Blood flow analysis in a constricted tube with permeable walls

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Abstract - In the present paper Jefferey fluid flow through a tube with an overlapping stenosis and permeable walls is investigated. The analytical solutions are obtained for velocity, resistance to the flow and wall shear stress. The effects of various parameters are anlyzed through the graphs.

Keywords: Jefferey fluid, Permeable walls, overlapping stenosis.

I. INTRODUCTION

The study of stenosis gained huge popularity among researchers due to its close association with cardiovascular system. Cardiovascular diseases such as heart attacks, strokes, chest pain etc are due to the abnormality of blood flows in an arteries. Abnormality in blood flows due to accumulation of fatty substances or plaque in an artery is called stenosis. Arterial walls may be elastic, movable or permeable and the presence of stenosis in an artery may disturb the normal functioning of blood flow. Hence extensive research has been done on fundamental study of blood flow analysis in stenosed arteries.[1-6] Jeffrey fluid is a generalization of Newtonian fluid and it serves as a better model for physiological fluids (Hayat et al., [7]). Hence Newtonian fluid can be deduced from the Jeffrey fluid model. Several researchers have investigated Jeffrey fluid flows under different conditions. Vajravelu et al., [8] studied the heat transfer effect on peristaltic flow of Jeffrey fluid. Akbar and Nadeem [9] have investigated the effect of variable viscosity on blood flow through a tapered artery with stenosis by considering blood as Jeffrey fluid. Bhuvana et al., studied the effect of magnetic field on Jeffrey fluid [10].

The formation of the stenosis maybe regular, irregular or maybe of different shapes. Riahi et al., [11] analyzed the flow of blood in an artery in the presence of overlapping stenosis. Ranadhir Roy and D.N. Riahi investigated the flow characteristics of blood through an artery with an overlapping unsteady stenosis [12]. Recently Maruthi Prasad et al., [13] developed a mathematical model on Herschel-Bulkley fluid through an overlapping stenosis.

With the motivation of above mentioned research, the present paper investigates Jeffrey fluid flow through a tube with permeable walls.

II. MATHEMATICAL FORMULATION

Consider the steady incompressible Jeffrey fluid flow through a uniform tube with overlapping stenosis as shown in Fig-1.

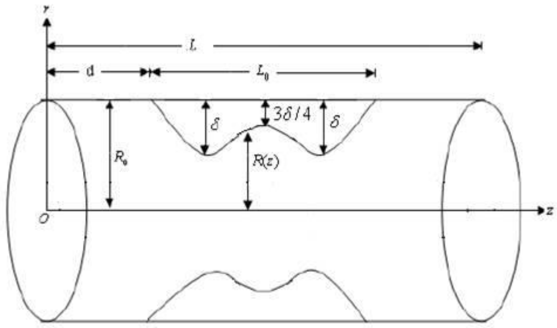

Figure 1: Physical model of a tube with overlapping stenosis.

The physical Model of the overlapping stenosis in an artery is represented mathematically as

$$h(z) = \frac{R(z)}{Ro(z)} = 1 - \frac{3}{2}\left(\frac{\delta}{RoLo^4}\left[11(z-d)Lo^3 - 47(z-d)^2Lo^2 + 72(z-d)^3Lo - 36(z-d)^4\right]\right), \quad d \le z \le d + Lo,$$
$$= 1, \text{ Otherwise.} \qquad (1)$$

Radius of the artery in the constricted region is $R(Z)$, Radius of the tube in the non-constricted region is $R_0(Z)$, length of the tube is L, Length of the stenosis is L_0 and the location of stenosis is d,at $Z = d + \frac{L_0}{6}$ and $Z = d + \frac{5L_0}{6}$ stenosis is attaining its maximum height which is denoted by δ. The critical height of the stenosis is $\frac{3\delta}{4}$ attained at $Z = d + \frac{L_0}{2}$ from the origin.

The constitutive equations for an incompressible Jeffrey fluid are

$$T = -pI + S \tag{2}$$

$$S = \frac{\mu}{1+\lambda_1}\left(\frac{\partial\gamma}{\partial t} + \lambda_2 \frac{\partial^2\gamma}{\partial t^2}\right) \tag{3}$$

Where T and S are Caushy stress tensor and extra stress tensor respectively, p is the pressure, I is the identity tensor, λ_1 is the ratio of the relaxation to retardation times, λ_2 is the retardation time, μ is the dynamic viscosity and $\bar{\gamma}$ is the shear rate.

The governing equations for the present problem are as follows

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

$$\rho\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)u = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rr}) + \frac{\partial}{\partial z}(S_{rz}) \tag{5}$$

$$\rho\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)w = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \frac{\partial}{\partial z}(S_{zz}) \tag{6}$$

With the extra tensor components

$$S_{rr} = \frac{2\mu}{1+\lambda_1}(1 + \lambda_2\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\frac{\partial u}{\partial r}$$

$$S_{rz} = \frac{\mu}{1+\lambda_1}\left(1 + \lambda_2\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\right)\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)$$

$$S_{zz} = \frac{2\mu}{1+\lambda_1}(1 + \lambda_2\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\frac{\partial w}{\partial z}$$

$$S_{\theta\theta} = \frac{2\mu}{1+\lambda_1}(1 + \lambda_2\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\frac{u}{r}$$

Where u and w are the velocities in r and z-directions respectively.

Using the following non-dimensional quantities

$$\bar{z} = \frac{z}{L}, \bar{\delta} = \frac{\delta}{R_0}, \bar{R} = \frac{R}{R_0}, \bar{P} = \frac{P}{\frac{\mu U L}{R_0^2}}, \bar{u} = \frac{u}{U}, \bar{w} = \frac{L}{U\delta}w, Re = \frac{\rho R_0 U}{\mu}, \tag{7}$$

in eqs. (4)-(6), and assuming mild stenosis, then these equations are reduced to the following form

$$\frac{\partial p}{\partial r} = 0 \tag{8}$$

$$\frac{\partial p}{\partial z} = \frac{1}{1+\lambda_1}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) \tag{9}$$

Since the flow is assumed axially symmetric, hence the velocity component in r- direction is zero.

The non-dimensional boundary conditions are

$$\frac{\partial w}{\partial r} = 0 \text{ at } r = 0 \tag{10}$$

$$w = w_B, \frac{\partial w}{\partial r} = \frac{\beta}{\sqrt{D_a}}(w_B - w_p) \qquad at\ r = h(z) \tag{11}$$

Where w_B is slip velocity, $w_p = -D_a\frac{\partial p}{\partial z}$, β is the slip parameter and D_a is Darcy number.

III. SOLUTION

Solving equation Eq. (9) under the the boundary conditions (10) & (11), we get the velocity as

$$w = \left((1 + \lambda_1)\left(\left(\frac{r^2 - h^2}{4}\right) + \frac{h}{2}\frac{\sqrt{D_a}}{\beta}\right) - D_a\right)\left(\frac{dp}{dz}\right) \tag{12}$$

The flow rate is defined by

$$Q = 2\pi\int_0^h wr\,dr \tag{13}$$

Integrating Eq. (13),

$$Q = 2\pi\left(\frac{dp}{dz}\right)\left((1 + \lambda_1)\left(\frac{h^3}{4}\frac{\sqrt{D_a}}{\beta} - \frac{h^4}{16}\right) - D_a\frac{h^2}{2}\right) \tag{14}$$

$$\frac{dp}{dz} = \frac{Q}{2\pi\left((1+\lambda_1)\left(\frac{h^3\sqrt{D_a}}{4\ \beta} - \frac{h^4}{16}\right) - D_a\frac{h^2}{2}\right)} \tag{15}$$

The pressure drop Δp across the stenosis between $z = 0$ to $z = 1$ is obtained by integrating Eq. (15), as

$$\Delta p = \int_0^1 \frac{dp}{dz}\,dz \tag{16}$$

$$\Delta p = \int_0^1 \frac{Q}{2\pi\left((1+\lambda_1)\left(\frac{h^3\sqrt{D_a}}{4\ \beta} - \frac{h^4}{16}\right) - D_a\frac{h^2}{2}\right)}\,dz \tag{17}$$

the resistance to the flow, λ, is defined by

$$\lambda = \frac{\Delta p}{Q} = \frac{1}{Q}\int_0^1 \frac{Q}{2\pi\left((1+\lambda_1)\left(\frac{h^3\sqrt{Da}}{4\ \beta} - \frac{h^4}{16}\right) - Da\frac{h^2}{2}\right)}\ dz$$

(18)

The pressure drop in the absence of stenosis $(h = 1)$ is denoted by Δp_N, is obtained from Eq. (15).

$$\Delta P_N = \int_0^1 \frac{Q}{2\pi\left((1+\lambda_1)\left(\frac{1\sqrt{Da}}{4\ \beta} - \frac{1}{16}\right) - Da\frac{1}{2}\right)}\ dz$$

(19)

The resistance to the flow in the normal artery is denoted by λ_N is obtained from Eq.(19),as

$$\lambda_N = \frac{\Delta P_N}{Q} = \frac{1}{Q}\int_0^1 \frac{Q}{2\pi\left((1+\lambda_1)\left(\frac{1\sqrt{Da}}{4\ \beta} - \frac{1}{16}\right) - Da\frac{1}{2}\right)} dz$$

(20)

The resistance to the flow denoted by

$$\bar{\lambda} = \frac{\lambda}{\lambda_N}$$

(21)

The wall shear stress S_{rz} is given by

$$S_{rz} = -\frac{h}{2}\frac{dp}{dz}$$

(22)

IV. RESULTS AND DISCUSSIONS

The velocity profile (u) is plotted against the radial axis (r) for different heights of stenosis (δ), Jeffrey fluid parameter (λ_1), Darcy number($\sqrt{D_a}$) and slip parameter (β) in Figs.2-5. It is observed that the velocity is symmetrical for all the parameters and is extreme at the center of the tube and lowest at walls. The velocity increases with the height of stenosis, Jeffrey fluid parameter, and slip parameter but it decreases with Darcy number.

From Figs. 6-9,It is seen that, the resistance to the flow raised with δ (stenosis height), length of stenosis, Jeffrey fluid parameter(λ_1), slip parameter , Darcy number.

It is observed from Figs.10-12, the wall shear stress (τ_h) increases proportionately with height (δ) and is maximum at the two stenosis throats $(z = 0.26, z = 0.53)$, minimum at its critical height $(z = 0.4)$. The shear stress increases with λ_1 and decreases with the slip parameter.

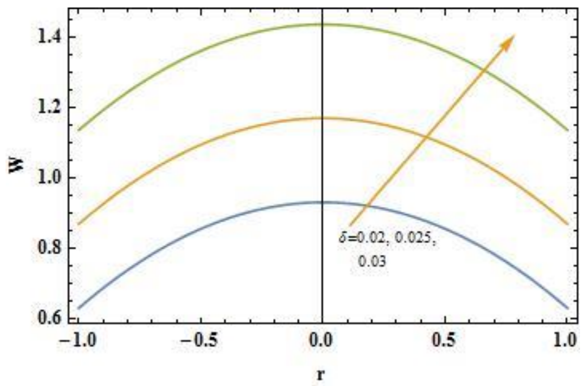

Fig. 2. Effect of δ on w

$(d = 0.2, L_0 = 0.2, \sqrt{D_a} = 0.01, \beta = 0.1, z = 0.1, \lambda_1 = 0.2, L = 1)$

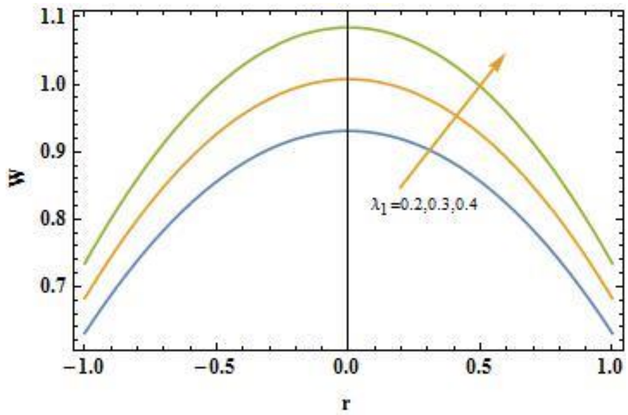

Fig. 3. Effect of λ_1 on w

$(d = 0.2, L_0 = 0.2, \delta = 0.02, \sqrt{D_a} = 0.01, \beta = 0.1, z = 0.1, L = 1)$

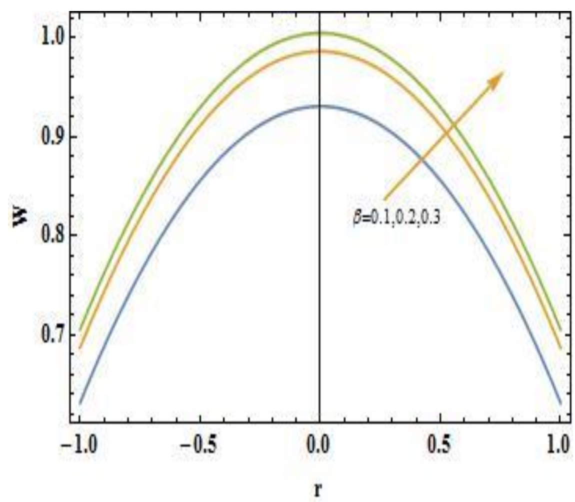

Fig. 4. Effect of β on w

$(d = 0.2\ , L_0 = 0.2, \delta = 0.02, \sqrt{D_a} = 0.01, \lambda_1 = 0.2, z = 0.1, L = 1)$

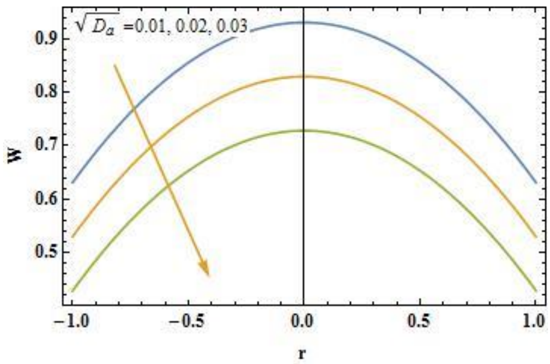

Fig. 5. Effect of $\sqrt{D_a}$ on w

$(d = 0.2\ , L_0 = 0.2, \delta = 0.02, \beta = 0.1, \lambda_1 = 0.2, z = 0.1, L = 1)$

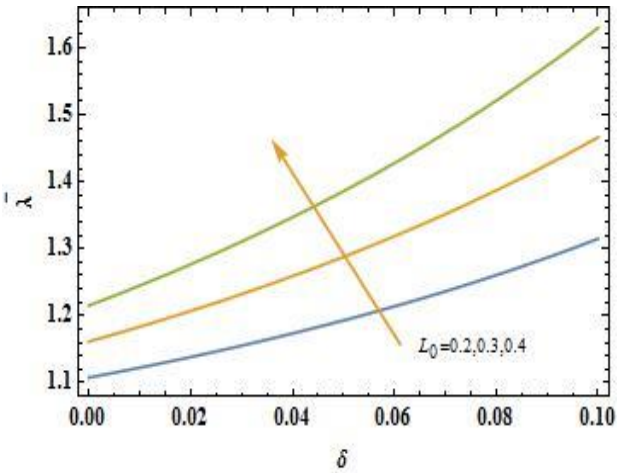

Fig. 6. . Effect of δ and L_0 on $\bar{\lambda}$

$(d = 0.2\ , \beta = 0.1, \lambda_1 = 0.2, \sqrt{D_a} = 0.01, L = 1)$

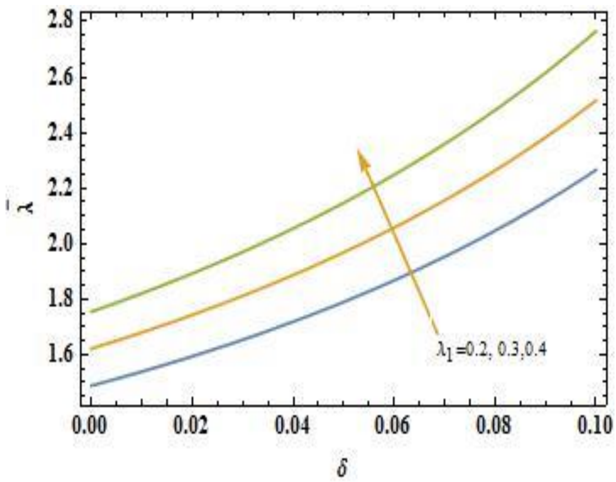

Fig. 7. Effect of δ and λ_1 on $\bar{\lambda}$

$(d = 0.2\ , L_0 = 0.2, \beta = 0.1, \sqrt{D_a} = 0.01, L = 1)$

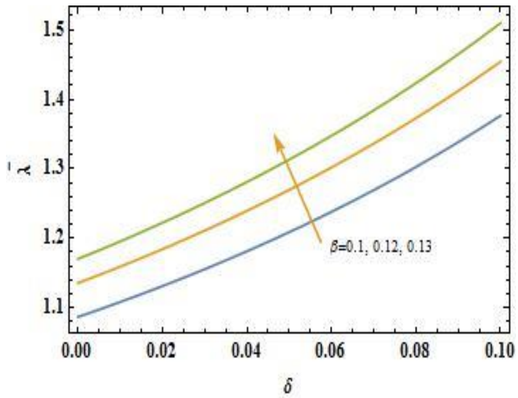

Fig. 8. Effect of δ and β on $\bar{\lambda}$

$(d = 0.2\ , L_0 = 0.2, \lambda_1 = 0.2, \sqrt{D_a} = 0.01, L = 1)$

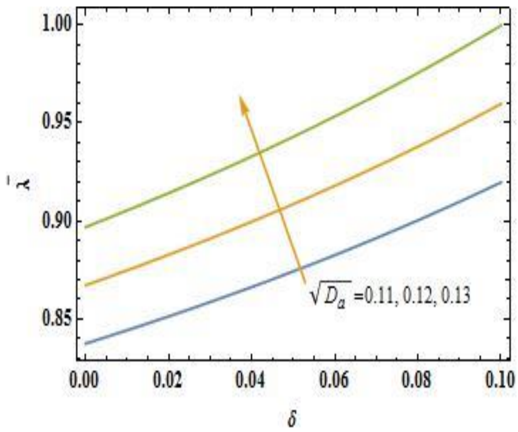

Fig. 9. Effect of δ and $\sqrt{D_a}$ on $\bar{\lambda}$

$(d = 0.2\ , L_0 = 0.2, \lambda_1 = 0.2, \beta = 0.1, L = 1)$

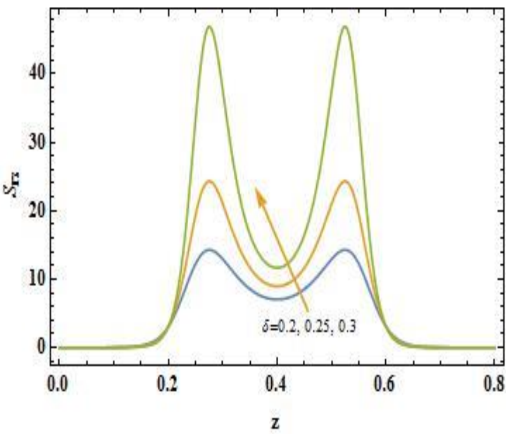

Fig. 10. Effect of λ_1 on S_{rz}

$(d = 0.2, L_0 = 0.2, \lambda_1 = 0.2, \sqrt{D_a} = 0.01, \beta = 0.1, L = 1, Q = 0.1)$

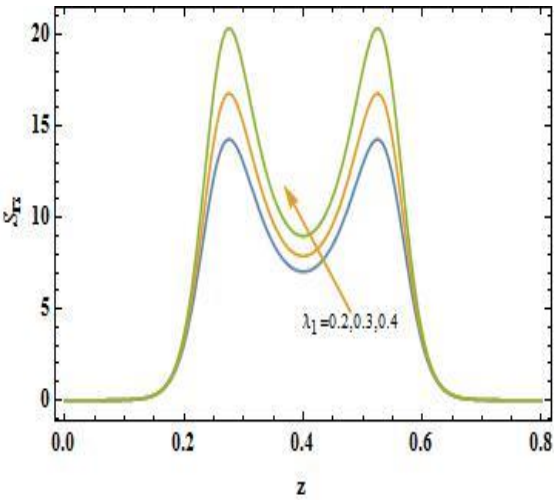

Fig. 11. Effect of λ_1 on S_{rz}

$(Q = 0.1, d = 0.2, L_0 = 0.2, \delta = 0.2, \sqrt{D_a} = 0.01, \beta = 0.1, L = 1)$

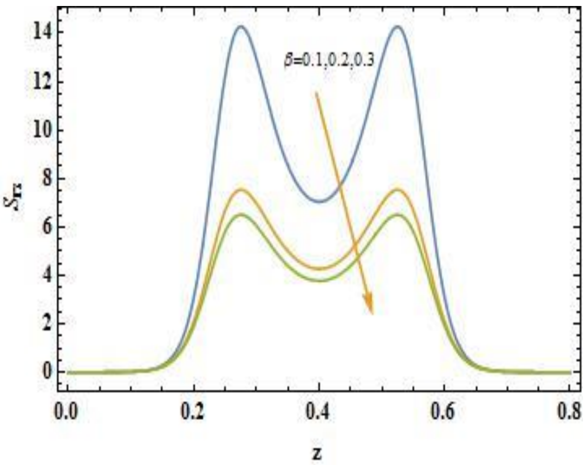

Fig. 12. Effect of β on S_{rz}

$(Q = 0.1, d = 0.2, L_0 = 0.2, \delta = 0.2, \sqrt{D_a} = 0.01, \lambda_1 = 0.2, L = 1)$

V. CONCLUSIONS

In the present mathematical analysis, the steady flow of Jeffrey fluid flow through a tube with permeable walls is examined. It is observed that the resistance to the flow increases with the height, length of the stenosis and Jeffrey fluid parameter and slip parameter. The wall shear stress increases with the height of stenosis Jeffrey fluid parameter, but it decreases with the slip parameter.

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