# Math 1496 Calc I

## More Rules

Consider $\frac{d}{dx} x^2 \cdot x^7 \stackrel{?}{=} \frac{dx^2}{dx} \cdot \frac{dx^7}{dx}$

$\frac{d}{dx} x^9 = 9x^8$

$2x \cdot 7x^6$

$= 14x^7$

clearly not the same

so in general $\frac{d}{dx} f(x)g(x) \neq \frac{df(x)}{dx} \cdot \frac{dg(x)}{dx}$

## Product Rule

$$\frac{d}{dx} f(x)g(x) = \frac{df(x)}{dx} \cdot g(x) + f(x) \frac{dg(x)}{dx}$$

Proof $\lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$ trick

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \cdot g(x+h) + f(x) \cdot \frac{g(x+h)-g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \cdot \lim_{h\to 0} g(x+h)$$

$$+ f(x) \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$= f'(x)g(x) + f(x)g'(x) \quad \boxtimes.$$

Ex $\frac{d}{dx} x^2 \cdot x^7 = 9x^8$

$f = x^2 \quad f' = 2x$

$g = x^7 \quad g' = 7x^6$

$$f'g + fg' = 2x \cdot x^7 + x^2 \cdot 7x^6$$

$$= 2x^8 + 7x^8 = 9x^8 \checkmark$$

ex $\frac{d}{dx} x^3 e^x$

$f = x^3 \qquad g = e^x$

$f' = 3x^2 \qquad g' = e^x$

$\frac{d}{dx} x^3 e^x = x^3 e^x + 3x^2 e^x$

ex $\frac{d}{dx} \frac{\sin x}{x} = \frac{d}{dx} x^{-1} \cdot \sin x$

$f = x^{-1} \qquad g = \sin x$

$f' = -1x^{-2} \qquad g' = \cos x$

$= x^{-1} \cos x - 1x^{-2} \sin x$

$= \frac{\cos x}{x} - \frac{\sin x}{x^2} = \frac{x \cos x - \sin x}{x^2}$

Quotient Rule

$$\frac{d}{dx}\frac{x^7}{x^2} = \frac{d}{dx}x^5 = 5x^4$$

$$\stackrel{?}{=} \frac{\frac{d}{dx}x^7}{\frac{d}{dx}x^2} = \frac{7x^6}{2x} = \frac{7}{2}x^5 \neq$$

Quot. Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

Proof

$$= \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h\,g(x)g(x+h)}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h\,g(x)g(x+h)}$$

$$= \lim_{h \to 0} \frac{\frac{f(x+h) - f(x)}{h} \cdot g(x) - f(x) \cdot \frac{g(x+h) - g(x)}{h}}{g(x)\, g(x+h)}$$

$$= \frac{f'(x) g - f(x) g'(x)}{g^2(x)} \quad \boxed{\checkmark}$$

Check

$$\frac{d}{dx} \frac{x^7}{x^2} = \frac{d}{dx} x^5 = 5x^4$$

$f = x^7 \qquad g = x^2$

$f' = 7x^6 \qquad g' = 2x$

$$\frac{gf' - g'f}{g^2} = \frac{x^2 \cdot 7x^6 - 2x \cdot x^7}{x^4} = \frac{7x^8 - 2x^8}{x^4}$$

$$= \frac{5x^8}{x^4} = 5x^4 \checkmark$$

# Derivative of Remaining Trig fcts

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{\cos x \frac{d}{dx}\sin x - \sin x \frac{d}{dx}\cos x}{\cos^2 x}$$

$$= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \sec^2 x$$

$$\boxed{\frac{d}{dx}\tan x = \sec^2 x}$$

$$\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} = \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} = \sec x \tan x$$

so $\boxed{\frac{d}{dx}\sec x = \sec x \cdot \tan x}$

Also

$\frac{d}{dx}\cot x = -\csc^2 x$ $\qquad$ $\frac{d}{dx}\csc x = -\csc x \cot x$

# Higher Order Derivatives

if $f(x) = x^2$ then $f'(x) = 2x$

can we take a derivative of a derivative?

Yes, then $f''(x) = \frac{d}{dx} 2x = 2$

we can continue in this fashion

Ex: $y = \sin x$

$y' = \cos x$

$y'' = -\sin x$

$y''' = -\cos x$

$y^{(4)} = \sin x$

notation

$\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$ etc

we will use these in the next chapter.