the question]
- What do I know?
  Circle the important numbers.
  Underline labels

Plan
- Choose a strategy.
  • Draw a Picture or Make a Table.

JUG SUG CUG JEG SEG SEG

+   −   −   ×   ÷   ÷

Solve
Show all your work.
Label your answer.
  3
+5
8 crayons

✓ Check
Explain & Justify

Is your answer is REASONABLE?

Steps to Problem Solving — UPS-Check

U Understand   P Plan

S Solve   ✓ Check, Justify & Explain

1 Don't tie **key words** to **operations**

2 Have an **attack strategy**

3 Teach word-problem **schemas**

Instruction Using Schemas

A **schema** refers to the structure of the word problem

Total

Difference

Change

Additive Schemas

Problem type	Definition	Examples			Equation	Graphic organizer
Total		Total unknown	Part unknown			
Difference		Difference unknown	Greater unknown	Lesser unknown		
Change (increase)		End unknown	Start unknown	Change unknown		
Change (decrease)		End unknown	Start unknown	Change unknown		

Additive Word Problems

Parts put together into a **total**

◦ Emily saw **4** cardinals and **5** blue jays. How many birds did Emily see?

　◦ **4** + **5** = **?**

◦ Emily saw **9** birds. If **4** of the birds were cardinals, how many were blue jays?

　◦ **4** + **?** = **9**

◦ Emily saw **9** birds. **5** of the birds were blue jays, how many were cardinals?

　◦ **5** + **?** = **9**

Total

P1 + P2 = T

(part) + (part) = (total)

Total
?

Part	Part
20	16

T Megan baked 28 sugar cookies and 24 chocolate chip cookies. Enter the total number of _cookies_ Megan baked in all.

$$P1 + P2 = T$$

$$28 + 24 = ?$$

$$28 + 24 = 52$$

$$X = 52 \text{ cookies}$$

Total

"Are parts put together for a total?"

A banana farm received a total of 12 millimeters of rain in March and April. If 11 millimeters of rain fell on the farm in March, how many _millimeters_ of rain fell on the farm in April?

12	
11	?

$$11 + 1 = 12$$

$$12 - 11 = 1$$

? = 1 millimeter

Additive Word Problems

TOTAL

with the **total** unknown

with a **part** unknown

DIFFERENCE

CHANGE (INCREASE)

CHANGE (DECREASE)

Problem type	Definition	Examples			Equation	Graphic organizer
Total		Total unknown	Part unknown			
Difference		Difference unknown	Greater unknown	Lesser unknown		
Change (increase)		End unknown	Start unknown	Change unknown		
Change (decrease)		End unknown	Start unknown	Change unknown		

Additive Word Problems

Difference

Greater and less amounts compared for a difference

- Shinead has **9** apples. Amanda has **4** apples. How many more apples does Shinead have? (How many fewer?)

  - **9** − **4** = **?**

- Shinead has **5** more apples than Amanda. If Amanda has **4** apples, how many does Shinead have?

  - **?** − **4** = **5**

- Amanda has **5** fewer apples than Shinead. Shinead has **9** apples. How many apples does Amanda have?

  - **9** − **?** = **5**

Difference

B − S = D

(bigger) − (smaller) = (difference)

Quantity
12

Quantity
5
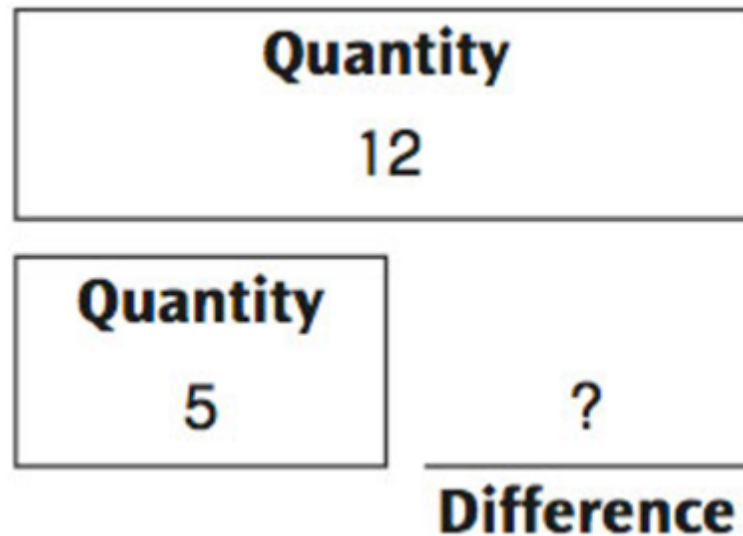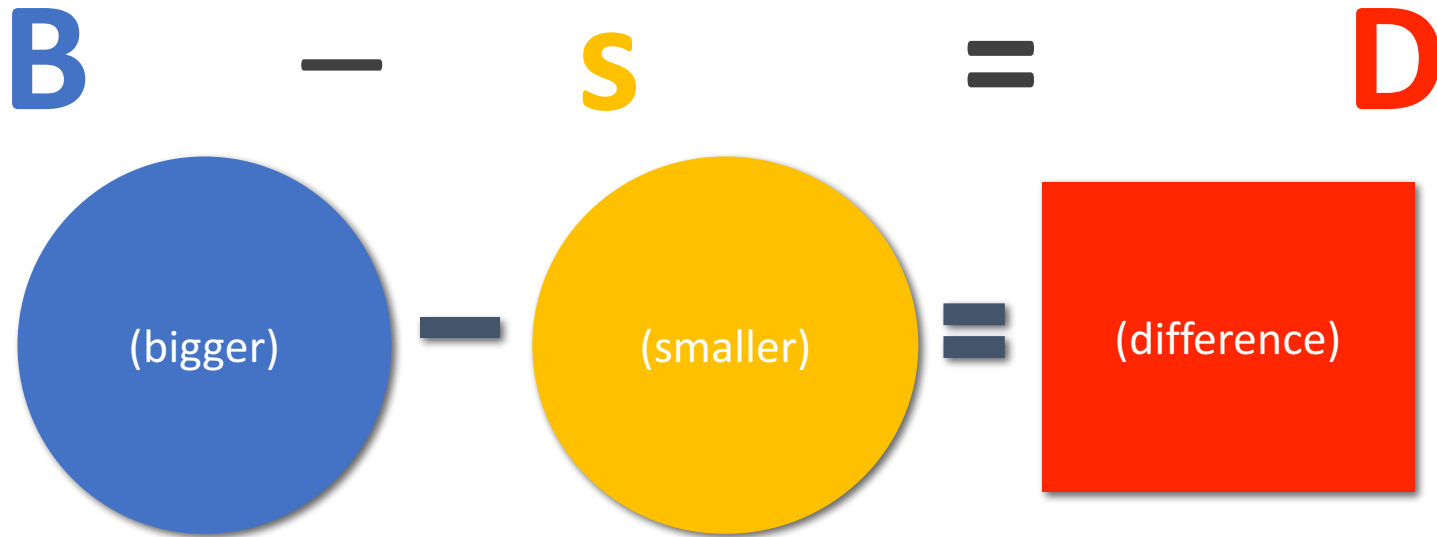
?
Difference

D Jana has 107 wooden beads and 68 glass beads. How many more wooden beads than glass beads does Jana have?

$$G - L = D$$

$$107 - 68 = B$$

$$107 - 68 = 39$$

$$\begin{array}{r} 107 \\ - 68 \\ \hline \end{array}$$

$$\begin{array}{r} 68 \\ + \ ? \\ \hline 107 \end{array}$$

W = 39 more beads

Total

"Are parts put together for a total?"

Difference

"Are amounts compared for a difference?"

Farmer Hank has 6 more cows than horses. He has 4 horses. He also has 9 chickens. How many cows does he have?

Additive Word Problems

TOTAL

DIFFERENCE

with the **difference** unknown

with a **greater** amount unknown

with a **lesser** amount unknown

CHANGE (INCREASE)

CHANGE (DECREASE)

Problem type	Definition	Examples			Equation	Graphic organizer
Total		Total unknown	Part unknown			
Difference		Difference unknown	Greater unknown	Lesser unknown		
Change (increase)		End unknown	Start unknown	Change unknown		
Change (decrease)		End unknown	Start unknown	Change unknown		

Additive Word Problems

An amount that **increases** or decreases

◦ Shannah had $**4**. Then she got $**3** for cleaning her room. How much money does Shannah have now?

  ◦ **4** + **3** = **?**

◦ Shannah has $**4**. Then she earned money for cleaning her room. Now Shannah has $**7**. How much money did she earn?

  ◦ **4** + **?** = **7**

◦ Shannah had some money. Then she made $**3** for cleaning her room. Now she has $**7**. How much money did Shannah start with?

  ◦ **?** + **3** = **7**

Change

ST   +/−   C   =   E

(change)

(begin)

(end)

Change

Start
20

+6

End
?

Fuchs et al. (2008); Griffin & Jttendra (2009)
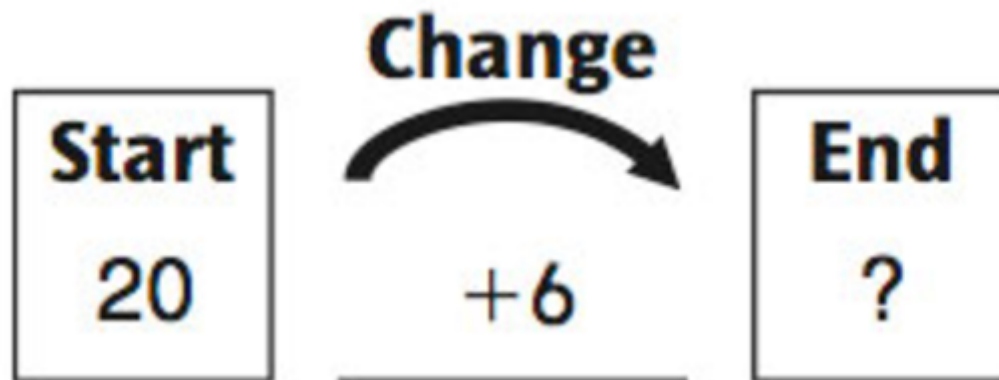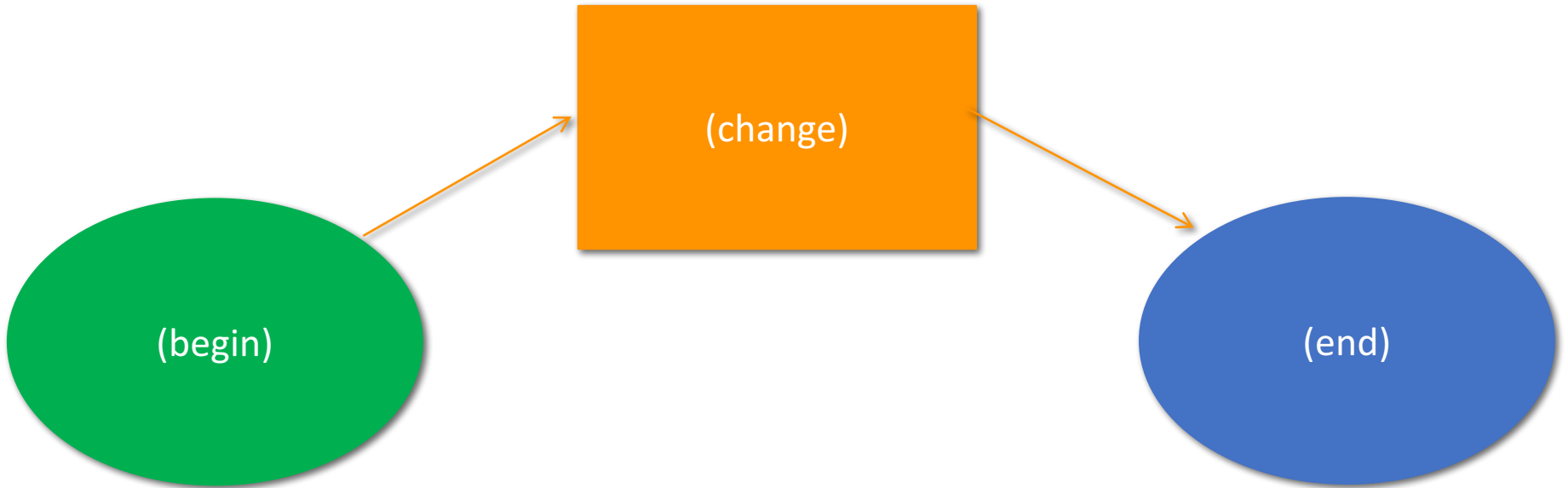
+ C A bus had 13 passengers. At the next stop, more passengers got on the bus. Now, there are 28 passengers. How many passengers got on the bus?

$$ST + C = E$$

$$13 + ? = 28$$

$$13 + 15 = 28$$

$$? = 15 \text{ passengers}$$

$$
\begin{array}{r}
28 \\
- \underline{13}
\end{array}
\qquad
\begin{array}{r}
13 \\
+ \ ? \\
\hline
28
\end{array}
$$

Total

"Are parts put together for a total?"

Difference

"Are amounts compared for a difference?"

Change

"Does an amount increase or decrease?"

Additive Word Problems

TOTAL

DIFFERENCE

CHANGE (INCREASE)

with the **end** unknown

with the **start** unknown

with the **change** unknown

CHANGE (DECREASE)

Change

An amount that increases or **decreases**

- Micaela had $**9**. Then she spent $**2** on candy. How much money does Micaela have now?

  - **9** − **2** = **?**

- Micaela had $**9**. She spent some money on candy. Now Micaela has $**7**. How much money did Micaela spend on candy?

  - **9** − **?** = **7**

- Micaela had some money. Then she spent $**2** on candy. Now she has $**7**. How much money did Micaela start with?

  - **?** − **2** = **7**

Change

ST +/− C = E

(change)

(begin)

(end)

Change

Start 20 +6 End ?
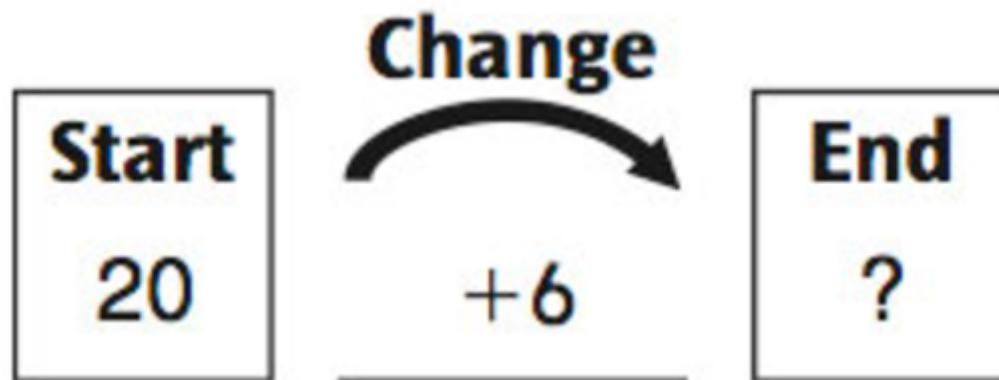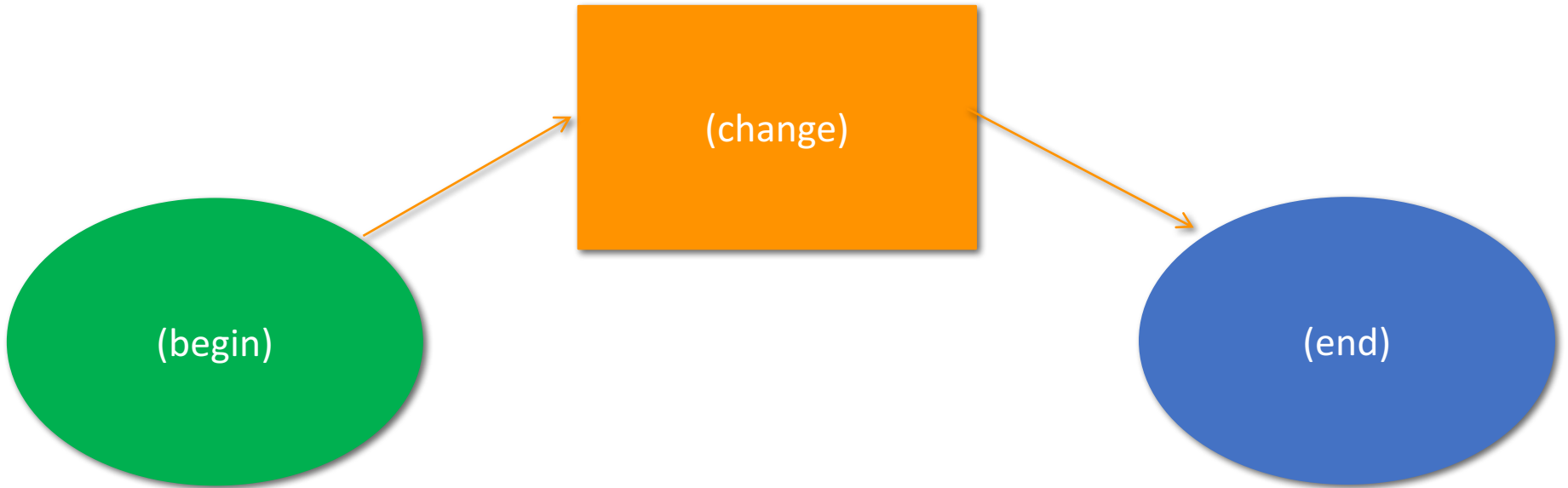
Martina had some money. Then, she spent $42 on a sweater. Now, she has $13. How much money did she have to start with?

Additive Word Problems

TOTAL

DIFFERENCE

CHANGE (INCREASE)

CHANGE (DECREASE)

with the **end** unknown

with the **start** unknown

with the **change** unknown

Problem type	Definition	Examples			Equation	Graphic organizer
Total		Total unknown	Part unknown			
Difference		Difference unknown	Greater unknown	Lesser unknown		
Change (increase)		End unknown	Start unknown	Change unknown		
Change (decrease)		End unknown	Start unknown	Change unknown		

Additive Word Problems

Ramon has a total of 815 sheep in two fields. He has 348 sheep in one of the fields. How many sheep does Ramon have in the other field?

Angelina looked in her closet and saw a container of markers. She took 42 markers out of the container and counted 88 left. How many markers were in the container when she found it in the closet?

The grocery store had 517 jars of crunchy peanut butter and 434 jars of creamy peanut butter. How many more jars of crunchy peanut butter were there?

Additive Word Problems

I.

The grocery store had 517 jars of crunchy peanut butter and 434 jars of creamy peanut butter. How many more jars of crunchy peanut butter were there?

J.

The animal park has 12 zebras, 25 monkeys, and some giraffes. If the total number of zebras, monkeys, and giraffes at the park is 50, how many giraffes are there?

K.

There were some people on a train. 19 people get off the train at the first stop. 17 people get on the train. Now there are 63 people on the train. How many people were on the train to begin with?

L.

Mrs. Lanier saved $617 in January. In February, she spent $249 of the money she saved. She saved $291 more in March. How much has Mrs. Lanier saved by the end of March?

The animal park has 12 zebras, 25 monkeys, and some giraffes. If the total number of zebras, monkeys, and giraffes at the park is 50, how many giraffes are there?

$$P1 + P2 + P3 = T$$

There were some people on a train. 19 people get off the train at the first stop. 17 people get on the train. Now there are 63 people on the train. How many people were on the train to begin with?

$$ST - C + C = E$$

Mrs. Lanier saved $617 in January. In February, she spent $249 of the money she saved. She saved $291 more in March. How much has Mrs. Lanier saved by the end of March?

$$ST - C + C = E$$

Additive Word Problems

TOTAL

DIFFERENCE

CHANGE (INCREASE)

CHANGE (DECREASE)

Let's Review

Why is an attack strategy helpful?

What's a Total problem?

What's a Difference problem?

What's a Change problem?

Why should students learn schemas?

A baker has 159 cups of brown sugar and 264 cups of white sugar. How many total cups of sugar does the baker have?

Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

The graph below shows the number of pounds of plastic the Keller family recycled for five months.

Recycled Plastic

Each ⬚ means 20 pounds.
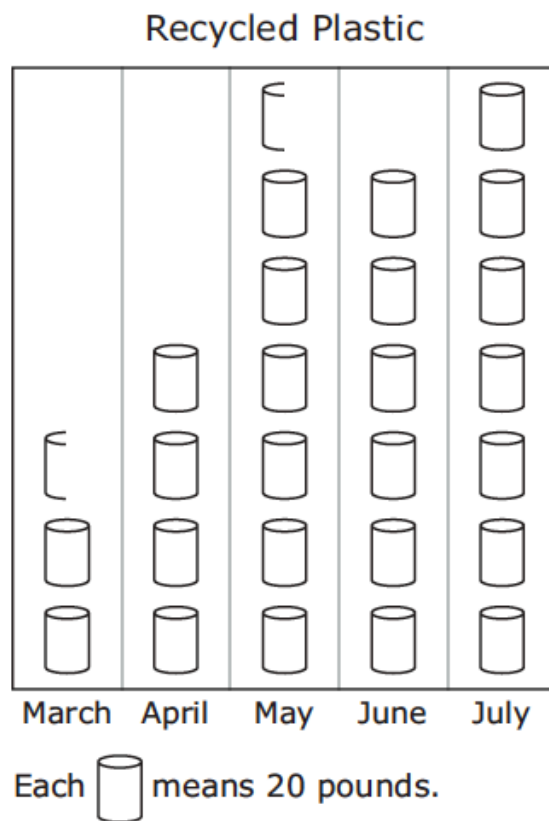
Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Roland's family drove $4\frac{6}{10}$ kilometers from their home to the gas station.

They drove $2\frac{30}{100}$ kilometers from the gas station to the store.

Which expression can be used to determine the number of kilometers Roland's family drove altogether?

At the beginning of June, a bean plant was $3\frac{4}{5}$ inches tall.

By the beginning of July, the plant was $6\frac{2}{5}$ inches tall.

How many inches did the plant grow during June? Enter your answer in the response box.

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

Package Delivery

10. **Part A**

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

Ⓐ 300

Ⓑ 340

Ⓒ 350

Ⓓ 360

Farmer Hank has 6 more cows than horses. He has 4 horses. He also has 9 chickens. How many cows does he have?

Change

Shelby's teacher gives out gold stars for great math work. Yesterday, Shelby earned 4 gold stars. Today, she earned 3 more. How many gold stars did Shelby earn in all?

Difference

Mrs. Taylor's class is measuring the wingspans of butterflies, in inches.

They record their results in this table.

Butterfly	Wingspan (in)
Monarch	$3\frac{2}{4}$
Zebra	$2\frac{3}{4}$
Checkered White	$1\frac{2}{4}$
Tiger	?

The wingspan of the Tiger Butterfly is $\frac{3}{4}$ inch longer than that of the Monarch Butterfly.

What is the difference, in inches, between the longest and shortest wingspans?

Jana gets a sticker for every 5 minutes she spends on her chores each day. She puts them on a picture graph as shown.

Jana's Chores

Day	Minutes of Chores
Monday	☆ ☆ ☆ ☆
Tuesday	☆ ☆ ☆ ☆ ☆
Wednesday	☆ ☆ ☆ ☆
Thursday	☆ ☆ ☆ ☆ ☆ ☆
Friday	

KEY
☆ = 5 minutes

Jana spends a total of 130 minutes doing chores during the week. How many stickers should Jana get on Friday?

Change

You have 18 marbles but you gave your friend 9. Now how many marbles do you have?

Ana starts eating lunch at 12:15 p.m. She finishes eating lunch 40 minutes later.

Which clock shows the time that Ana finishes eating lunch?

Difference

Mr. Thompson sold 247 meals on Tuesday at his restaurant. He sold 516 meals on Wednesday. What is the difference between the numbers of meals Mr. Thompson sold on these two days?

There were 58 kittens at the pet shop on Friday. 29 of them were sold on Saturday. How many kittens were left?

The table shows the number of pennies Nolan saved each week for four weeks.

Pennies Saved Each Week

Week	Number of Pennies
Week 1	18
Week 2	40
Week 3	32
Week 4	25

What is the total number of pennies Nolan saved during the four weeks? Show your work.

Enter your answer and your work in the space provided.

Change

Adyssen started with $87 in her bank account. She put $213 into her account last week and another $137 this week. What is the total amount Adyssen now has in her bank account?

Carla buys apples and peaches at the store. The mass of the apples is 724 grams. The mass of the peaches is 471 grams.

How much greater is the mass, in grams, of the apples than the mass of the peaches?

Change

7. There were some people on a train.

19 people get off the train at the first stop.

17 people get on the train.

Now there are 63 people on the train.

How many people were on the train to begin with?

Marks

Show your method.

Mrs. Green has 60 terriers. 25 of them are boys. How many terriers are girls?

Mary's cat had kittens. She gave 3 to her friends.

She now has 6 kittens left. How many kittens did she have to start with ?

Last month Jim drove his car 2,718.3 miles. That brought the car's total mileage to 87,416 miles. What was the car's total mileage before last month?

Vicente hung three posters in his bedroom.

- The first poster had a length of 59 centimeters.
- The second poster had a length of 92 centimeters.
- The third poster had a length of 127 centimeters.

What is the best estimate of the total length of these three posters in centimeters?

A 260 cm

B 350 cm

C 240 cm

D 280 cm

Multiplicative Schemas

Problem type	Definition	Examples			Equation	Graphic organizer
Equal Groups						
Comparison						
Combinations						
Ratios and Pro-portions						

Multiplicative Word Problems

Instruction Using Schemas

A **schema** refers to the structure of the word problem

Equal Groups

Comparison

Ratios/Proportions

Equal Groups

Groups multiplied by **number in each group** for a **product**

- Mark has **2** bags of apples. There are **6** apples in each bag. How many apples does Mark have altogether?
  - **2** × **6** = **?**
- Mark has **12** apples. He wants to share them equally among his **2** friends. How many apples will each friend receive?
  - **2** × **?** = **12**
- Mark has **12** apples. He put them into bags containing **6** apples each. How many bags did Mark use?
  - **?** × **6** = **12**

Equal Groups

$$G \times N = P$$

(groups) × (number in each group) = (product)

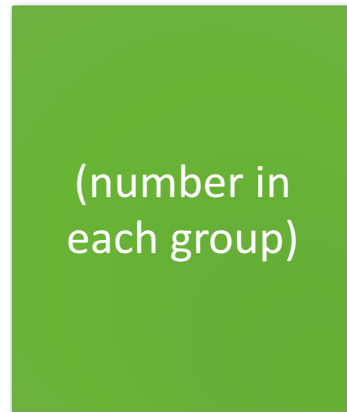## A.

Ms. Thompson sold 6 cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?

$$G \times N = P$$

$$6 \times 25 = ?$$

$$6 \times 25 = 150$$

$$? = 150 \text{ cherries}$$

"Are there groups with an equal number in each group?"

Equal Groups

Jane bought 24 light bulbs. The light bulbs come in packs of 4.

How many packs of light bulbs did Jane buy?

$$? \times 4 = 24$$

$$? \times \frac{4}{4} = \frac{24}{4}$$

$$? = 6 \text{ packs}$$

Comparison

Set multiplied by a number of **times** for a **product**

- Jill picked **6** apples. Mark picked **2** times as many apples as Jill. How many apples did Mark pick?
  - **6** × **2** = **?**
- Mark picked **12** apples. He picked **2** times as many apples as Jill. How many apples did Jill pick?
  - **?** × **2** = **12**
- Mark picked **12** apples, and Jill picked **6** apples. How many times as many apples did Mark pick as Jill did?
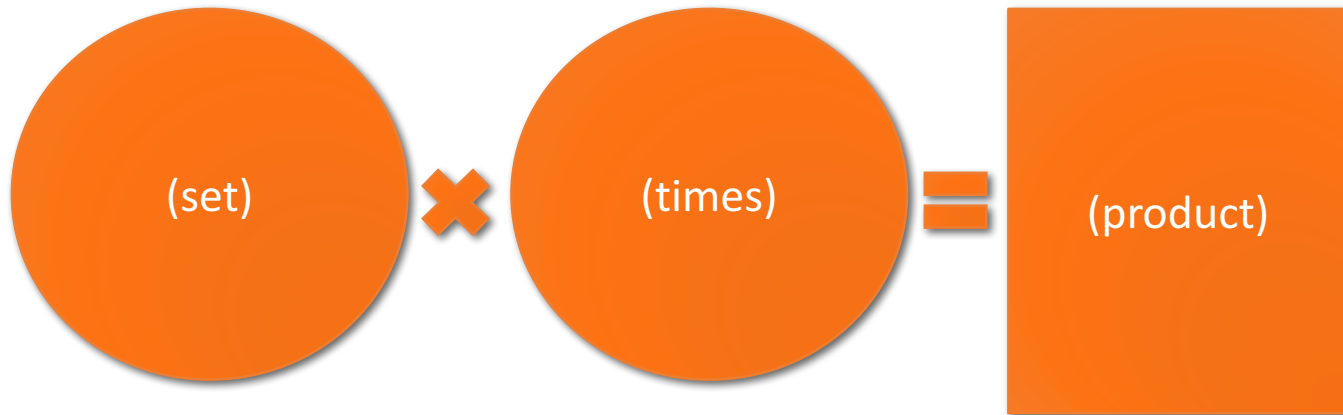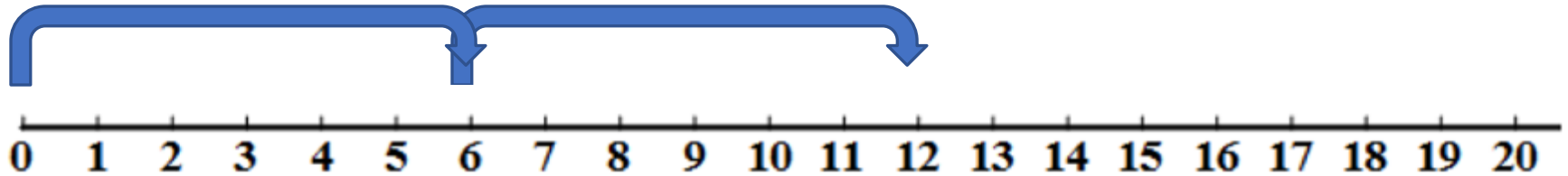 - **6** × **?** = **12**

If a scuba diver could carry 36 coins to the surface in one trip, how many trips would it take to carry 108 coins?

Isabella has 2 times as many DVDs as Emma. Emma has 6 DVDs.

$2 \times 6 = ?$

$? = 12$ DVDs

Equal Groups

"Are there groups with an equal number in each group?"

Comparison

"Is a set compared a number of times?"

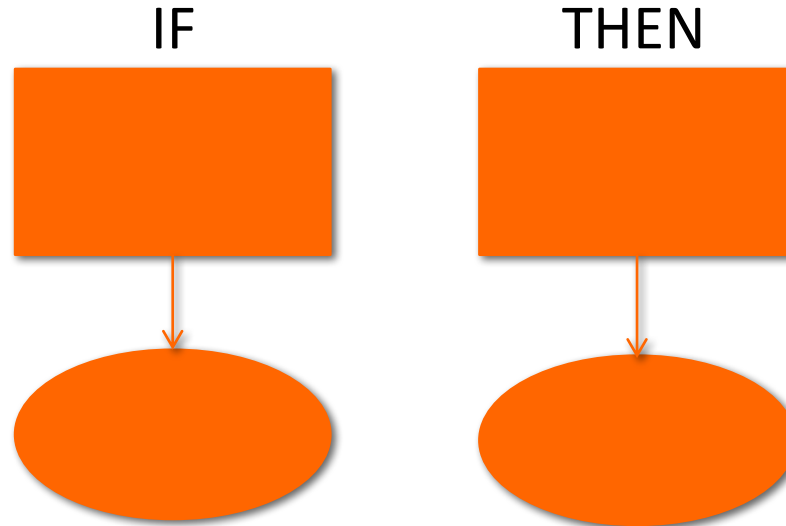# Combinations

Set multiplied by a **set** for a **product**

Examples

◦ Sam bought 4 pairs of pants and 3 jackets, and they can all be worn together. How many different outfits consisting of a pair of pants and a jacket does Sam have?
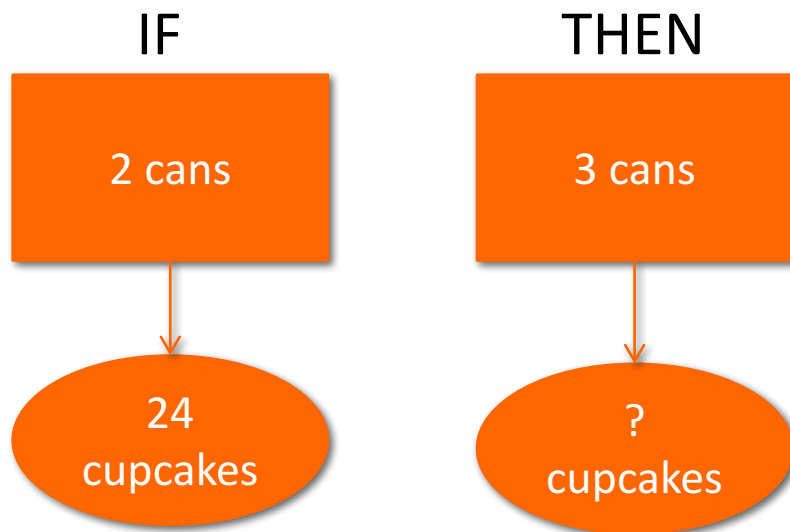
◦ **4 × 3 = ?**

Ratios/Proportions

Description of relationships among quantities

IF   THEN

Ratios/Proportions

IF

THEN

2 cans

3 cans

24 cupcakes

? cupcakes

Sylvia used 2 cans of icing to ice 24 cupcakes. How many cupcakes can she ice with 3 cans of icing?

$$\frac{2 \text{ cans}}{24 \text{ cupcakes}} = \frac{3 \text{ cans}}{? \text{ cupcakes}} \qquad ? = 36 \text{ cupcakes}$$

There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slices of bread are in 5 loaves?
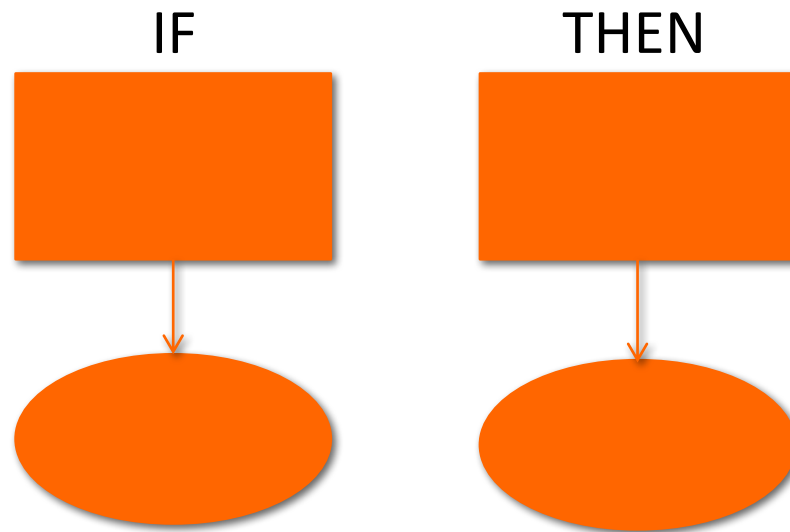
IF   THEN

Equal Groups

"Are there groups with an equal number in each group?"

Comparison

"Is a set compared a number of times?"

Ratios/Proportions

"Are there relationships among quantities - if this, then this?"

Multiplicative Word Problems

EQUAL GROUPS

COMPARISON

COMBINATIONS

RATIOS/PROPORTIONS

Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

Comparison

Susan has 3 times as many books as Mary. Mary has 18 books. Which equation can be solved to figure out how many books Susan has?

Equal Groups

There are 1,092 people who work in an office building. The building has 4 floors, and the same number of people work on each floor. How many people work on each floor?

Ratios/Proportions

Ethan correctly answers 80% of the total questions on his history test. He correctly answers 32 questions.

Equal Groups

The members of a gym use 98 towels every day. How many towels are used in 7 days?

Equal Groups

Ryan makes 6 backpacks. He uses $\frac{3}{4}$ yard of cloth to make each backpack. What is the total amount of cloth, in yards, Ryan uses to make all 6 backpacks?

A. $1\frac{1}{2}$

B. $2\frac{1}{4}$

C. $4\frac{1}{2}$

D. $6\frac{3}{4}$

Comparison

A basketball team scored a total of 747 points for the season. This was 9 times the number of points scored in the first game. How many points were scored during the first game?

Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?

Equal Groups

Eric has 158 action figures to put in display cases. Each display case can hold 8 action figures. How many cases does Eric need to hold all his action figures?

Ratios/Proportions

There are 7 math folders on a classroom shelf. This is $\frac{1}{3}$ of the total number of math folders in the classroom.

Ratios/Proportions

A bus travels 36 miles in 45 minutes.

Enter the number of miles the bus travels in 60 minutes at this rate.

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Ratios/Proportions

The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes.

36. Part A

On Monday, the baker makes 36 blueberry muffins.

What is the total number of muffins that the baker makes on Monday?

Enter your answer in the box.

Equal Groups

A basketball team plays 82 games each year. How many games will the team play in 25 years?

Multiplicative Word Problems

EQUAL GROUPS

COMPARISON

COMBINATIONS

RATIOS/PROPORTIONS

Let's Review

What's an Equal Groups problem?

What's a Comparison problem?

What's a Ratio/Proportions problem?

1. Don't tie **key words** to **operations**

2. Have an **attack strategy**

3. Teach word-problem **schemas**

Contact Information

Sarah Powell
- University of Texas at Austin

✉ srpowell@austin.utexas.edu

🐦 @sarahpowellphd