Products of m- polar Fuzzy Graphs: A Review

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Abstract - To solve various combinatorial problems of Computational intelligence and computer science rely on graph theory. Upon the usage of the normal and tensor products of fuzzy graphs, this paper discusses these operations in a new light on m- polar fuzzy graphs. In addition, Cartesian product, composition of the resultant graph that is obtained from m – polar fuzzy graphs are discussed.

Keywords: Cartesian product, composition, normal and tensor product of an m-polar fuzzy graph.

I. INTRODUCTION

The problem of Konigsberg Bridge in the eighteenth century made mathematicians look towards graph theory for a solution. Later, L. Euler proved with the help of graph theory that Konigsberg Bridge problem has no solution at all. Values between 0 and 1 are used to develop a set theory based on fuzziness by L.A. Zadeh [1-3] in 1965. In 1994, J. Chen et al [4] proposed the generalized concept of the bipolar fuzzy set to obtain an m – polar fuzzy graph. The notions of an m-polar fuzzy set are an advance in nature than fuzzy sets and eliminate doubtfulness more absolutely. G. Ghori, M. Pal studied some operations, properties of strong, self-complementary, homomorphism, weak, co-weak, isomorphism, cross product, composition, density on m-polar fuzzy graphs and also studied m-Polar fuzzy planar graphs, defined faces and the dual nature of m-polar fuzzy planar graphs [5-7]. H. Rashmanlou, S. Samanta [8] discussed some properties of its results are investigated.

II. PRELIMINARIES

Definition 2.1 Let W be an m-polar fuzzy set on X . An m-polar fuzzy relation on W is an m-polar fuzzy set F of $X \times X$ such that $F(uv) \le \min\{W(u), W(v)\}$ for all $u, v \in X$ i.e., for each $i = 1, 2, \dots m,$ for all $u, v \in X$, $p_i \circ F(uv) \le \min\{p_i \circ W(u), p_i \circ W(v)\}$.

We assume the following: For a given set V , define an equivalence relation $\sim$ on $V \times V - \{(s,s) : s \in V\}$ as follows: $(s_1, t_1) \sim (s_2, t_2) \Leftrightarrow$ either $(s_1, t_1) = (s_2, t_2)$ or $s_1 = t_2$ and $t_1 = s_2$. The quotient set obtained in this way is denoted by $\tilde{V}^2$.

Definition 2.2 The Cartesian product of G_1 and G_2 is defined as a pair $G_1 \times G_2 = (V_1 \times V_2, W_1 \times W_2, F_1 \times F_2)$ such that for $i = 1, 2, \dots, m$

(i) $p_i \circ (W_1 \times W_2)(s_1, s_2) = \min\{p_i \circ W_1(s_1), p_i \circ W_2(s_2)\}$ for all $(s_1, s_2) \in V_1 \times V_2$

(ii) $p_i \circ (F_1 \times F_2)((s, s_2)(s, t_2)) = \min\{p_i \circ W_1(s), p_i \circ F_2(s_2 t_2)\}$ for all $s \in V_1, s_2 t_2 \in E_2$,

(iii) $p_i \circ (F_1 \times F_2)((s_1, r)(t_1, r)) = \min\{p_i \circ F_1(s_1 t_1), p_i \circ W_2(r)\}$ for all $r \in V_2, s_1 t_1 \in E_1$,

(iv) $p_i \circ (F_1 \times F_2)((s_1, s_2)(t_1, t_2)) = 0$ for all for all $(s_1, s_2)(t_1, t_2) \in \widetilde{(V_1 \times V_2)}^2 - E.$

Definition 2.3 The composition of G_1 and G_2 is defined as a pair $G_1[G_2] = (V_1 \times V_2, W_1[W_2], F_1[F_2])$ such that for $i = 1, 2, \dots, m$

(i) $p_i \circ (W_1[W_2])(s_1, s_2) = \min\{p_i \circ W_1(s_1), p_i \circ W_2(s_2)\}$ for all $(s_1, s_2) \in V_1 \times V_2$.

(ii) $p_i \circ (F_1[F_2])((s, s_2)(s, t_2)) = \min\{p_i \circ W_1(s), p_i \circ F_2(s_2 t_2)\}$ for all $s \in V_1, s_2 t_2 \in E_2$,

(iii) $p_i \circ (F_1[F_2])((s_1, r)(t_1, r)) = \min\{p_i \circ F_1(s_1 t_1), p_i \circ W_2(r)\}$ for all $r \in V_2, s_1 t_1 \in E_1$,

(iv) $p_i \circ (F_1[F_2])((s_1, s_2)(t_1, t_2)) = \min\{p_i \circ W_2(s_2), p_i \circ W_2(t_2), p_i \circ F_1(s_1 t_1)\}$

for all $(s_1, s_2)(t_1, t_2) \in E^0 - E,$ where $E^0 = E \cup \{(s_1, s_2)(t_1, t_2) | s_1 t_1 \in E_1, s_2 \ne t_2\}.$

Definition 2.4 The normal product of G_1 and G_2 is defined as an m-polar fuzzy graph $G_1 \bullet G_2 = (V_1 \times V_2, W_1 \bullet W_2, F_1 \bullet F_2)$ on G^* where $E = \{((s,s_2)(s,t_2)) | s \in V_1, s_2t_2 \in E_2\} \cup \{(s_1,r)(t_1,r) | s_1t_1 \in E_1, r \in V_2\} \cup \{(s_1,s_2)(t_1,t_2) | s_1t_1 \in E_1, s_2t_2 \in E_2\}$ such that for $i = 1,2,...,m$

(i) $p_i \circ (W_1 \bullet W_2)(s_1,s_2) = \min\{p_i \circ W_1(s_1), p_i \circ W_2(s_2)\}$ for all $(s_1,s_2) \in V_1 \times V_2$,

(ii) $p_i \circ (F_1 \bullet F_2)((s,s_2)(s,t_2)) = \min\{p_i \circ W_1(s), p_i \circ F_2(s_2t_2)\}$ for all $s \in V_1, s_2t_2 \in E_2$,

(iii) $p_i \circ (F_1 \bullet F_2)((s_1,r)(t_1,r)) = \min\{p_i \circ F_1(s_1t_1), p_i \circ W_2(r)\}$ for all $r \in V_2, s_1t_1 \in E_1$,

(iv) $p_i \circ (F_1 \bullet F_2)((s_1,s_2)(t_1,t_2)) = \min\{p_i \circ F_1(s_1t_1), p_i \circ F_2(s_2t_2)\}$ for all $s_1s_2 \in E_1$ and $t_1t_2 \in E_2$.

Definition 2.5 The tensor product of G_1 and G_2 is defined as an m-polar fuzzy graph $G_1 \otimes G_2 = (V_1 \times V_2, W_1 \otimes W_2, F_1 \otimes F_2)$ on G^* where $E = \{((s_1,s_2)(t_1,t_2)) | s_1t_1 \in E_1, s_2t_2 \in E_2\}$ such that for $i = 1,2,...,m$

(i) $p_i \circ (W_1 \otimes W_2)(s_1,s_2) = \min\{p_i \circ W_1(s_1), p_i \circ W_2(s_2)\}$ for all $(s_1,s_2) \in V_1 \times V_2$.

(ii) $p_i \circ (F_1 \otimes F_2)((s_1,s_2)(t_1,t_2)) = \min\{p_i \circ F_1(s_1t_1), p_i \circ F_2(s_2t_2)\}$ for all $s_1s_2 \in E_1$ and $t_1t_2 \in E_2$.

III. DEGREE OF A VERTEX IN M- POLAR FUZZY GRAPH

Definition 3.1 Let $G = (V,W,F)$ be an m-polar fuzzy graph. Then the degree of a vertex s in G is defined as

$$d_G(s) = < \sum_{st \in E\, s \neq t} p_1 \circ F(st), \sum_{st \in E\, s \neq t} p_2 \circ F(st), \sum_{st \in E\, s \neq t} p_3 \circ F(st), ... \sum_{st \in E\, s \neq t} p_m \circ F(st) >.$$

3.2 Degree of a vertex in Cartesian product

From the definition, for every vertex $(s_1,s_2) \in V_1 \times V_2$, for $i = 1,2,..., m$ $d_{G_1 \times G_2}(s_1,s_2) = \sum_{(s_1,s_2)(t_1,t_2)} p_i \circ (W_1 \times W_2)((s_1,s_2)(t_1,t_2))$

$$= \sum_{s_1 = t_1, s_2t_2 \in E_2} \min\{p_i \circ W_1(s_1), p_i \circ F_2(s_2t_2)\} + \sum_{s_2 = t_2, s_1t_1 \in E_1} \min\{p_i \circ W_2(s_2), p_i \circ F_1(s_1t_1)\}.$$

3.3 Degree of a vertex in Composition

From the definition, for every vertex $(s_1,s_2) \in V_1 \times V_2$, for $i = 1,2,..., m$ $d_{G_1[G_2]}(s_1,s_2) = \sum_{(s_1,s_2)(t_1,t_2) \in E} p_i \circ (F_1[F_2])((s_1,s_2)(t_1,t_2))$

$$= \sum_{s_1 = t_1, s_2t_2 \in E_2} \min\{p_i \circ W_1(s_1), p_i \circ F_2(s_2t_2)\} + \sum_{s_2 = t_2, s_1t_1 \in E_1} \min\{p_i \circ W_2(s_2), p_i \circ F_1(s_1t_1)\}$$
$$+ \sum_{s_2 \neq t_2, s_1t_1 \in E_1} \min\{p_i \circ W_2(t_2), p_i \circ W_2(s_2), p_i \circ F_1(s_1t_1)\}.$$

3.4 Degree of a vertex in normal product

From the definition, for every vertex $(s_1,s_2) \in V_1 \times V_2$ for $i = 1,2,...,m$

$$d_{G_1 \bullet G_2}(s_1,s_2) = \sum_{(s_1,s_2)(t_1,t_2) \in E} p_i \circ (F_1 \bullet F_2)((s_1,t_1)(s_2,t_2))$$

$$= \sum_{s_1=t_1, s_2t_2\in E_2} \min\{p_i \circ W_1(s_1), p_i \circ F_2(s_2 t_2)\} + \sum_{s_2=t_2, s_1t_1\in E_1} \min\{p_i \circ W_2(s_2), p_i \circ F_1(s_1t_1)\} + \sum_{s_2t_2\in E_2, s_1t_1\in E_1} \min\{p_i \circ F_1(s_1t_1), p_i \circ F_2(s_2t_2)\}.$$

3.5 Degree of a vertex in tensor product

From the definition, for every vertex $(s_1, s_2) \in V_1 \times V_2$ for $i = 1, 2, ..., m$

$$d_{G_1 \otimes G_2}(s_1, s_2) = \sum p_i \circ (W_1 \otimes W_2)((s_1, s_2)(t_1, t_2)) = \sum_{s_1t_1\in E_1} \min\{p_i \circ F_1(s_2t_2), p_i \circ F_2(s_1t_1)\}.$$

Example 3.6 Tensor product $G_1 \otimes G_2$ of two m-polar fuzzy graphs G_1 and G_2 is shown in Figure 1.

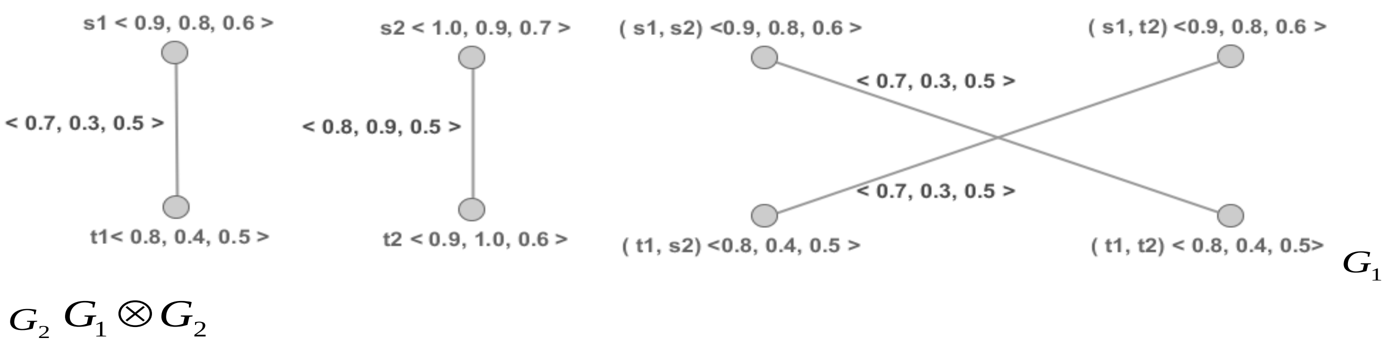

Figure 1. Tensor product of two 3- polar fuzzy graphs G_1 and G_2

Since $p_i \circ F_2 \geq p_i \circ F_1$, by theorem 3.6, we have

$$d_{G_1 \otimes G_2}(s_1, s_2) = d_{G_1}(s_1) = <0.7, 0.3, 0.5>, \; d_{G_1 \otimes G_2}(s_1, t_2) = d_{G_1}(s_1) = <0.7, 0.3, 0.5>,$$
$$d_{G_1 \otimes G_2}(t_1, s_2) = d_{G_1}(t_1) = <0.7, 0.3, 0.5>, \; d_{G_1 \otimes G_2}(t_1, t_2) = d_{G_1}(t_1) = <0.7, 0.3, 0.5>.$$

IV. M- POLAR FUZZY GRAPHS AND THEIR APPLICATION

Guidance in decision making is an important application of m- polar fuzzy graphs. For example, let us consider a scenario where a college wants to select a principal. Let W= {Adithya, Phani, Mahi, Nayak} be the set of doctorates shortlisted and F = {Krishna, Srinu, Manoj, Ramesh} be the set of college management members. The management has to make a selection taking into account the set of qualities Q = {management skills, attitude, patents and research}. Each member from set F can assign a value between 0 and 1 to each element listed in the set Q for each candidate mentioned in set W, such as, $W(K) = <0.3, 0.4, 0.6, 0.2>$, $W(L) = <0.9, 0.5, 0.7, 0.3>$, $W(M) = <0.4, 0.7, 0.3, 0.5>$, $W(N) = <0.3, 0.6, 0.7, 0.6>$. Then we have a 4-polar fuzzy graph, in which each member gives his opinion to w belongs to W on the basis of his qualities. So $W(K), W(L), W(M)$ and $W(N)$ denote the degree of management skills, attitude, patents and research of each person given by the members of college management and edges denote the same qualities of two persons as shown in Figure 2.

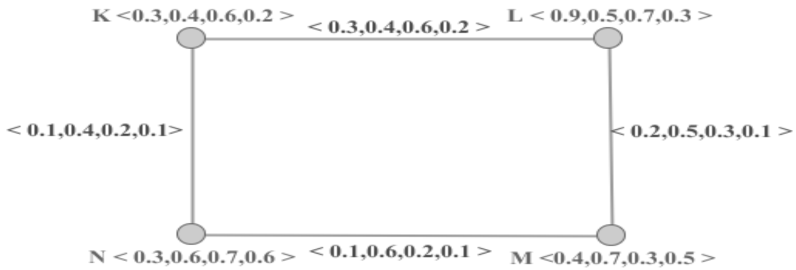

Figure 2. 4-polar fuzzy graph in decision making

The membership degree of an edge KL denotes the members K and L decides that Adithya is 30% eligible, Phani is 40% eligible, Mahi is 60% eligible, Nayak is 20% eligible. So according to KL Mahi is an eligible person for the post of principal. In the same way LM decides Phani, MN decides Phani, NK also decides Phani. Hence according to all persons, Phani has a higher value of desirability among the shortlisted candidates.

V. CONCLUSIONS

Many real time applications can be solved by using the theory of a fuzzy graph like Number theory, Algebra, Topology, Operation research are few areas of research. It solves the day to day problems that are faced in the society related to probabilistic data. The novel approach discussed here enables decision makers to formulate better preferred option with the help of unique case pattern than the conventional fuzzy graph approaches. The concept of Cartesian product, composition, tensor product, normal products are introduced and discussed with some examples..

VI. REFERENCES

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