Math 1497 Calc 2

Consider

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \qquad \text{converges}$$

we see the common ratio $r = \frac{1}{2} < 1$

$$\sum_{n=1}^{\infty} 2^n = 2 + 4 + 8 + 16 + \cdots \qquad \text{diverges}$$

here the common ratio $r = 2 > 1$

so comparing consecutive terms in a series might give us a clue on convergen

Ratio Test

$\sum a_n \quad a_n > 0$

series

if $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = L$ if $L < 1$ converges

$L > 1$ diverges

$L = 1$ no conclusion

Ex1 $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

$a_n = \frac{1}{n!} \qquad a_{n+1} = \frac{1}{(n+1)!}$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} = \frac{n!}{(n+1)!} = \frac{\cancel{n!}}{(n+1)\cancel{n!}}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{n+1} = 0 < 1$$

so by ratio test, the series conv.

Ex2 $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^n}$

$$\lim_{n\to\infty} \frac{(n+1)!}{2^{n+1}} \Big/ \frac{n!}{2^n} = \lim_{n\to\infty} \frac{(n+1)!}{2^{n+1}} \cdot \frac{2^n}{n!}$$

$$= \lim_{n\to\infty} \frac{n+1}{2} \to \infty > 1$$ so series diverges

Ex $\sum \frac{1}{n}$ we know it div

$$\lim_{n\to\infty} \frac{\frac{1}{n+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n+1} = 1 \quad \text{no conclusion}$$

Ex $\sum \frac{1}{n^2}$ $p = 2$ conv

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{(n+1)^2} = 1 \quad \text{no conclusion}$$

so the ratio test is good when there are factorials or powers in the generator

Consider $\sum \frac{1}{2^n} = \sum \left(\frac{1}{2}\right)^n$ ← note power of n

<u>Root Test</u>

if $\lim_{n\to\infty} \sqrt[n]{a_n} = L$

series

$L > 1$ diverges

$L < 1$ conv.

$L = 1$ no conclusion

Ex $\sum_{n=1}^{\infty}\left(\frac{1}{2}+\frac{1}{n}\right)^n$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{1}{2}+\frac{1}{n}\right)^n}=\lim_{n\to\infty}\frac{1}{2}+\frac{1}{n}=\frac{1}{2}<1$$

so by root test the series conv.

Ex $\sum_{n=1}^{\infty}\left(\frac{5n+1}{2n+1}\right)^n$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{5n+1}{2n+1}\right)^n}=\lim_{n\to\infty}\frac{5n+1}{2n+1}=5/2>1$$

so by root test the series div.

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$\sum_{n=1}^{\infty}\left(2\sqrt[n]{n}+1\right)^n$

LH $\lim_{n\to\infty} n^{\frac{1}{n}}=\lim_{n\to\infty}e^{\ln n^{\frac{1}{n}}}$

$$=\lim_{n\to\infty}e^{\frac{\ln n}{n}}=e^0=1$$

$\lim_{n\to\infty}\left(2n^{\frac{1}{n}}+1\right)=3>1$ so by root test

the series div