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PERSPECTIVES & VIEWS

Darkness: What comprises empty space?

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The problem of dark energy is perhaps the most profound issue facing fundamental physics at present. This essay deals with a speculation that extends the problem in what appears to me to be a new direction.

Even for the most remote regions of space in the middle of cosmic voids, far away from ordinary matter and dark matter, it is very arguable that there is still a lot of something. I will call this "something", for want of a better term, "darkness". I will argue that darkness can be quantified, and that there are roughly 10^{38} units of darkness per liter in the emptiest of empty space, and much more than that elsewhere in space. It is not a novelty to suggest that empty space is full of some kind of stuff. Sometimes this stuff is called vacuum fluctuations, sometimes it is called virtual particles, sometimes it is called spacetime foam, and sometimes it is called spin networks. But the natural scale for a basic unit of such stuff is usually taken to be the Planck scale of 10^{-33} cm, leading to a density of about 10^{102} units per liter. What is different here is the supposition that this kind of language might be relevant, even in the context of pure gravitation theory, at a much larger scale -which I will call the Zeldovich scale- of about 10^{-12} cm.

The motivation for this supposition lies in a certain version of gravitational theory, called the MacDowell-Mansouri extension of the first-order Einstein-Cartan formalism for general relativity [1, 2]. This formalism, which has very deep mathematical roots [24], requires the existence of a nonvanishing, positive dark energy density (i.e. a cosmological term in the action). In addition it requires the existence of a curious topological term in the action, known as the Euler or Gauss-Bonnet invariant. The presence *per se* of such a term is by itself unremarkable. But the coefficient in front of this term is the famous 10^{120}, which permeates all discussions of the problem of the magnitude of the dark energy density observed in nature. It is this feature which creates in my mind novelty, and which motivates me to pursue possible consequences of the existence of this term in the action.

To make such a suggestion is to live dangerously, because we have a huge amount of evidence regarding what goes on at the Zeldovich scale. Real physical effects of "darkness" must be hidden very effectively; otherwise the idea is wrong. But if the effects of "darkness" are everywhere negligible at this scale, the idea is not even wrong - it is, operationally speaking, irrelevant. But it is conceivable, albeit a long shot, that there is a middle ground. This note is based on that notion.

1. Dark energy and the expanding universe

General relativity, supplemented with Einstein's cosmological term, accounts for large-scale cosmological phenomena. The Einstein equations contain two parameters, Newton's constant G_N and the cosmological constant Λ. Quantum theory and special relativity provide two others -Planck's constant $\hbar$ and the speed of light c. (Hereafter we choose units such that $\hbar = c = 1$.) With the help of these, we may construct two fundamental lengths, one extremely small and the other extremely large. The Planck length is constructed from Newton's constant

$$l_{Pl}^2 = G_N^{-1} = (1.6 \times 10^{-33}\text{cm})^2$$

It defines the short distance boundary of state-of-the-art descriptions of nature. The Hubble length is constructed from the cosmological constant

$$l_H^2 = \frac{3}{\Lambda} = (1.5 \times 10^{+28}\text{cm})^2 \equiv H^{-2}$$

Its inverse H, as defined above, will be prominent in what follows; it describes the expansion rate of dark-energy-dominated spacetime. This Hubble scale serves as a quantitative measure of the size (as well as the age) of the observable part of our universe. And for a long time to come the Hubble length will be the biggest distance scale relevant to observational cosmology. Beyond that distance scale, we will, for the foreseeable future, have no observational information about the structure of our universe.

These two extreme distance scales define the outer boundaries

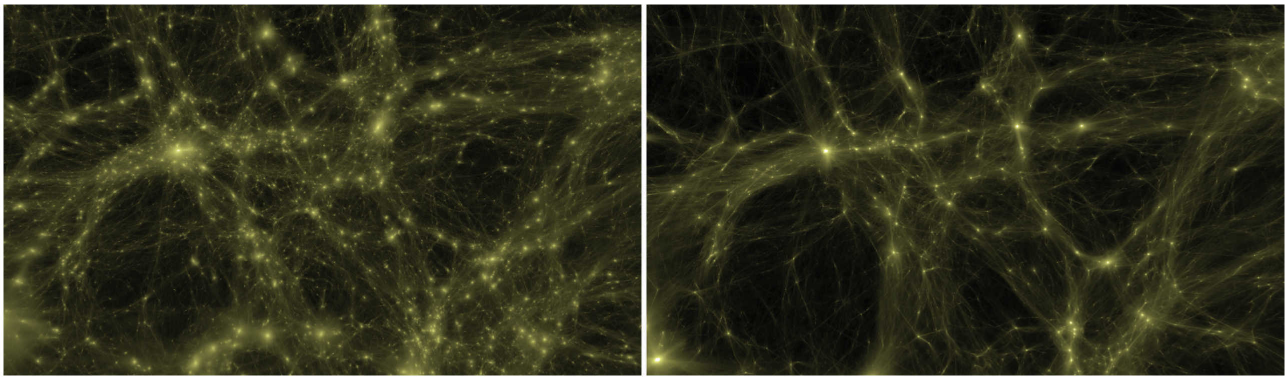

Figure 1 Simulations of the cosmic web (a) as of now, and (b) in the future, when the universe has expanded tenfold. The distance scale for Fig. 1a is roughly 50 megaparsecs. (The simulations were carried out by Tom Abel and Oliver Hahn, and the visualizations were carried out by Ralf Kaehler and Tom Abel, all at the Kavli Institute for Particle Astrophysics and Cosmology, SLAC)

of scientific inquiry. They are separated by about 61 powers of ten. But while the subject matter of gravitation theory and cosmology is bracketed by these two extreme distance scales, there exists an intermediate distance scale which is induced from the other two. We may call this the dark-energy scale. It occurs from examining the dynamics of the expanding universe. If we look at the structure of the cosmos at the largest scales, we see what is known as the cosmic web: a complex pattern of massive nodes (galaxies, stars, black holes, etc.) connected by sheets of matter as well as by filaments of matter. In between these structures are large voids containing almost nothing. If one puts well-separated, small bits of matter into these voids, the bits of matter will accelerate away from each other, with constant acceleration. This is due to the presence of the dark energy inhabiting such voids. And indeed the size of these voids will themselves expand (exponentially) as time progresses, until they take over almost all of space. Our local group of galaxies, within which there are no such voids, will not expand. But all the neighboring voids will expand and coalesce around us. Our local group will therefore become (after perhaps 200 billion years or so) an island surrounded by a rapidly expanding sea of dark energy. How should we view this situation? It is not hard to provide some intuition, and my preferred choice is as follows. Imagine being in a sea in a dead calm, with no wind or ocean currents. We are one of many boats floating in the sea, and we see neighboring boats receding away from us. We learn that the inhabitants of the other boats experience the very same phenomenon. What is the simplest explanation? It is simply that the sea level is rising. (In this analogy we are subtracting one space dimension, assuming that the sea lies on a spherical earth, and that we are viewing things from the point of view of “flatlanders”.) There are plenty of mechanisms which can be brought forward to account for this sea level rise, One of the simplest is that there exist springs at the bottom of the sea. Another is that extra water flows in from the boundaries of the sea, out of sight from the observers in the boats. Implicit in many such interpretations is the presumption of extra dimensions (beyond the flatlanders’ two space dimensions, in the analogy). My own favorite is that there are six extra dimensions of spacetime, very similar to what is envisioned by the string theory community. But my motivation, differs sharply from that of the string theorists. To quantify what happens in the real universe, put a few atoms, very well separated, into the void, all at rest relative to each other. If we wait long enough (that means something like 10 billion years!),we will see them moving away from each other at ever increasing speeds. Indeed the Einstein equations, with dark energy included, lead to a very simple equation of motion:

$$\frac{d^2 r}{dt^2} = H^2 r, \text{ with } r(t) = r(0)e^{Ht}$$

Here $r(t)$ represents the distance between any two of the atoms. If one creates a (big) box by putting an atom at each corner, then as time goes on this box will increase in volume exponentially. But the density of dark energy inside the box does not change. Therefore the total amount of dark energy in the box also increases exponentially with time. At this point we must say a few words about what this phrase “dark energy” really signifies. In the Einstein equations, the cosmological term can be interpreted as a source of gravitational field which acts like a uniform fluid with a small, uniform, positive energy density, but with negative pressure equal in magnitude to the energy density. While this sounds bizarre, such fluids do exist in the form of superconductors or superfluids like liquid Helium at low temperatures. The energy density ρ_{DE} (or pressure if one prefers) is given by the formula

$$|p| = \rho_{DE} = \frac{U}{V} = \frac{\Lambda}{8\pi G_N} = \frac{3}{8\pi} H^2 M_{Pl}^2$$

With this analogy, one is invited to view even the emptiest of empty space that we can in principle find in the observable universe as a medium which contains something, in particular energy, and this is for sure a mysterious medium. The central question for me is the origin of the exponentially increasing energy in the comoving box we constructed above. In any case, we do see that this dark-energy density is characterized by a mass scale m_{DE} and/or a distance scale l_{DE} which lies halfway, logarithmically speaking, between the cosmological scale and the Planck scale:

$$m_{DE} = \left(\frac{3}{8\pi}\right)^{1/4} \sqrt{H M_{Pl}} = 2.4\,\mathrm{meV}$$
$$l_{DE} \equiv m_{DE}^{-1} = 8 \times 10^{-3}\mathrm{cm}$$

This scale is a very delicate one, and it is very easy to overwhelm the effects of dark energy by putting some matter nearby. For example, consider a single proton put into the middle of a dark-energy-dominated void. A test particle (e.g. something with the mass of an electron but which is electrically neutral) will feel the gravitational force of the proton if it is close enough, and for example able to orbit around the proton, like the earth around the sun, But if it is far away from the proton, it will feel the dark energy and accelerate away from the proton. Call the boundary surface, where this transition occurs, the sphere of influence of the proton [3]. Its radius r is given by the formula

$$\frac{G_N M_{\mathrm{proton}}}{r^2} = H^2 r$$

with

$$r = \left(\frac{M_{\mathrm{proton}}}{M_{Pl}^2 H^2}\right)^{1/3} \approx 30\,\mathrm{cm}$$

The total dark energy inside the sphere of influence turns out to be of the same magnitude as the rest energy of the source, as might be expected already by dimensional analysis.

$$U_{\mathrm{inside}} = \frac{4}{3}\pi r^3 \rho_{DE} = \frac{4}{3}\pi \left(\frac{M_{\mathrm{proton}}}{M_{Pl}^2 H^2}\right)\left(\frac{3}{8\pi} H^2 M_{Pl}^2\right) = \frac{M_{\mathrm{proton}}}{2}$$

Given this idea of sphere of influence, there is another interesting way to define the dark energy scale,. Were the test particle to have a really small mass, the uncertainty principle would imply that its sphere of influence would lie within its own Compton wavelength, rendering the very concept of sphere of influence with radius r –at a classical level of description– irrelevant. The crossover mass is of order the dark-energy mass scale:

$$r \sim \frac{1}{m} \sim \left(\frac{m}{M_{Pl}^2 H^2}\right)^{1/3}$$

and thus

$$m^4 \sim M_{Pl}^2 H^2 \sim m_{DE}^4$$

This value is tantalizingly close to the mass scale of the three neutrino species [4]. Given this induced dark-energy mass and distance scale, our next task is to motivate another such induced scale, to be associated with the concept I have called “darkness”. To do so properly requires some discussion of general relativity at a more technical level, to be given in the online Appendix A. The bottom line in that discussion is that there is a third term to be added to the action of general relativity. This is the aforementioned Euler or Gauss-Bonnet topological term. We will encounter a special case of this result, and explicitly see its effects, in the following section.

2. Darkness in the void

Let us imagine ourselves in the center of a large void, with the nodes, sheets, and filaments of matter comprising the “cosmic web” receding away from us at an accelerating rate. Now define a box by inserting, say single atoms at the corners of the box. We imagine this to be a very big box, of kilometer scale or greater, so that the effects of gravity induced by the fiducial atoms at the corners is negligible inside almost all of the box. The exception is of course near the corners where the spheres of influence of the individual atoms (which have a scale of about a meter or so) exist. This volume expands exponentially with time. The spacetime within the box is described by the Friedmann-Robertson-Walker (FRW) metric tensor:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

Given that the spacetime in the interior of the box is dominated by dark energy, the solution of the Einstein equations is very simple:

$$a(t) = a(0)e^{Ht}$$

This means that, if the initial dimensions of the box were L_1, L_2 , L_3, then at later times one will have for the physical dimensions $l_1(t)$, $l_2(t)$, $l_3(t)$

of the box

$$l_i(t) = L_i a(t) = L_i e^{Ht}$$

Of interest to us is how the action function S for the contents of the box behaves, and in particular how the Gauss-Bonnet term in that action behaves. Given the above FRW form for the metric tensor, and the form of the action given in online Appendix A, it is straightforward to write down S:

$$S_{FRW} = -\frac{3M_{Pl}^2}{8\pi H^2}\int_0^t dt \times (\ddot{a} - H^2 a)(\dot{a}^2 - H^2 a^2)\, V(0)$$

Here the volume factor is evidently

$$V(0) = L_1 L_2 L_3 \ \textit{and}\ V(t) = V(0)e^{3Ht}$$

Note that the total action vanishes when the equations of motion are satisfied. This form of the above expression for S is also valid for a more general FRW cosmological expansion. When expanded out, one finds

$$S_{FRW} = -\frac{3M_{Pl}^2 V(0)}{8\pi H^2}\int_0^t dt \times \left(\frac{1}{3}\frac{d}{dt}\dot{a}^3 - \frac{H^2}{6}\left[\frac{d^2}{dt^2}a^3 + 3\ddot{a}a^2\right] + H^4 a^3\right)$$

Evidently the left-hand term, a total time derivative, is the Gauss-Bonnet contribution of interest. The two middle terms are contributed by the Einstein-Hilbert action, and the term on the right is contributed by the dark energy (cosmological constant). Now let us look at the Gauss-Bonnet term in more detail. Because it is a total time derivative (this is a general statement, not limited to this FRW application), we write it as follows:

$$S_{GB} = \int_0^t dt\frac{df}{dt} = 2\pi\,[N(t) - N(0)]$$

This is because, at the quantum level, the phase of a semiclassical wave function is in fact just the action S, and for topological terms such as the Gauss-Bonnet term the phase tends to accumulate in discrete units of 2π. Consequently, we define "the amount of darkness" $N(t)$ as the phase accumulation of such a wave function, in units of 2π, which is supposed to describe the contents of the spacetime within the box, due to the presence of the Gauss-Bonnet term in the action. We find that $N(t)$ is an extensive quantity, proportional to the time-dependent volume of the box $V(t)$. Therefore one may define an intensive quantity, the density of darkness n, which will, like the dark-energy density itself, be time-independent:

$$n = \frac{N(t)}{V(t)} = \frac{HM_{Pl}^2}{16\pi^2} \equiv \Lambda_Z^3$$

From this result, we have defined above a mass scale Λ_Z and/or a distance scale l_Z. Putting in the numbers, we obtain

$$\Lambda_Z \approx 10\,\mathrm{MeV}$$
$$l_Z \approx 2\times 10^{-12}\,\mathrm{cm}$$

Logarithmically speaking, this scale lies about two thirds of the way from the cosmological distance scale to the Planck distance scale. (The subscript Z stands for Zeldovich [6], who to my knowledge is the first one to identify this scale in more or less the same way as described above. There may be some prehistory associated with Dirac's large number hypothesis. There is definitely some posthistory; this relationship has been independently rediscovered by several others in the interim [7].) I must emphasize that this Zeldovich scale has been inferred from the gravitational sector alone. It is, like the dark-energy scale, an induced scale. Both this scale and the dark-energy scale emerge from the fundamental input scales, cosmological and Planck, which are defined in terms of Einstein's cosmological constant Λ and Newton constant G_N. Nevertheless, it is very tempting to associate this Zeldovich scale with the QCD scale of the standard model of elementary particles. It is something which I myself did for several years before coming around to the point of view laid out in this note. The bottom line is that the amount of "darkness" in a liter of "nothing at all", such as found in the midst of a cosmic void, is presumed to be huge, of order 10^{38}. The challenge is to sharpen up what is really meant by such words.

3. Darkness outside the void

The expression for darkness defined in the previous section can be generalized into regions where the spacetime curvature is dominated by matter rather than by dark energy. In particular, as already mentioned, we can trace its evolution during the big bang, as described by the simple FRW cosmology. For that case, the relevant formula has already been given:

$$n(t) = \frac{N}{V} = \frac{M_{Pl}^2}{16\pi^2 H^2}\left(\frac{\dot{a}}{a}\right)^3$$

As we go back in cosmological time, $\dot{a}/a$ increases. Therefore, the density of darkness was much larger in the early universe. In fact, it was already Planckian at a rather late stage of cosmological evolution, when the temperature of the cosmic background radiation was only about 50 MeV. To see this we solve the FRW equation for the expansion of the universe:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_{Pl}^2}\rho(t)$$

Since the solution occurs during the radiation-dominated epoch of cosmological expansion, the energy density $\rho(t)$ can be expressed in terms of temperature via the Stefan-Boltzmann law:

$$\rho(t) = \frac{\pi^2 g^*}{30} T^4$$

The quantity g* is the effective number of degrees of freedom of the plasma, and is about 15 [8]. Putting together the above equations exhibits the structure of the solution for the temperature at which the density of darkness was Planckian[1]:

$$T_{\text{critical}} \approx 5\Lambda_Z \approx 50\,\text{MeV}$$

Evidently this temperature is of order the Zeldovich mass scale. I interpret this feature as defining the limits of the mass scale, or equivalently of the distance scale, for which the MacDowell-Mansouri (MM) description is applicable. In other words, for earlier stages of cosmological evolution, some kind of modification of the MM description is required. I would like to emphasize that this does not necessarily imply a modification of the dynamics, i.e. the Einstein equations for gravity. We can also explore how the density of darkness behaves as one approaches a matter source at the center of some sphere of influence which resides in the middle of a void. An easy description of this configuration is in terms of the Painleve-Gullstrand metric for Schwarzschild-deSitter spacetime [9]. One writes for the metric tensor:

$$ds^2 = dt^2 - (dr - v(r)dt)^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

[1] See online material Appendix B

The "velocity function" $v(r)$ characterizes the structure of the spacetime, and is identical to the velocity of a Newtonian test particle of zero total energy moving radially toward (or away from) a source with mass M - even in the fully relativistic limit:

$$\frac{v^2}{2} = \frac{G_N M}{r} + \frac{H^2 r^2}{2}$$

When the two contributions to the velocity are equal, one is near the surface of the sphere of influence. This is in fact another way of defining where it is. It is not too difficult, given the expression for the Painleve-Gullstrand metric and the rules for computing the Riemann tensor from it, to determine the form of the density of darkness. One finds

$$n(r) = \frac{M_{Pl}^2}{16\pi^2 H^2}\left(\frac{v^2}{r^2}\frac{\partial v}{\partial r}\right)$$

For distances r much larger than the radius of the sphere of influence, one is in a region of space dominated by dark energy, and the velocity is simply

$$v = Hr$$

Therefore

$$n(r) \longrightarrow \frac{H^2 M_{Pl}^2}{16\pi^2} \text{ when } r \to \infty$$

Happily, this agrees with what we previously computed for dark-energy-dominated spacetime. On the other hand, for distances r much smaller than the radius of the sphere of influence, the density of darkness increases rapidly, as the inverse 9/2 power of the distance from the source:

$$n(r) \approx \frac{M_{Pl}^2}{32\pi^2 H^2}\left(\frac{2M}{M_{Pl}^2 r^3}\right)^{3/2}$$

Therefore we can again anticipate a critical radius at which the darkness density becomes Planckian. It turns out that this distance is most cogently characterized for sources which are of nuclear matter density, such as a neutron star, a large nucleus, or for that matter a proton or neutron. For a source of radius R and atomic number A, we write

$$M \cong m_{\text{proton}} A \cong 100\Lambda_Z A$$

The radius of an object of nuclear matter density scales as $A^{1/3}$, with coefficient R_0 empirically determined to be

$$R_0 = 1.2 \times 10^{-13}\,\text{cm}$$
$$R = R_0 A^{1/3} \cong \frac{A^{1/3}}{16\Lambda_Z}$$

When the density of darkness goes critical, i.e. is of the Planck scale, we have

$$n_{\text{critical}} \equiv M_{Pl}^3 = \frac{\sqrt{2} M^{3/2}}{16\pi^2 H^2 M_{Pl} r_{\text{critical}}^{9/2}}$$

Therefore

$$\frac{r_{\text{critical}}}{R} \cong 2.5$$

It is again clear that the Zeldovich scale is playing an important role in this result. Again, it appears that the MacDowell-Mansouri description becomes incomplete in regions of space where the energy density of matter exceeds that of nuclear matter. Note also that the amount of darkness residing outside (but quite near!) the critical radius is easily found to be of order $(M_{Pl}/H)A \sim 10^{60} A$.

4. A digression: The history of ideal gases

Back in the days of James Watt, it was realized that a hot gas contained energy which could be converted into useful work. But at that time even the concept of energy was not crisp, not to mention the concept of temperature as understood by a scientist today. In addition, half a century would pass before Carnot introduced the concept of entropy, an essential ingredient in analyzing the maximum efficiency of the steam engine itself. And it would take nearly another century before the microscopic basis of these concepts was firmly established. Now, instead of ideal gases, we must confront empty space, which turns out to be not completely empty. Not only does it contain a positive energy density and a nonvanishing negative pressure, but perhaps a darkness density. We have in previous sections already used the language of thermodynamics, in particular the identification of extensive and intensive variables and the general relations between them. The hallmark of the nineteenth-century history of ideal gases was the development of thermodynamics and the role it played in elucidating the general properties of ideal gases even in the absence of a microscopic understanding of them. This may serve as a useful object lesson in approaching the dark energy problem today. Are there other thermodynamics-like parameters that should be ascribed to empty space? There certainly are candidates. Thanks to Bekenstein and Hawking, among others, temperature and entropy are commonplace concepts used to describe black hole and cosmological horizons. Can they be adapted to the description of spacetime in the bulk? While most of the attention has been focused on horizon structure, there have been adventurous attempts to do just that. The most recent is the idea of entropic force introduced by Eric Verlinde [10], which builds on previous work by Jacobson and Padmanabhan [11]. But there are serious difficulties, and the present situation, at least to me, appears to be very unclear. In thermogravity, the concept of entropy is associated with area. And the concept of temperature is associated with acceleration. According to Unruh [11], detectors undergoing constant acceleration behave as if they are at a finite temperature. The formula relating acceleration a and temperature T is simple:

$$T = \frac{a}{2\pi}$$

However, what this formula really means is far from simple, and there is plenty of room for debate among the experts. Nevertheless, the very existence of such a connection invites a further step. I here cross the line and enter a regime of personal, rather toxic speculation. Consider again the comoving box in the void which we discussed already. Identify the relative acceleration of pairs of opposite walls with Unruh temperatures:

$$a_i = L_i H^2$$

Again, L_1, L_2 and L_3 are the dimensions of the box. Now identify the areas of the walls with entropies S, according to the Bekenstein rule:

$$S_1 = \frac{M_{Pl}^2}{4} L_2 L_3, \quad S_2 = \frac{M_{Pl}^2}{4} L_3 L_1,$$
$$S_3 = \frac{M_{Pl}^2}{4} L_1 L_2$$

It follows that

$$\begin{aligned} T_1 S_1 &= T_2 S_2 = T_3 S_3 \\ &= \frac{M_{Pl}^2 H^2}{8\pi} L_1 L_2 L_3 \\ &= \frac{1}{8\pi}\left(H^2 M_{Pl}^2\right) V(t) = \frac{1}{3} U(t) \end{aligned}$$

Evidently the sum adds up to the total energy of the box. It is also amusing that this contribution can be identified with the $\ddot{a}a^2$ term in the Einstein-Hilbert part of the action (S_{FRW}). However, I must freely admit that I have not been able to assemble these vague hints into a scientific argument. What about the Gauss-Bonnet term itself, responsible for the purported “darkness”? As best as I can surmise, it seems better not to associate it with temperature and entropy. In the ideal-gas analogy, it appears to me somewhat closer to the concepts of mole fractions and chemical potentials. In other words, if empty space has several kinds of “constituents”, analogous to the air we breathe, then “darkness” may define one of these components, or some relationship between them. It is very arguable that empty space contains at the very least a Higgs condensate, a complex quantum chromodynamics (QCD) vacuum structure associated with quark confinement, the vacuum amplitudes of all the other field degrees of freedom, the “Dirac sea” of quarks and leptons, and perhaps an axion condensate responsible for dark matter. Consequently, a multicomponent description of the contents of a cosmic void may not be unreasonable. But attaining a full microscopic description may be as elusive as nineteenth-century attempts to attain a full microscopic description of the classical ideal gas.

Another argument for considering the concept of chemical potential in the context of empty-space properties has been given repeatedly by Volovik [12]. He considers the vacuum state as analogous to a quantum liquid like helium at very low temperatures. Imagine stuffing some of that into a box, and then expanding the box. Eventually the box will be big enough that a droplet of the liquid is formed. The condition for this to occur is that, ignoring surface effects, the pressure vanishes. But if the liquid is in a universality class which admits special relativity as a low-energy emergent symmetry, this implies that the energy density must also vanish-up to corrections which scale with surface area, not volume. This behavior does mimic the situation regarding dark-energy-dominated spacetime. However, there is a catch. In the analogy, one has implicitly kept the baryon number of the liquid constant while expanding the box. In the thermodynamic description, this is tantamount to introduction of a chemical potential. And indeed, Volovik's detailed thermodynamics-based line of argument does introduce it via the Gibbs-Duhem equation.

5. Effective field theory and the MacDowell-Mansouri description

Most quantum field theories have only a limited domain of applicability. But in elementary particle physics and in gravitation theory, theorists' ambitions have been traditionally higher than that. Nevertheless, precious few if any of our theories satisfy the goal of unlimited applicability. And the MacDowell-Mansouri description, the centerpiece of this note, definitely does not fall into that category. Quantum field theories which do not fill the bill as fundamental are designated as "effective field theories". The degrees of freedom in an effective-field-theory action quite often turn out to be not so fundamental after all. And the various terms in the action play the role of phenomenological, descriptive placeholders, often expressing some features of a more fundamental underlying theory - especially with respect to symmetries.

A very relevant example of such an effective field theory is called the Gasser-Leutwyler chiral effective action [13]. Its ingredients are the raw material of nuclear physics, namely the Yukawa π-mesons, the proton, and the neutron. It most importantly expresses the symmetry properties of the strong interactions at large distances (in particular its chiral symmetries), as well as much of the dynamics. But the form of the action is anything but simple, and in any case is inapplicable for the short-distance, high-energy applications associated with the more fundamental theory QCD. Under those circumstances the Gasser-Leutwyler action gets replaced by the action of QCD, which looks very much like the Maxwell action of electrodynamics. It is an unmet theoretical challenge to fully derive the Gasser-Leutwyler effective action from the QCD action.

One way to think of that action is in terms of a field theory designed to only describe the fields within a box of finite size, such as the comoving boxes we introduced in previous sections of this note. If the dimensions of such a box are very large compared to the size of a nucleon, and if one restricts oneself to low enough energies, the Gasser-Leutwyler action provides the most appropriate description. However, when the box is small compared to the size of a nucleon, one is forced to the QCD description in terms of quarks and gluons. There is a kind of "phase transition" which is implied when the box size is of order the confinement scale of the strong interactions. I find this point of view quite germane when contemplating the role of the darkness, or Zeldovich, scale in gravitation theory. The MacDowell-Mansouri description is only viable if the box is chosen to be large compared to the Zeldovich scale. The description for boxes very small in comparison to the Zeldovich scale can be expected to be as different from the MacDowell-Mansouri description as the quark-gluon description is from the Gasser-Leutwyler pion-nucleon description. Nevertheless, the description of gravitational forces, not to mention the standard-model forces and other standard-model properties, must not be affected in a fundamental way. Those descriptions must be applicable on both sides of the darkness scale, perhaps in a way similar to how the weak and electromagnetic interactions are successfully described on each side of the strong-interaction, QCD-confinement boundary. In that case, a very appropriate language is that of Gell-Mann's current algebra, and perhaps something similar can be constructed for the case of darkness. But there is a clear need to elucidate in more detail the nature of this purported phase boundary at the Zeldovich scale. Clearly, the novel feature in the MacDowell-Mansouri description lies in the huge coefficient of 10^{120} multiplying the action. Where does such a big number come from? Here I offer a highly speculative suggestion—six extra dimensions. Seeing how this works requires a close look at the MacDowell-Mansouri formalism. A sketch of the argument is given in the online Appendix B.

An interesting consequence of this hypothesis is that the formalism expands to gargantuan size. For the original version of the theory, there are 40 field degrees of freedom. In the extra-dimensions version, this number increases to 550. When this generalized MacDowell-Mansouri action is expanded out in terms of the Riemann tensor, vierbein, etc., many new terms in the action are generated. Most of these terms are not total time derivatives and express nontrivial dynamics. They are analogous to the Einstein-Hilbert terms which were induced alongside the Gauss-Bonnet and cosmological terms in the original MacDowell-Mansouri formalism. Those terms describe gravitons. There is a good chance that some or all of these extra terms can be identified with terms in the standard-model action describing, e.g., gluons or other physical degrees of freedom. This is because the group structure implied by the geometry of the extra dimensions might be identifiable with standard-model internal symmetries. Indeed, there has existed for some time a small theory subculture devoted to the exploitation of essentially this idea [14]. In summary, the speculative picture that emerges is that the MacDowell-Mansouri description is limited to scales large compared to the Zeldovich scale because the properties of the six small, compact extra dimensions cannot be resolved at the larger scales. The description breaks down because at much smaller scales the six extra dimensions must in some way be included. The biggest challenge remaining is to show that, for standard-model phenomenology, there exists a description at scales small compared with the Zeldovich scale which remains completely—or almost completely—insensitive to the presence of those six extra dimensions, while remaining formally distinct from the MacDowell-Mansouri description valid at large distance scales.

6. An intuitive view of some topological actions

In searching for a better understanding of darkness, I crave a better intuitive understanding of the meaning of topological terms in an action function. This section is devoted to two such examples. The first example comes from classical electrodynamics, and the idea goes all the way back to Gauss. But the variant I discuss here is from the twentieth century, and features the Dirac string. A Dirac string is essentially a very thin solenoid containing a finite amount of magnetic flux, even in the limit of vanishing solenoid diameter. If the string is open, the magnetic field emerging from the end is radial and inverse square. It describes a magnetic monopole; the other end of the string is the source of a magnetic antimonopole. If the string is closed, there is no magnetic field outside the string, and classically the presence of the string is unobservable. Quantum-mechanically, the presence of the closed string can be detected via an Aharanov-Bohm experiment [15]. Now consider two loops of string. They can either be linked or unlinked. The distinction can be determined, even classically, via the magnitude of a topological term in the action. This topological term is the spacetime integral of $\vec{E}\cdot\vec{B}$:

$$\begin{aligned}\int dt \int d^3x\,(\vec{E}\cdot\vec{B}) &= -\int dt \int d^3x\,\dot{\vec{A}}\cdot(\vec{\nabla}\times\vec{A})\\ &= -\frac{1}{2}\int dt\,\frac{d}{dt}\int d^3x\,\vec{A}\cdot\vec{B}\end{aligned}$$

(We here choose temporal gauge, $A_0=0$, for convenience.) It is a short, pleasurable calculation to show that the space integral of $\vec{A}\cdot\vec{B}$ is twice the product of the fluxes if the two loops of Dirac string are linked, and is zero otherwise. This is called the Gauss linking theorem. The result is independent of the shape, position, and circumference of the strings; it measures a topological property of the string/field configuration. It is even fun to model darkness in this language. Build a long chain out of Dirac string, make each link rigid, with a size of order the Zeldovich scale, and arrange each pair of adjacent links to give a positive contribution to the topological action. Make the field strength inside the string of Zeldovich scale, and make the diameter of the string the Planck scale. Pack this chain into our comoving box, assuming that each link occupies a Zeldovich-scale volume. Another pleasurable calculation shows that these specifications suffice to provide a dark energy density and a darkness density of the right orders of magnitude.

The second example comes from quantum chromodynamics (QCD). It again features the $\vec{E}\cdot\vec{B}$ term, but now with the complications induced by the replacement of the single photon of quantum electrodynamics with the eight gluons of quantum chromodynamics. Instead of the unit of topology being represented by a closed loop of Dirac string, the conventional description introduces what is often called a "hedgehog". It is a nonsingular gauge potential which gives rise to zero field strength, and can be written as a 3 x 3 matrix in color space, as follows:

$$A_0=0,\quad \vec{A}=U^{-1}\vec{\nabla}U,$$
$$U=e^{i\vec{\tau}\cdot\vec{r}\,f(r)}$$

with

$$f(r)\longrightarrow\begin{cases}0 & r\to\infty\\ \pi & r\to 0\end{cases}$$

The Pauli matrices $\vec{\tau}$ in the above expression act in color space and rotate a chosen pair of the three colors into each other. Antihedgehogs can be built by replacing the unitary matrix U by its inverse. In this case, unlike that of the Dirac string, the gauge potential $\vec{A}$ is everywhere nonsingular.

Now imagine that the universe is filled with a "condensate" of hedgehogs. Choosing a density of order the Zeldovich, darkness, scale will lead to a total hedgehog number N of order 10^{120} for the visible universe. However, there is a complication. While one may expect that classically that N is conserved, this is not true at the quantum level. Tunnelling processes, called instanton events, can create or destroy hedgehogs and antihedgehogs at a nominal rate of one event per darkness volume per darkness unit of time. Over all of observable cosmological space and cosmological time there have been of order 10^{160} such instanton events, leading to an uncertainty ΔN in the hedgehog number N of order 10^{80}. But this number is much smaller than $\langle N \rangle$, so no serious damage has been done.

The conventional, textbook QCD wisdom regarding hedgehogs differs from the above version. Instead of $\langle N \rangle \sim 10^{120}$, $\langle N \rangle = 0$ is assumed instead. Instead of $\Delta N \sim 10^{80}$, conventional wisdom sets $\Delta N = \infty$. The variable quantum-mechanically conjugate to N is the well-known CP-violating parameter θ of QCD. It is conventionally regarded as sharp, i.e. $\Delta\theta = 0$. In my version $\Delta\theta > 10^{-80}$. It is unlikely that this causes phenomenological difficulties.

But it is not fully clear whether this variant causes difficulties elsewhere in the QCD formalism. Certainly the version I have described would better match the overall theme of this note than the standard version. What I take away from these examples is not a realistic model of darkness. Instead, the message to me is that topological information like darkness is in a sense nonlocal. In our examples, the topological information does not depend upon the local properties of strings and hedgehogs, such as size, shape, and position. Those properties are in fact not gauge invariant. Therefore they will not represent "elements of physical reality", unless at some deep level gauge invariance (and quite likely Lorentz covariance as well) is broken. Such a situation is anticipated in the emergent-gravity approach, based on analogues to condensed-matter behavior [12].

7. Distance scales in the standard model

The action function which describes the standard model of elementary particles is widely regarded as an effective action. It contains over two dozen input parameters. Many of these introduce intrinsic distance (or energy) scales into the description. In this section we investigate whether, in some future improved version of the standard model, some or all of such distance scales might in fact be traceable to the darkness and dark-energy scales and therefore to G_N and Λ only. Many of the dimensionful parameters of the standard-model action have to do with quark masses and mixings. These spread out over five orders of magnitude, from the electron mass to the top quark mass. Understanding them is still for the future. However, the centroid of this broad distribution, logarithmically speaking, is very near the Zeldovich scale. Perhaps this is no accident. It is important in this context to note that conventional wisdom provides no real clue as to why quark and lepton masses should be anywhere near this value.

We have already commented that the QCD confinement scale also lies near the Zeldovich scale. Perhaps this is also no accident. Here the relevant standard model parameter is the dimensionless running coupling constant $\alpha_s(\mu^2)$. In the far ultraviolet, near the Planck scale, this parameter is somewhere around $1/40$. Its inverse decreases by about 2 units per order of magnitude as the scale-parameter μ is reduced, becoming of order unity at the confinement (or darkness!) scale. The structure of the QCD vacuum at the confinement scale is complex. Perhaps darkness most peacefully coexists with the QCD vacuum structure if the darkness density matches the density of the QCD vacuum structures responsible for confinement. Gravity needs only to whisper to QCD at the Planck scale in order to create the necessary linkage of the confinement scale to the darkness scale.

The remaining dimensionful parameters in the standard-model action have to do with the Higgs sector. These include the vacuum condensate value of the Higgs field (242 GeV), the top quark mass (173 GeV), and the newly-discovered Higgs-boson mass of 125 GeV. It is curious that these cluster together, and even more curious that the top quark mass is the geometric mean of the other two, which in turn are in the ratio 2/1. Perhaps a future theory can relate these to each other. If so, there might be a tenuous link to the Zeldovich scale. I regard this as a marginal possibility, given that this scale is at least 1000 times the darkness scale. However, it may be that the Higgs sector itself has family structure, containing a variety of states which span mass scales much like the quarks and leptons do, and extending all the way down to the regime inhabited by the proposed Peccei-Quinn axion. If so, the Higgs sector might be rich enough to include the dark matter sector as well.

In this regard, I find the ideas of Hong-Mo Chan and his collaborators especially relevant and provocative [16]. They suggest that the mass matrices of quarks and leptons evolve with scale as the renormalization scale μ descends from the ultraviolet toward the infrared, becoming

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very rapid as one approaches the Zeldovich scale. Here I interpret "scale" in terms of the size of the quantization volume. They have considerable success in providing a phenomenological description of quark and lepton masses and mixings, especially the second-generation parameters.

I have constructed my own version of their ideas. It is a fragile construction, and this is not the place to describe or defend it in detail. But I do get quite good formulae for first and second generation masses and mixings in terms of the input values of third generation quark and lepton masses, plus a crucial characteristic input mass parameter $m \cong 7$ MeV. For large mass scales μ, I relate the basic, scale-dependent, mixing angle $\alpha(\mu)$ of the scheme to this parameter m in a simple way [17]: $\alpha(\mu) \approx \sqrt{m/\mu}$. The output formulae which I obtain are listed in Table 1.

The main reason for including all of this here is the ubiquity of the 7 MeV mass parameter. It lies right in the middle of the darkness scale. And it seems clear that it represents a "phase boundary" which separates the description at scales well below 7 MeV from scales well above it. However, the arguments leading to the above formulae still remain fragile, albeit provocative. As I view this rotating-mass-matrix idea, it requires the existence of low-mass particles to create the necessary "rotation". This in turn reinforces the notion expressed above that the Higgs sector has a rich family structure and dynamic range—enough to encompass the problem of dark matter. Indeed, ideas not too dissimilar to this are in fact politically correct, and there is considerable activity nowadays in searching for dark-matter candidates having masses near the darkness, or Zeldovich scale [18].

8. CP violation

The phenomenon of CP violation is subtle, and is seen only in the context of elementary particle processes. The fundamental origin of this phenomenon is not at all clear at this time. CP violation can also be characterized as violation of time-reversal symmetry of the fundamental laws of physics, and this way of thinking about it may be especially appropriate in the context of field theory. The standard Einstein-Hilbert metric theory of gravity does not admit time-reversal violation. However, the first-order Einstein-Cartan version, and especially the MacDowell-Mansouri extension thereto, does admit this possibility in a natural way [19]. And I see a possible, albeit very speculative, link to QCD and the dark-matter sector as well, thanks to the presence of the darkness scale. Addition of CP violating terms in the action can be accomplished in the MacDowell-Mansouri formalism in a very simple way [22]. When this is done and the action is expanded out in a way analogous to what was described earlier in this text, three new terms are generated [2]. Two of them are topological. One is called the Pontryagin term, and is very similar to the $\vec{E} \cdot \vec{B}$ term present in QCD. The other topological term is called the Nieh-Yan invariant, and is not very familiar, even to many experts in the business.

The third term, widely known as the Holst term [20], is not topological. When written down in Einstein-Cartan language, it bears a close kinship to the Einstein-Hilbert term in the action. Nevertheless, for almost all practical purposes, it does not affect the Einstein equations themselves. Activation of the Holst term requires activation of the degrees of freedom called torsion which were briefly mentioned earlier. For most practical applications of general relativity—falling apples, planetary motion, cosmology—the torsion vanishes as a consequence of the Einstein-Cartan version of the variational principle. But when quarks and leptons are included it is possible—but not very simple in practice—to activate these torsion degrees of freedom. There is a small theory subculture that explores this option [21]. This includes me; my own attempt to activate torsion led to a fermion condensate of Zeldovich density [22]. It would contribute significantly to the dark-energy budget and would also lead, in addition to CP violation, to a tiny amount of Lorentz violation—roughly a billion times less than present-day experimental sensitivity.

While the Einstein-Cartan and MacDowell-Mansouri formalisms only admit the possibility of CP violation, the phenomenon clearly exists. It must be an integral part of a future theory of masses and mixings of quarks and leptons. In addition, the QCD theory of the strong interactions of quarks and gluons also naturally admits CP violation via the $\vec{E} \cdot \vec{B}$ term discussed earlier. This issue is most successfully dealt with via the Peccei-Quinn mechanism. It involves adding to the theory Higgs-like degrees of freedom which screen the CP violation from the strong interactions, at the expense of introducing an almost massless boson called the axion. The axion, in turn, is an attractive dark-matter candidate and is the object of experimental search nowadays [23].

It should be clear that this CP violation issue impinges on the subject matter of this note—especially the previous section, and in particular the rotating-mass-matrix ideas of Hong-Mo Chan and his collaborators. An important feature of that

Table 1 First and second generation masses, as well as mixings in terms of the input values of third generation quark and lepton masses. Experimental values are in parentheses. The asterisks are a Michelin scale for the quality of the theoretical arguments leading to the quoted results.

	proposed value	exp. value
First-generation masses	$m_u \lesssim m = 7$ MeV	$(2.3 \pm 0.6$ MeV$)$
	$m_d \lesssim m = 7$ MeV	$(4.8 \pm 0.5$ MeV$)$
	$m_e = m^2/m_\mu = 0.44$ MeV*	(0.51 MeV)
Second-generation masses	$m_c = \sqrt{mm_t} = 1.1$ GeV**	(1.3 GeV)
	$m_s = \sqrt{mm_b} = 170$ MeV**	$(100 \pm 30$ MeV$)$
	$m_\mu = \sqrt{mm_\tau} = 110$ MeV**	(106 MeV)
CKM mixings	$\lvert V_{cb}\rvert \cong \sqrt{m/m_b} = .040^{**}$	(.041)
	$\lvert V_{td}\rvert \cong m/\sqrt{m_b m_s} = .0080^*$	(.0081)
	$\lvert V_{ub}\rvert \cong m/\sqrt{m_b m_c} = .0032^*$	(.0039)
Unitary-triangle vertex angle	$\alpha = \pi/2$	$(89^\circ \pm 4^\circ)$

description, both in the original version and in the variant which I explored, is that—at least for mass scales μ large in comparison to the critical scale of 7 MeV—the mass matrix is assumed to be rank 2 or less. At least one eigenvalue must vanish. Under such circumstances, it is well-established that strong CP-violating effects vanish. Only below the 7 MeV scale will the strong CP violation emerge phenomenologically. And Hong-Mo Chan et. al. and I argue that below that scale the strong CP violating effects are expressed only in the electroweak sector according to the usual Cabibbo-Kobayashi-Maskawa (CKM) description. In order for this to happen, the primordial value of the coefficient θ in front of the QCD $\vec{E} \cdot \vec{B}$ term in the action is expected to be large, of order unity, and some variant of the Peccei-Quinn mechanism must be operative.

The bottom line is that perhaps one way of characterizing the distinction between the "macroscopic" phase describing phenomena at distance scales large compared to the darkness scale, and the opposite "microscopic" phase describing phenomena at distance scales small compared to the darkness scale, is that CP violation is present in the former case and absent in the latter. For better or worse, I generalize this to include the lepton number violation associated with the standard picture of neutrino mixing. This gives rise to interesting implications for the description of neutrino masses and mixings. But these are beyond the scope of this note.

9. Darkness and the foundations of quantum theory

We have seen that the radius of a sphere of influence for a single proton is about 30 cm. The spheres of influence of larger, ordinary objects scale as the cube root of their atomic numbers. That means that my personal sphere of influence is about a million kilometers. Consequently, if I try to observe an elementary phenomenon occurring in the midst of a cosmic void, such as a typical elementary-particle-physics scattering process, it may make a difference if I distance myself (and all other measuring apparatus of a similar size) by an amount large compared to the distance from here to the moon. Otherwise I am ostensibly contributing a lot of "background darkness" to the region of space where the elementary process occurs.

For example, imagine an 8 TeV proton-proton collision which produces, amidst the collision debris, a Higgs boson which decays into two gamma rays. Let us suppose it occurs in the midst of a cosmic void. On approach, each proton presumably carries with it a Lorentz-contracted "darkness disc", within which there are of order 10^{60} units of darkness. In addition, when the separation of the two protons is small compared to 30 cm, there must also appear an ephemeral sphere of influence with radius appropriate to the total center-of-mass energy of the two protons; the radius of that sphere of influence is evidently about six meters. Eventually, long after the collision is over, the sundry fragments carry away their own Lorentz-contracted “darkness discs”, and the six-meter sphere of influence disappears. A serious generalization of the MacDowell-Mansouri description would endeavor to follow the detailed evolution of the darkness as such a collision evolves, perhaps in analogy to how statistical mechanics and kinetic theory flesh out a thermodynamic description of the evolution of a gas or fluid.

If I were to take a close look at such a collision process, the darkness within my gravitational field would swamp the other contributions and arguably influence the flow of all the darkness associated with the collision itself. In other words, at the MacDowell-Mansouri level of description, the presence of nearby macroscopic observers and nearby macroscopic measuring devices will for sure alter the description. However, we do not expect that this will affect the practicalities involving the prediction of the outcome of the collision process.

But it is arguable that in a sense it does. We cannot predict in advance that an individual proton-proton collision will produce the Higgs boson, and in any case cannot predict its decay mode into two gamma rays. There are quantum choices to be made. Perhaps the details of the configuration of darkness do influence the quantum decisions. Even so, it may still be the case that, for all practical purposes, it does not matter whether such observers are present or not. In either case the density of darkness in the neighborhood of the Higgs boson is so large that its fate may be only determined statistically.

9.1 Final remarks

The above musings essentially assert that for some strange reason the gravitational fields of observers and even of elementary particles do have to be taken into account in order to fully understand present-day, mundane elementary-particle collision processes. Conventional wisdom, for very good reasons, says that such effects are negligible. So the chances are that the above arguments will be of interest, if at all, only to those natural philosophers who puzzle over the foundations of quantum mechanics and the theory of measurements. But I will add two remarks to this conclusion. One is that if darkness is in some sense related to quantum-mechanics hidden variables, then Bell’s theorem implies these are nonlocal. This meshes with our previous discussion, and again suggests that, if darkness at some deep level is emergent and does represent “elements of physical reality” , Lorentz covariance and/or gauge invariance will be broken at that deep level.

The second remark is also something of a repetition. It is curious that the quantum (Wheeler-deWitt) wave function of a piece of dark-energy dominated space at the semiclassical level is, in the MacDowell-Mansouri description, trivial—something that does not happen in the absence of the enormous Gauss-Bonnet topological term. Perhaps the darkness-dominated regions of cosmic voids are in some sense so close to “nothing at all” as to be “beyond the quantum theory”.

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