Calculus 3 - Quadratic Surfaces

Today we consider a special class of surfaces called *Quadratic Surfaces*. They are a 3D version of the quadratic equations in 2D. In 2d these are of the form

$$ax^2 + bxy + cy^2 + dx + ey + f = 0 \tag{1}$$

where $a - f$ are constants. The four basic curves are the straight line, parabola, ellipse and hyperbola whose equations are:

$$\begin{aligned} \text{line} \quad & ax + by + c = 0 \\ \text{parabola} \quad & y = ax^2 \\ \text{ellipse} \quad & \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \\ \text{hyperbola} \quad & \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \end{aligned}$$

the straight line, parabola, ellipse and hyperbola.

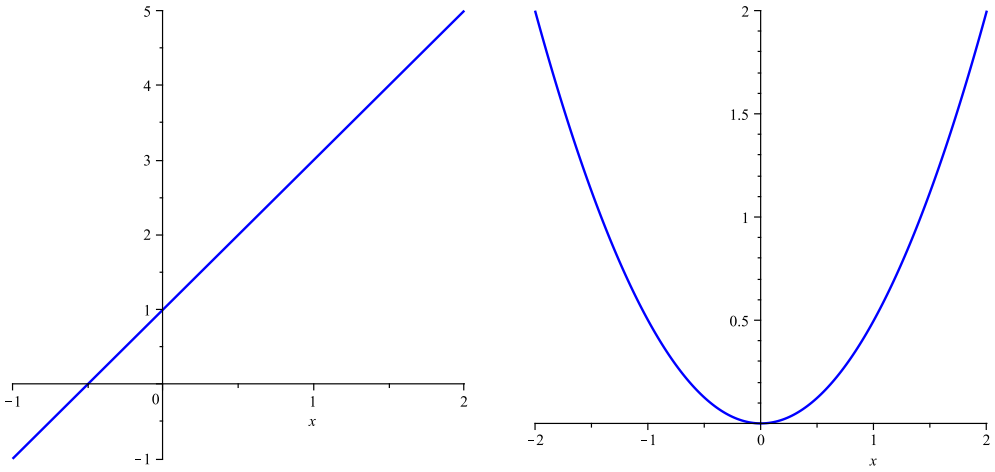


Figure 1: Line and Parabola.

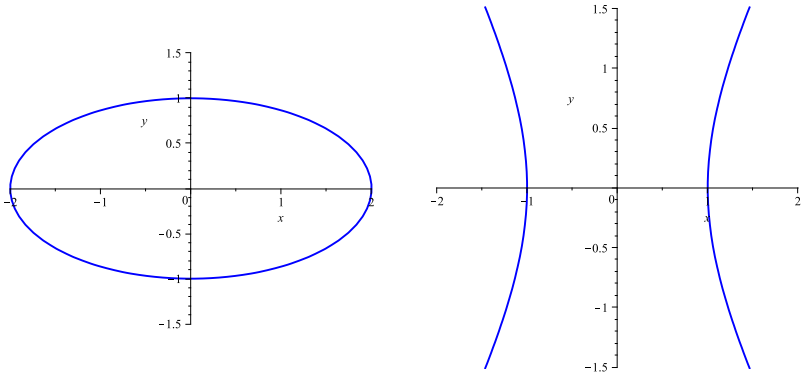


Figure 2: Ellipse and Hyperbola.

So new we extend these to 3D and consider equations of the form

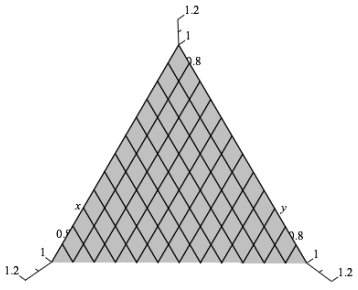
$$ax^2 + by^2 + cz^2 + dxy + exz + fyz + gx + hy + iz + j = 0 \tag{2}$$

where $a - j$ are constants. We omit the cross terms xy, xz and yz as they just represent rotation of the surface and will not change the picture. In total there are 10 basic surfaces

1. *Plane*

The equation is of the form

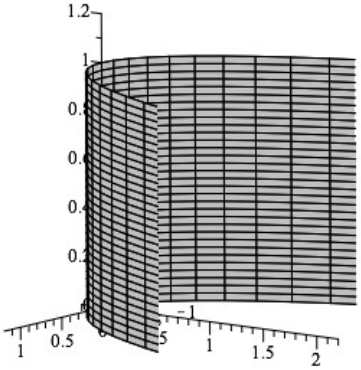
$$ax + by + cz = d \tag{3}$$



2. *Parabolic Cyliner*

The equation is of the form

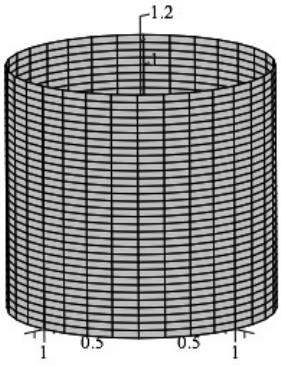
$$y = ax^2 \tag{4}$$



3. *Elliptic Cylinder*

The equation is of the form

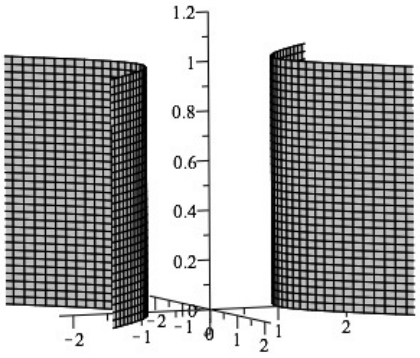
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



4. *Hyperbolic Cylinder*

The equation is of the form

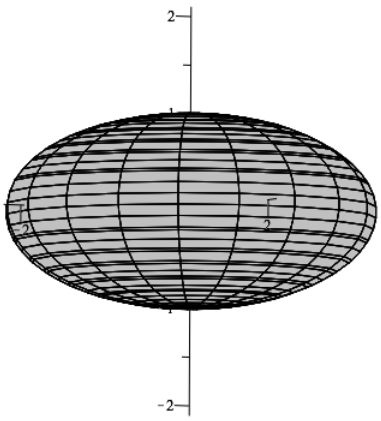
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



5. *Ellipsoid*

The equation is of the form

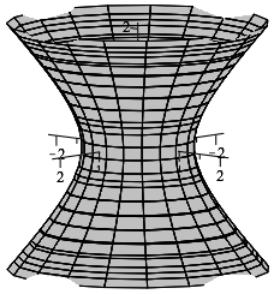
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



6. *Hyperboloid of 1 Sheet*

The equation is of the form

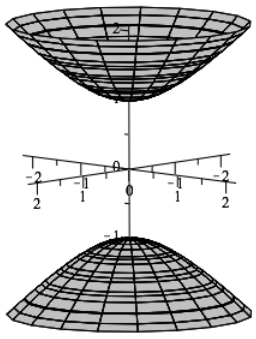
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



7. *Hyperboloid of 2 Sheet*

The equation is of the form

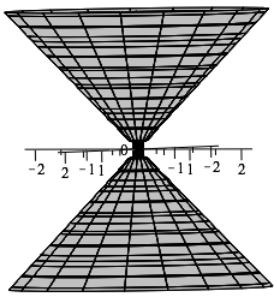
$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



8. *Cone*

The equation is of the form

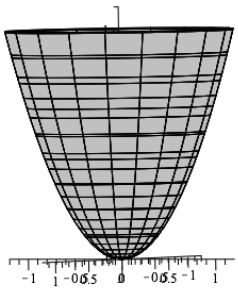
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$



9. *Paraboloid*

The equation is of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$



10. *Saddle*

The equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$$