Mathematical Modeling of Rayleigh Type Surface Waves

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ABSTRACT: Because of their practical importance in the field of seismology and electrodynamics, the studies related to surface waves always find attraction of concerned researchers. These studies are more important when we discuss theoretical models according to realistic situations. Practically, the earth surface is not homogeneous, isotropic and so it is reasonable to consider the solid layer underlying water as anisotropic one. In this paper, the Generalized Rayleigh type surface waves are studied for a three-layered medium having an anisotropic solid layer underlying a liquid layer and overlying a heterogeneous elastic solid half-space. The heterogeneity is considered to be of realistic type. The inclusion of anisotropic elastic layer and representation of substarta by heterogeneous half-space lead to vigorous and complex mathematical calculations. The interface between solid layer and half space is treated as an imperfect interface and suitable boundary conditions are applied thereat.

The dispersion equation is obtained in the form of determinant of order eight. The phase speed at attenuation at the interface between liquid and anisotropic layer can be obtained by solving the characteristic equation for different values of wave number. The surface waves studied are found to be of dispersive type.

Key Words: *Mathematical modeling, Rayleigh type surface, seismology, anisotropic and electrodynamics.*

1. INTRODUCTION

Surface waves in a material can reveal important information about the material. It has been found by many investigators that the wave speed as well as the attenuation property in material is affected when the material is subjected to fatigue or placed in hostile environment. For layered materials, the surface coating thickness and its properties can be obtained by measuring the Rayleigh waves speed through the layer. In addition, it has used to study the anisotropy in the material. Thus an accurate surface wave speed tells the material behavior non-destructively. It is known that the earth is far from being homogeneous and isotropic. The earth's interior is stratified with depth. Therefore, in the theoretical studies seismology, consideration of vertical as well as lateral inhomogeneity is essential. Anisotropy in the earth's crust and upper mantle have significant effects on the surface characteristics such as phase and group velocities [1,2 and 3].

Many investigators [4, 5 and 6] have propagation of elastic waves in anisotropic and inhomogeneous medium. Abubaker and Hudson[1] studied the dispersive properties of liquid overlying a semi-infinite homogeneous transversely isotropic half-space. Gogna[8] considered the surface wave propagation in homogeneous anisotropic layer over a homogeneous isotropic elastic half-space and under a uniform layer of liquid. Alenitsyn[3] investigated the sound wave propagation from a point time-harmonic source in an inhomogeneous stratified fluid layer on a homogeneous elastic half-space. Kaushik and Rana [12] studied the transmission of SH-waves through a linear visco-elastic layer sandwiched between two inhomogeneous anisotropic elastic half-spaces. Rokklin and Wang[15] described an ultrasonic technique for determination of elastic constants of anisotropic plates and demonstrated its applicability for thin aluminum oxide membranes. Rossikhin and Shitkova [16] focused on slightly inhomogeneous waves in two layered medium involving a slightly anisotropy layer and an isotropic half-space. Kiselev and Rogoff[13] found the explicit asymptotic expression for the dilation of the high-frequency S-waves in smoothly inhomogeneous isotropic media.

Robins[14] calculated the reflection coefficients of plane wave incident on an in inhomogeneous elastic solid layer with physical properties varying with depth. Gingold et al.[7] enunciated some local principles of wave propagation in inhomogeneous media. Kaushik and Chopra[11] studied the problem of transmission and reflection of inhomogeneous plane SH-waves at an interface between two horizontally and vertically heterogeneous linear viscous-elastic media[7,8 and 9].

In this paper, the problem of surface wave propagation of Rayleigh type is studied in a three-layered medium. The top most layer of the model is taken as liquid layer[10,11 and 12]. Beneath this, a homogeneous but anisotropic layer is considered. These two layers are lying over heterogeneous elastic solid half-space. The heterogeneity of vertical type is being considered. This model is significant in theoretical seismological studies, as realistic situations demand that the models incorporate anisotropy and heterogeneity. The dispersion equation for the waves, studied, is derived. The results corresponding to isotropic and homogeneous layers, studied by earlier authors, can be derived from present study as particular cases [13, 14 and 15].

2. BASIC EQUATIONS AND FORMULATION OF PROBLEM

We consider a transversely isotropic solid layer of thickness h, density d. The layer is confined between the planes z=0 and z=h. An inhomogeneous half-space with elastic parameters and density d occupies the domain z>h. Liquids layer of thickness h, density d and bulk modulus y lies over the solid layer:

For liquid layer, the displacement potential is written as

$$f = x\cos(y) + y\sin(x) \quad (1)$$

$$b_{yx} = \sum_{i \in s} \frac{(y_i - y')(x_i - x')}{(x_i - x')^2} \quad (2)$$

Where x is the velocity of dilational wave liquid, k b is the wave number and x' is the phase velocity [14, 15]. The displacement and pressure the medium are given by

$$u_1 = \frac{\partial y}{\partial x} + \frac{\partial y}{\partial z} \quad (3)$$

In transverselyisotropic solid layer, the strain energy volume density function can be written as

$$2W_2 = A_2 e_{xx}{}^2 + C_2 e_{zz}{}^2 \quad (4)$$

Since W2 is of positive definite form, therefore

$$A_2 C_2 - F_2{}^2 > 0 \quad (5)$$

Components of stress can be obtained by

$$t_{xx} = A_2 e_{xx} + F_2 e_{zz} \quad (6)$$

The equations of motion, when there are no body forces, are

$$A_2 \partial^2 u_2 + L_2 \partial^2 v_2 \quad (7)$$

$$L_1 \partial^2 u_2 + L_2 \partial^2 v_2 \quad (8)$$

Using equations (6), (7) and (8), we can obtain

$$-A_2 k^2 U + L_2 s^2 U - ks(F_2 + L_2) = 0 \quad (9)$$

$$-L_2 k^2 W + C_2 s^2 W - ks(F_2 + L_2) = 0 \quad (10)$$

To get non a non-trivial solution of (10), we must have

$$-L_2 s^2 W + A_2 k^2 W - ks(F_2 + L_2)U = 0 \quad (11)$$

The equation being quadratic in s^2 has the solution

$$s_j{}^2 = k^2 \frac{-r \pm \sqrt{R^2 - 4L_2 C_2} S}{2L_2 C_2} \quad (12)$$

Where

$$R = (F_2 + L_2)^2 - (A_2 - C^2) \quad (13)$$

The ratio of displacement amplitudes Uj and Wj, corresponding to these sj's is obtained as

$$\frac{R = (F_2 + L_2)^2 - (A_2 - C^2)}{(F_2 + L_2)} \quad (14)$$

We introduce the following notations

$$W_1 = e_1 U_1 \quad (15)$$

$$W_2 = -e_1 U_2 \quad (16)$$

$$W_3 = -e_1 U_3 \quad (17)$$

$$W_4 = -e_1 U_4 \quad (18)$$

The displacements can now be written as

$$u_2 = [u_1' \sinh(s_1 z) + u_2' \sinh(s_1 z)] \quad (19)$$

In half space H, considering the quadratic inhomogeneity varying with depth, the elastic constants are given by

$$l_3 = l_0 (1 + bz)^2 \quad (20)$$

All the variables in equation 20 can be treated as decision parameters[22, 23, 24 and 25] .

The compressional and shear wave speeds are given by

$$a^2 = a_0{}^2 (1 + bz)^2 \quad (21)$$

Parameters present in equation 21 indicates density of medium.

The equation of motion in the heterogeneous medium from equation (20) and (21) are obtained by

$$\gamma\Delta(\mu\overline{\nabla S}) - \nabla X(\nabla X\vec{S}) = \frac{\beta}{\theta} \quad (22)$$

Parameters present in equation 22 are called as scalar and vector potentials respectively. They satisfy the wave equation[16, 17 and 18]

$$\nabla^2\phi - \left(\frac{1}{\alpha^2}\right) \frac{\partial^2\phi}{\gamma\Delta(\mu\overline{\nabla S}) - \nabla X(\nabla X\vec{S}) = \dfrac{\beta}{\theta}} \quad (23)$$

The solution of equation (19) corresponding to surface waves are obtained as

$$\phi = P_3 K_{v1}(x) \Big/ (1+bz)e^{ik(x-ct)} \quad (24)$$

The expression for displacements and stresses are given by [19, 20 and 21]

$$u_3 = \frac{\partial \phi}{\partial x} - \frac{\partial \gamma}{\partial z} \quad (25)$$

$$\tau_{zz} = \mu_3[\partial^2 \phi \big/ \partial x^2 - (2b \big/ 1 + bz)(\partial \psi \big/ \partial x)] \quad (26)$$

Inserting the values of the parameters of equation (25) and (26) into (23) and (24), we get

$$u_3 = \mu_3[\partial^2 \psi \big/ \partial x^2 - (2b \big/ 1 + bz)(\partial \phi \big/ \partial x)] \quad (27)$$

3. BOUNDARY CONDITIONS AND SOLUTION

On the free surface z=-h, the sound pressure is taken to be zero. On the interface z=0, the normal and tangential components of stress and only the normal component of displacement are continuous.

These eight boundary conditions give rise to a set of eight homogeneous equations in U_1, U_2, U_3, U_4, A_1, B_1, P_3, Q_3. The period equation is given by

$$\text{Det}(a_{ij})=0 \quad (i, j=1,2,\ldots,8) \qquad (28)$$

The characteristic equation (28) relates phase velocity c to the wavelength $2\pi/k$. Since the wavelength is a multi-valued function of phase velocity, therefore, each value corresponds to a different mode of propagation and also indicates the dispersive nature of existing wave.

4. RESULTS

All the data sets are collected from BOOST seismological database and analyzed its elastodynamic properties under different media.

Fig.1. shows typical behavior Rayleigh wave properties under earthquake conditions and where as Fig.2. shows the behavior of Rayleigh surface in fluid-saturated media.

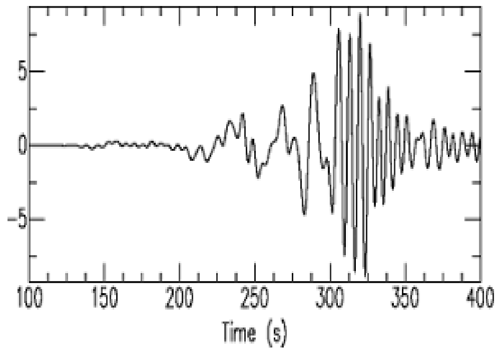

Fig.1. A dispersed Rayleigh wave generated by an earthquake

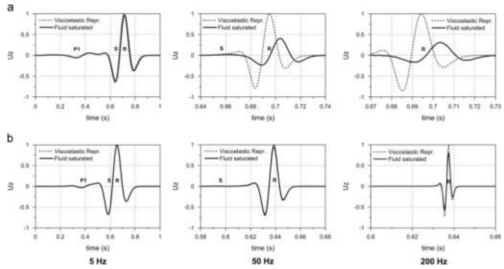

Fig2. Rayleigh surface in fluid-saturated media

Fig.3. exhibits the properties of LBAW analysis with respect to angle form and phase velocity under different constraints. All these figures show the behavior of Rayleigh wave properties with different seismological conditions. Table1 explains the result of bulk modulus, density and viscosity with respect to water and air media. Table2 describes bulk, shear, solid, material, porosity and permeability with respect to sand, sandstone and clay. Table3 analyses P waves, S waves, L waves and R waves with respect to sand, sandstone and clay. Wave analysis shows superior performance compare to media analysis under different constraints imposed by media.

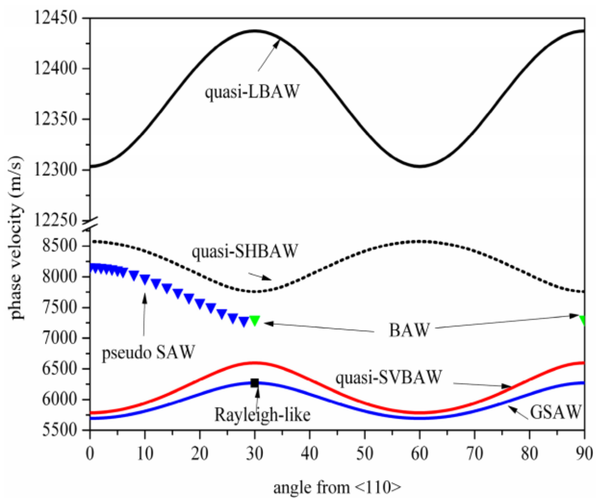

Fig3. LBAW Analysis of Rayleigh wave

Fluids	Water	Air
Bulk Modulus	4.32	1.86
Density	576	2.4
Viscosity	1	6.34

Table1 Properties of material

Media	Sand	Sandstone	Clay
Bulk	0.234	2.01	3.21
Shear	0.04	2.32	2.221
Solid	23	31	32
Material	2421	2134	2341
Porosity	0.42	0.54	0.3
Permeability	1100	7	6

Table2: Differnet media Analysis

Sesmic Waves	Sand	Sandstone	Clay
P waves	.1453	1.321	2.221
S waves	.321	2.431	3.321
L waves	.432	3.142	4.321
R waves	.521	2.431	5.321

Table3: Differnet waves Analysis

5. CONCLUSION

In this paper we studied generalized Rayleigh type surface waves are studied with help of P waves, S waves, L waves and R waves. Results shows that sand and sand stone are really affected by the behavior of P wave and S waves. In similar manner clay and sandstone affected by the media porosity and permeability. All the results of Rayleigh surface with respect to fluid-saturated media are summarized in the figure 2. Quasi LBAW Analysis of Rayleigh wave shows superior performance compare to other LBAW techniques as shown in the figure 3. All the results related to earth quake are analysed in the figure1 with the help of Rayleigh type surface waves.In the analysis of Rayleigh wave, the dispersion equation played vital role which is of order eight.

6. REFERENCES

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