Vertical Planning Vocabulary Representations

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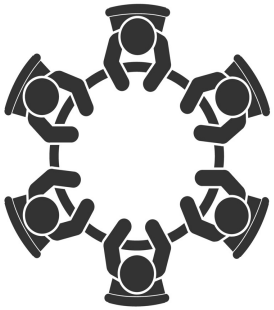
Introductions

Vertical Planning

Vocabulary

Representations



Say hello!

1. What are your strengths with teaching mathematics? What's an opportunity for growth?
2. What are your STUDENTS' strengths with mathematics? What's an opportunity for growth?

Introductions

Vertical Planning

Vocabulary

Representations

Vertical Planning
Vocabulary and Representations

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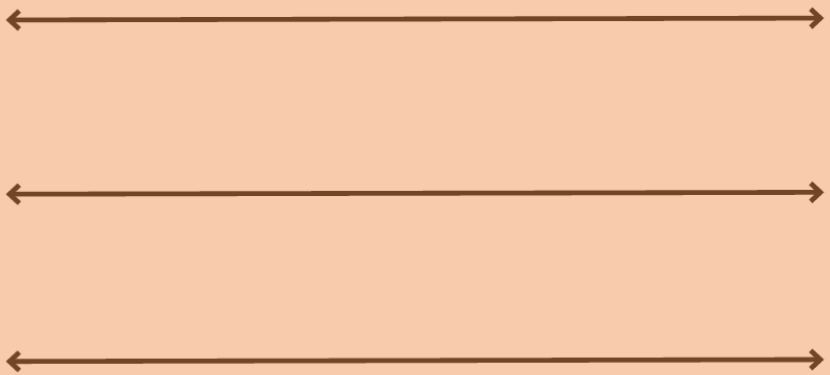
Mathematical Progressions

Critical Content

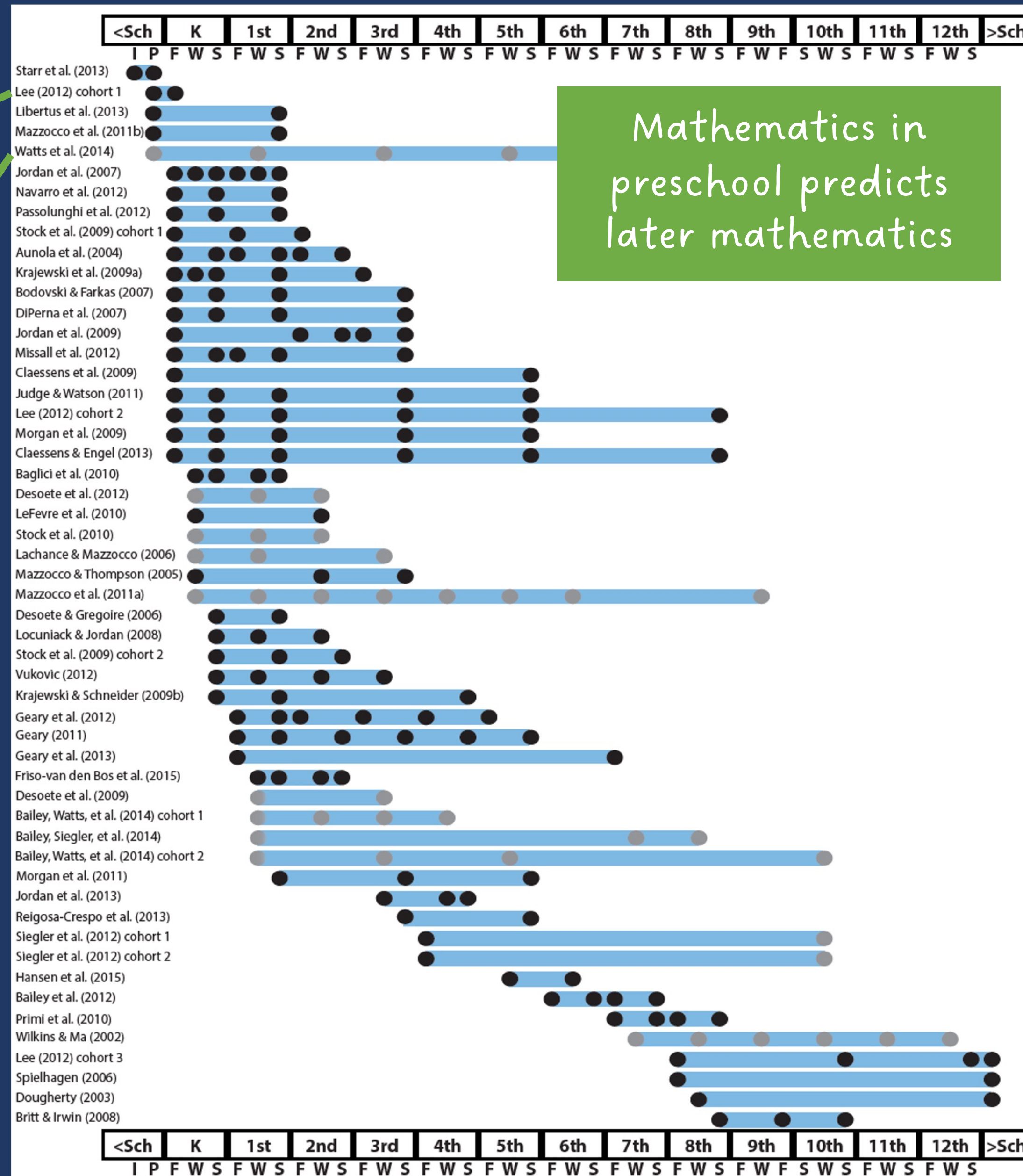


Mathematics is cumulative.
Early mathematics knowledge is essential for later mathematics.

Mathematics in preschool predicts later mathematics

Broad math in prek predicted K broad math

Broad math in prek predicted grade 10 broad math

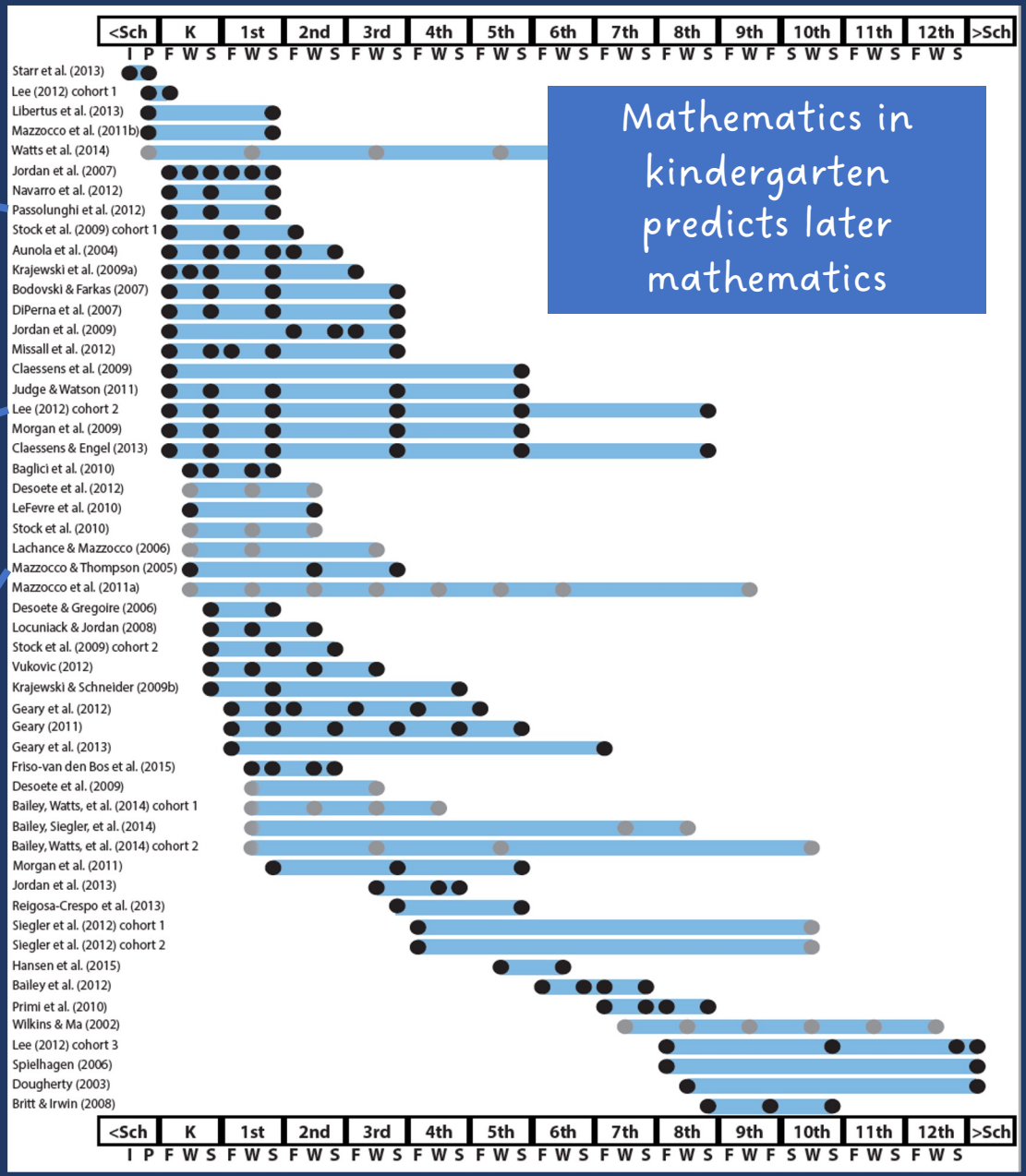


http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Counting in K predicted grade 1 broad math

Broad math in K predicted grade 8 broad math

K math accurately predicted math performance below 10th percentile in grades 2 and 3 with 84% correct classification

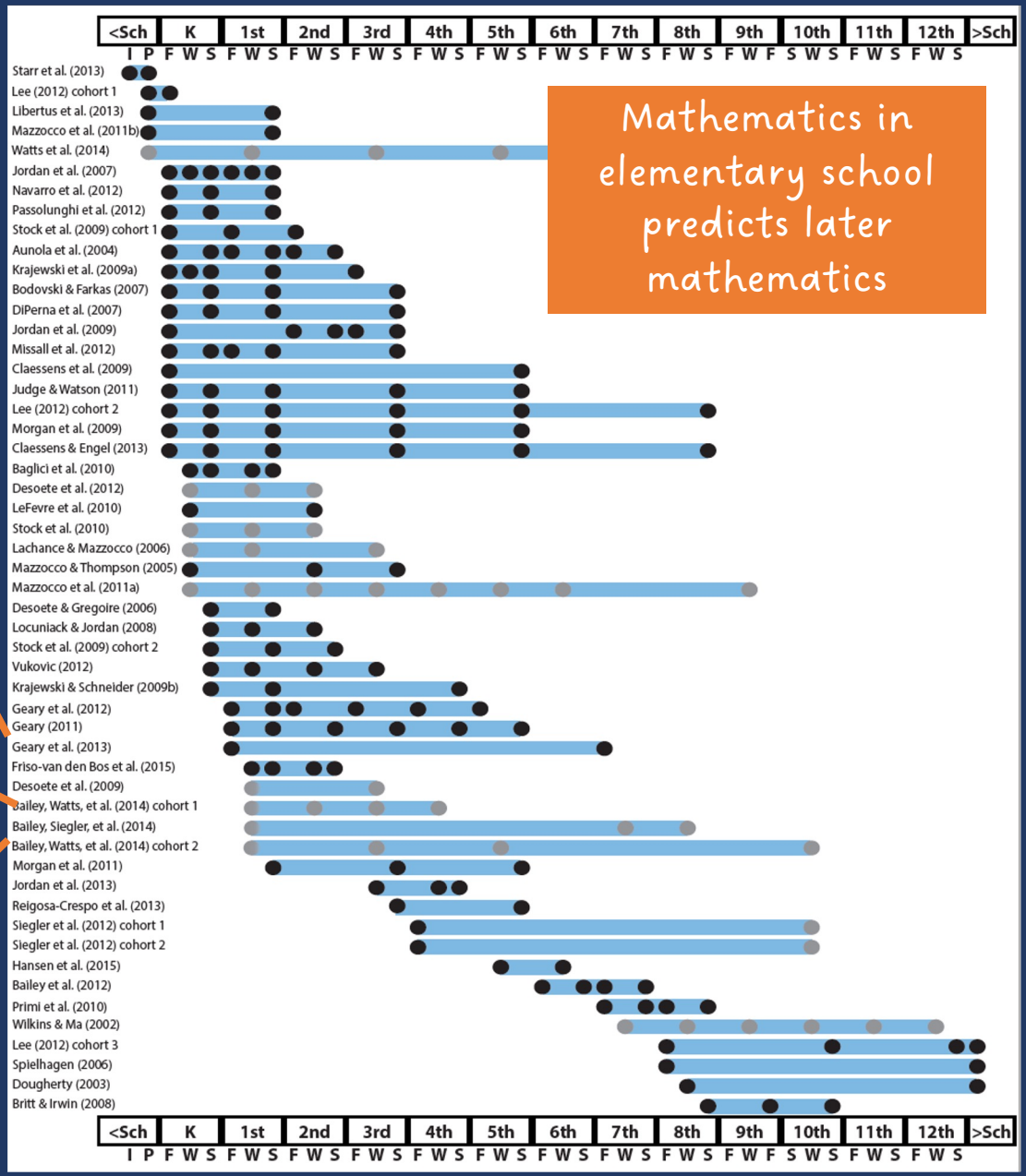


http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Addition influenced arithmetic with increasing importance from grades 1 to 5

Grade 1 arithmetic predicted arithmetic at grades 2, 3, and 4

Grade 1 broad math predicted broad math at grades 3, 5, and 10

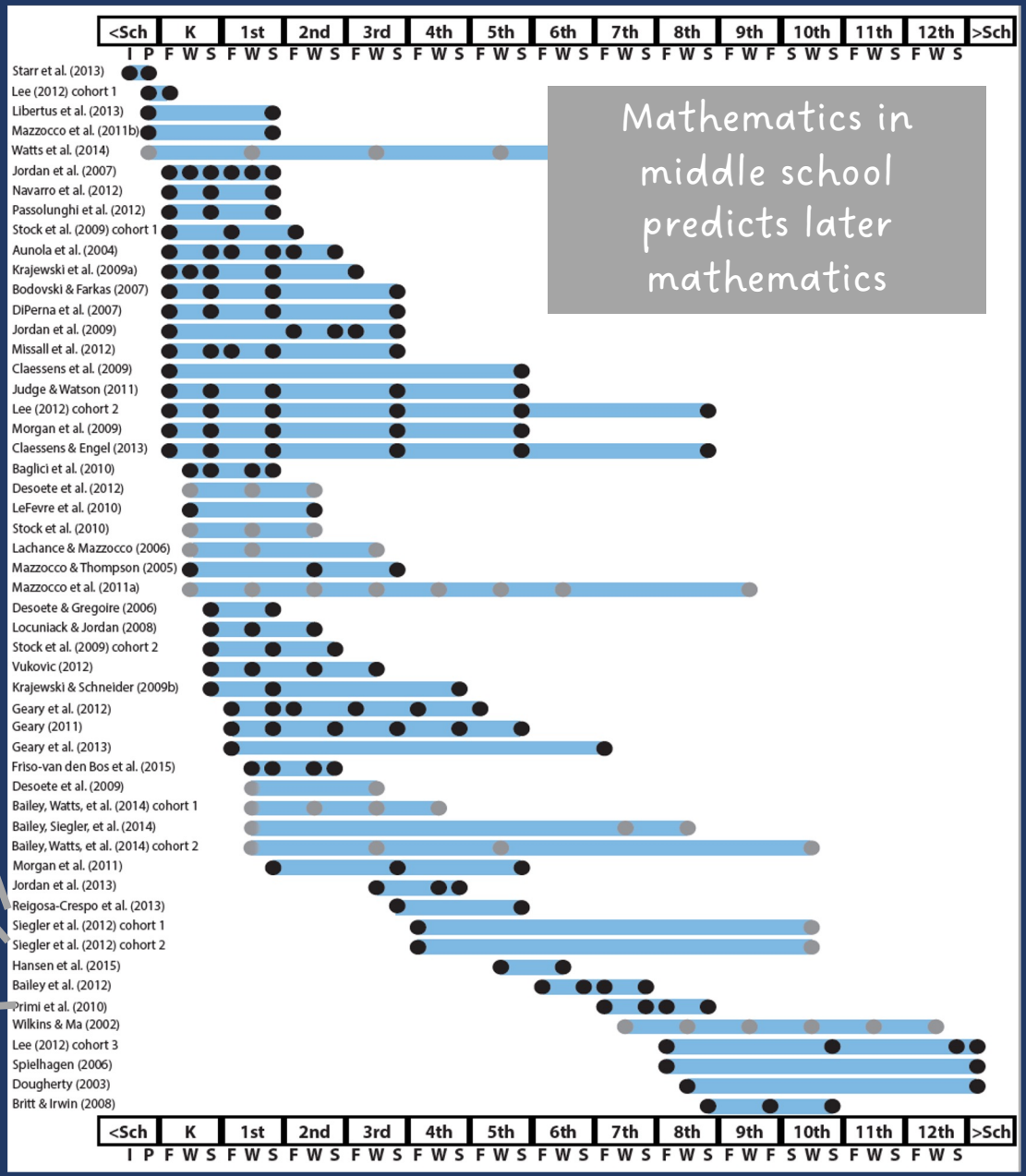


http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Counting and comparison in grades 2 or 4 predicted broad math 1 year later

Fractions at 10-12 years old predicted broad math 5 years later

Broad math in grade 7 predicted broad math in grade 8



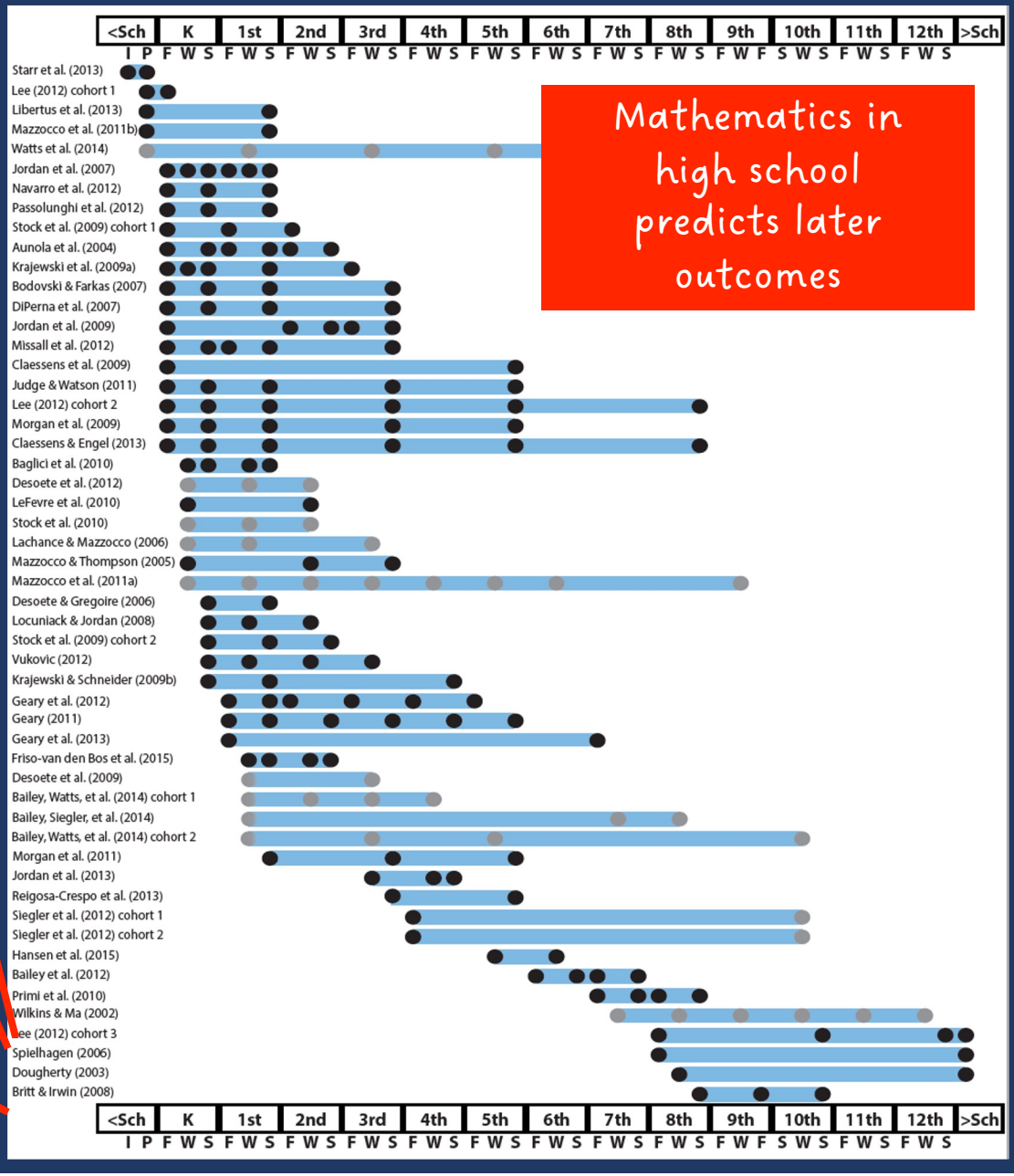
http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Mathematics in high school predicts later outcomes

Broad math in grade 8 predicted completion of 4-year college degree

Students who took algebra in grades 8 took more advanced math courses and enrolled in 4-year colleges more often than students who took algebra in grade 9

Numeracy measured in adolescence impacted hourly earnings 7 to 15 years later



http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

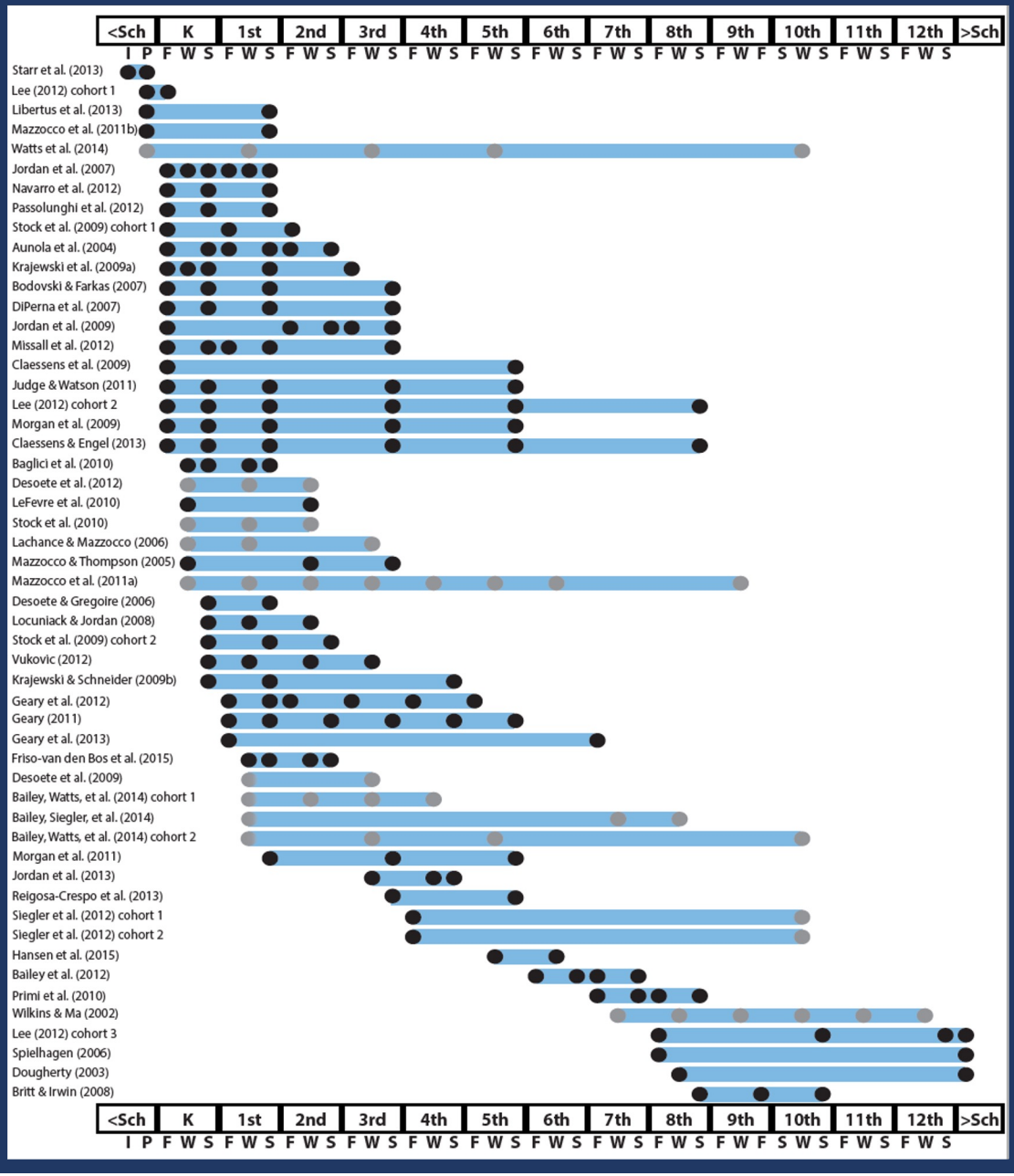
Mathematics in preschool predicts later mathematics

Mathematics in kindergarten predicts later mathematics

Mathematics in elementary school predicts later mathematics

Mathematics in middle school predicts later mathematics

Mathematics in high school predicts later outcomes



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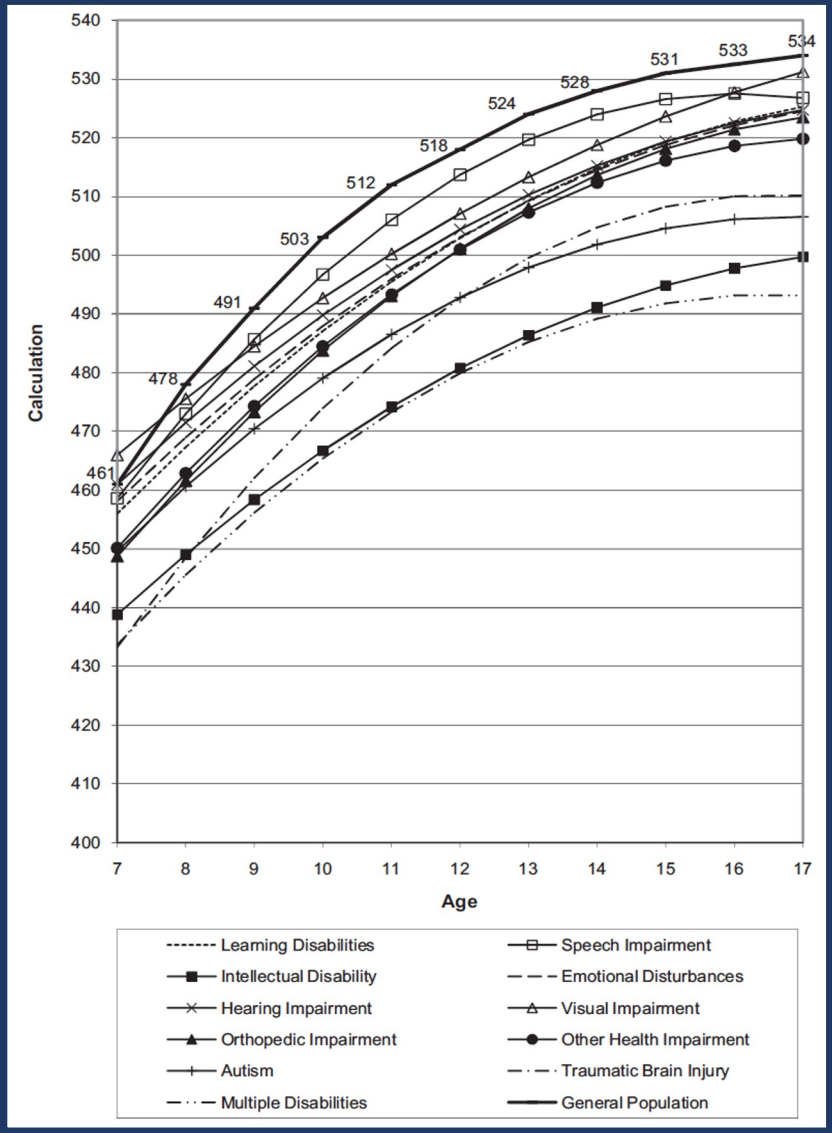
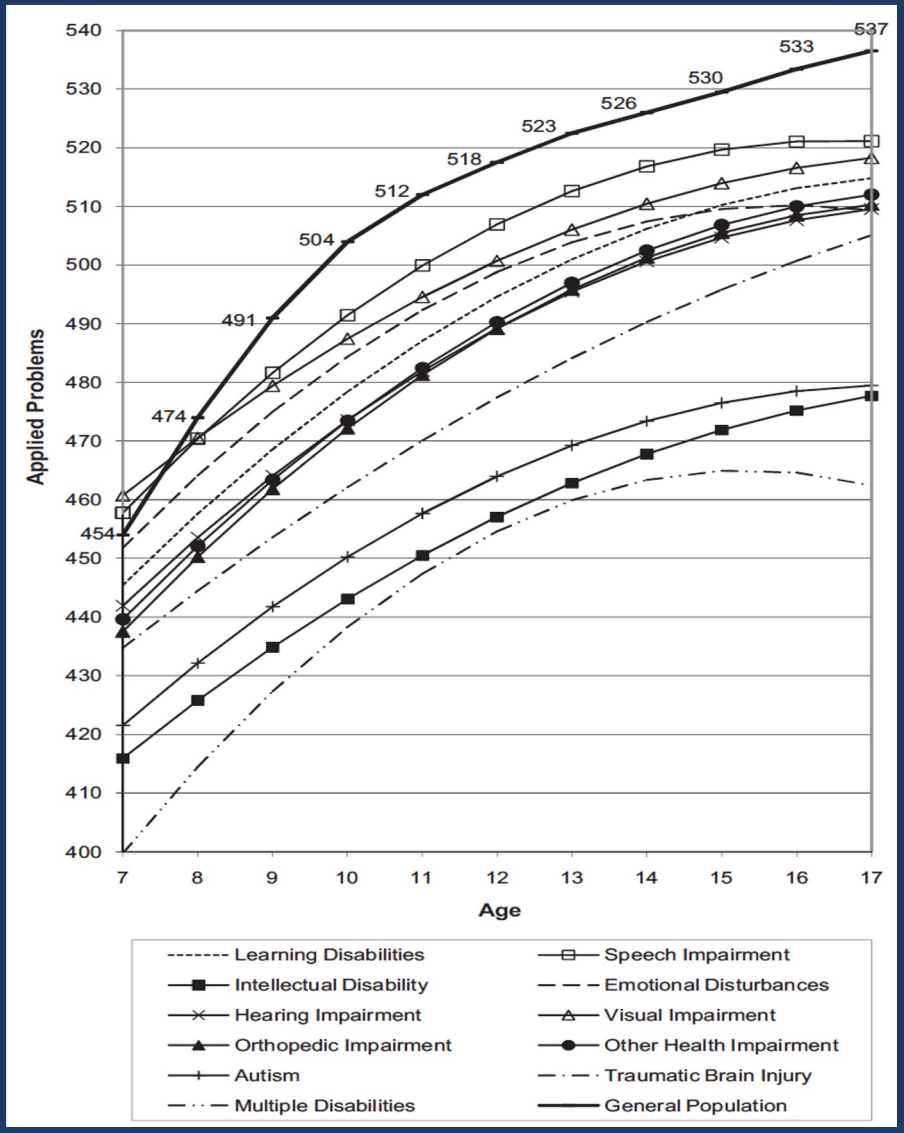
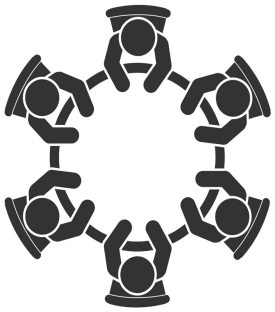


Computation



Problem Solving

Wei, Lenz, & Blackorby (2013)



Place this content in order from easier to more difficult.

continuum of mathematics learning

vertical planning

Vertical Planning
Vocabulary and Representations

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Mathematical Progressions

Critical Content

Fluently add and subtract multi-digit whole numbers

Fluently multiply and divide within 100

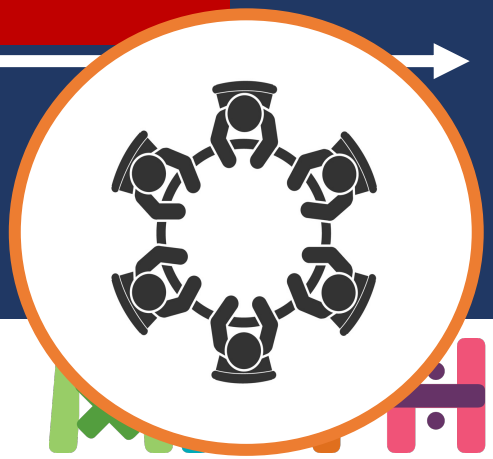
Fluently multiply multi-digit whole numbers

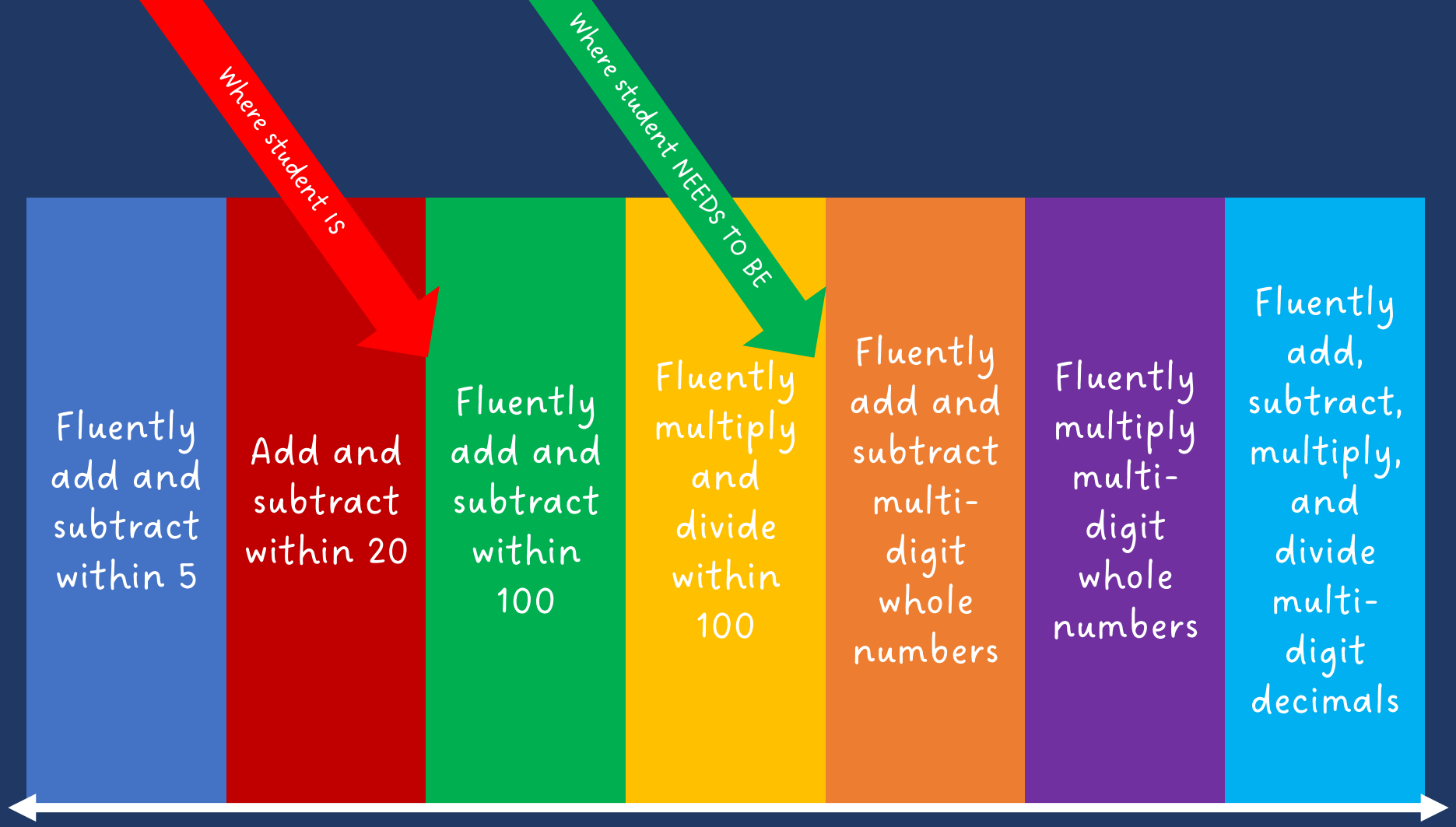
Fluently add and subtract within 100 using strategies

Fluently add and subtract within 5

Add and subtract within 20

Fluently add, subtract, multiply, and divide multi-digit decimals





Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

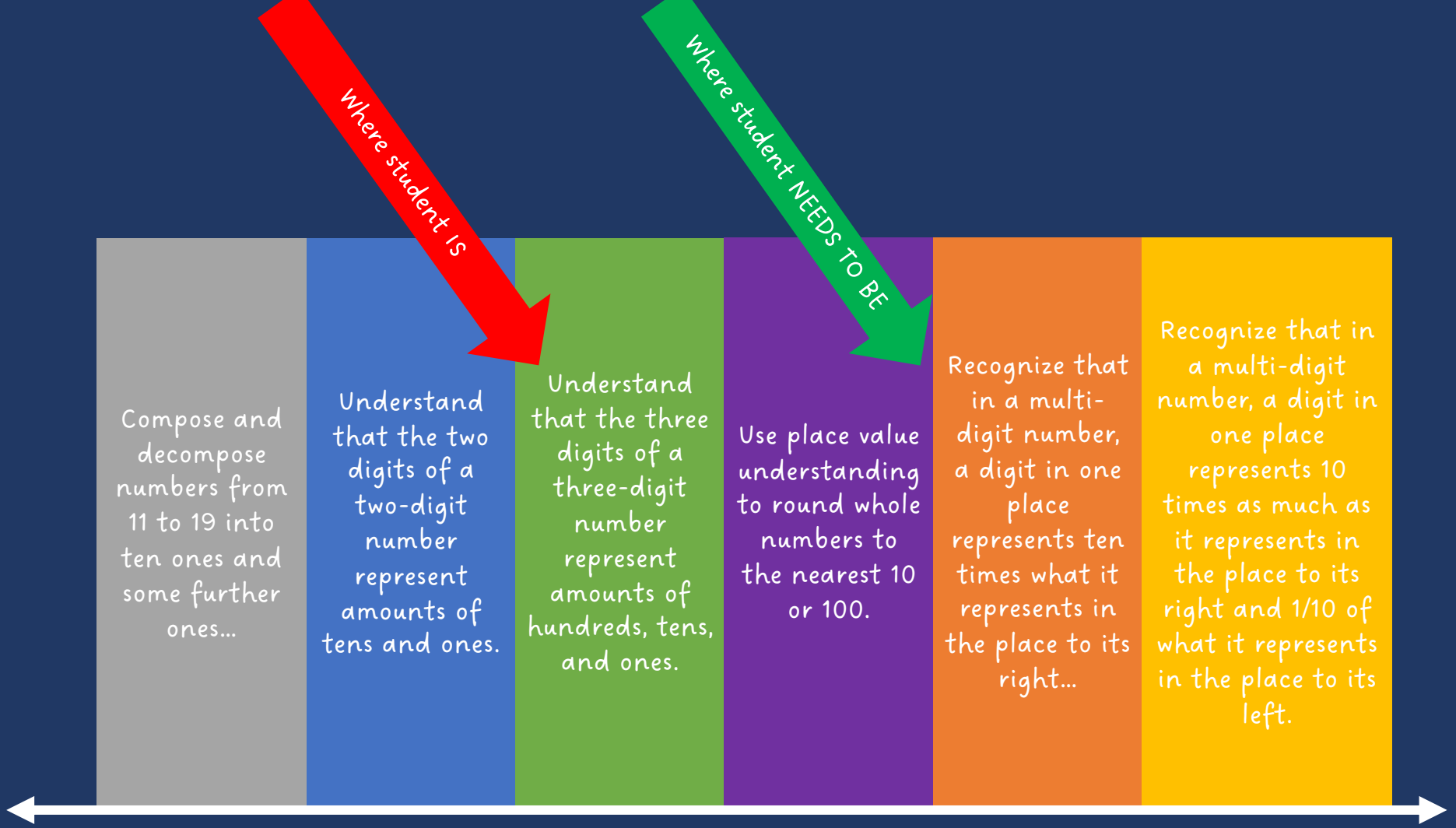
Understand that the two digits of a two-digit number represent amounts of tens and ones

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones

Compose and decompose numbers from 11 to 19 into ten ones and some further ones

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right

Use place value understanding to round whole numbers to the nearest 10 or 100



Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20

Solve real-world and math problems involving the four operations with rational numbers.

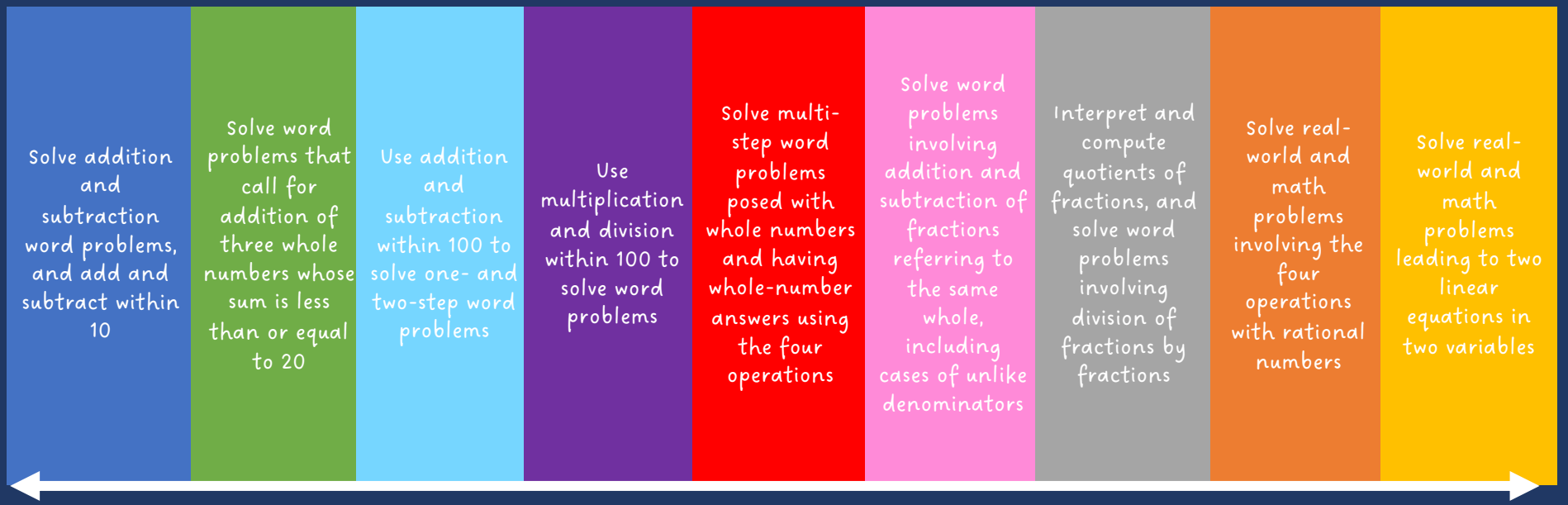
Use addition and subtraction within 100 to solve one- and two-step word problems

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators

Solve real-world and math problems leading to two linear equations in two variables

Use multiplication and division within 100 to solve word problems

Solve addition and subtraction word problems, and add and subtract within 10



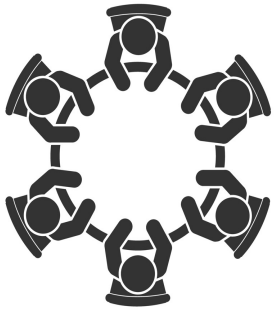
Where student IS

Where student NEEDS TO BE

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20

Use addition and subtraction within 100 to solve one- and two-step word problems

Use multiplication and division within 100 to solve word problems



What content would be important to include on this continuum?

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.
** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

https://achievethecore.org/category/774/mathematics-focus-by-grade-level

Curriculum Focal Points and Connections for Grade 2

The set of three curriculum focal points and related connections for mathematics in grade 2 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 2 Curriculum Focal Points	Connections to the Focal Points
Number and Operations: **Developing an understanding of the base-ten numeration system and place-value concepts** Children develop an understanding of the base-ten numeration system and place-value concepts (at least to 1000). Their understanding of base-ten numeration includes ideas of counting in units and multiples of hundreds, tens, and ones, as well as a grasp of number relationships, which they demonstrate in a variety of ways, including comparing and ordering numbers. They understand multidigit numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones).	***Number and Operations:*** Children use place value and properties of operations to create equivalent representations of given numbers (such as 35 represented by 35 ones, 3 tens and 5 ones, or 2 tens and 15 ones) and to write, compare, and order multidigit numbers. They use these ideas to compose and decompose multidigit numbers. Children add and subtract to solve a variety of problems, including applications involving measurement, geometry, and data, as well as nonroutine problems. In preparation for grade 3, they solve problems involving multiplicative situations, developing initial understandings of multiplication as repeated addition. ***Geometry*** and ***Measurement:*** Children estimate, measure, and compute lengths as they solve problems involving data, space, and movement through space. By composing and decomposing two-dimensional shapes (intentionally substituting arrangements of smaller shapes for larger shapes or substituting larger shapes for many smaller shapes), they use geometric knowledge and spatial reasoning to develop foundations for understanding area, fractions, and proportions. ***Algebra:*** Children use number patterns to extend their knowledge of properties of numbers and operations. For example, when skip counting, they build foundations for understanding multiples and factors.
Number and Operations **and** ***Algebra:*** **Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction** Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition (commutativity and associativity). Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multidigit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.	
Measurement: **Developing an understanding of linear measurement and facility in measuring lengths** Children develop an understanding of the meaning and processes of measurement, including such underlying concepts as partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units and use rulers and other measurement tools with that understanding. They understand the need for equal-length units, the use of standard units of measure (centimeter and inch), and the inverse relationship between the size of a unit and the number of units used in a particular measurement (i.e., children recognize that the smaller the unit, the more iterations they need to cover a given length).	

https://www.nctm.org/curriculumfocalpoints/

Curriculum Focal Points and Connections for Grade 3

The set of three curriculum focal points and related connections for mathematics in grade 3 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 3 Curriculum Focal Points	Connections to the Focal Points
Number and Operations **and** ***Algebra:*** **Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts** Students understand the meanings of multiplication and division of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). They use properties of addition and multiplication (e.g., commutativity, associativity, and the distributive property) to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts. By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.	***Algebra:*** Understanding properties of multiplication and the relationship between multiplication and division is a part of algebra readiness that develops at grade 3. The creation and analysis of patterns and relationships involving multiplication and division should occur at this grade level. Students build a foundation for later understanding of functional relationships by describing relationships in context with such statements as, "The number of legs is 4 times the number of chairs." ***Measurement:*** Students in grade 3 strengthen their understanding of fractions as they confront problems in linear measurement that call for more precision than the whole unit allowed them in their work in grade 2. They develop their facility in measuring with fractional parts of linear units. Students develop measurement concepts and skills through experiences in analyzing attributes and properties of two-dimensional objects. They form an understanding of perimeter as a measurable attribute and select appropriate units, strategies, and tools to solve problems involving perimeter. ***Data Analysis:*** Addition, subtraction, multiplication, and division of whole numbers come into play as students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems. ***Number and Operations:*** Building on their work in grade 2, students extend their understanding of place value to numbers up to 10,000 in various contexts. Students also apply this understanding to the task of representing numbers in different equivalent forms (e.g., expanded notation). They develop their understanding of numbers by building their facility with mental computation (addition and subtraction in special cases, such as 2,500 + 6,000 and 9,000 – 5,000), by using computational estimation, and by performing paper-and-pencil computations.
Number and Operations: **Developing an understanding of fractions and fraction equivalence** Students develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models, including the number line, to identify equivalent fractions.	
Geometry: **Describing and analyzing properties of two-dimensional shapes** Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space and the use of those attributes and properties in solving problems, including applications involving congruence and symmetry.	

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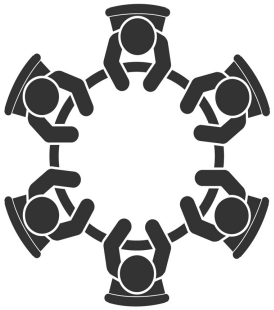
Five Essentials for Grade 2	Five Essentials for Grade 3



What are the 5 proficiency essentials for GRADE 2 students by the end of the school year?

What are the 5 proficiency essentials for GRADE 3 students by the end of the school year?

Introductions

Vertical Planning

Vocabulary

Representations

Vocabulary

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...

Significant correlation (r = .49) between mathematics vocabulary and mathematics performance. Mathematics vocabulary appears most important for word-problem performance (r = .58).

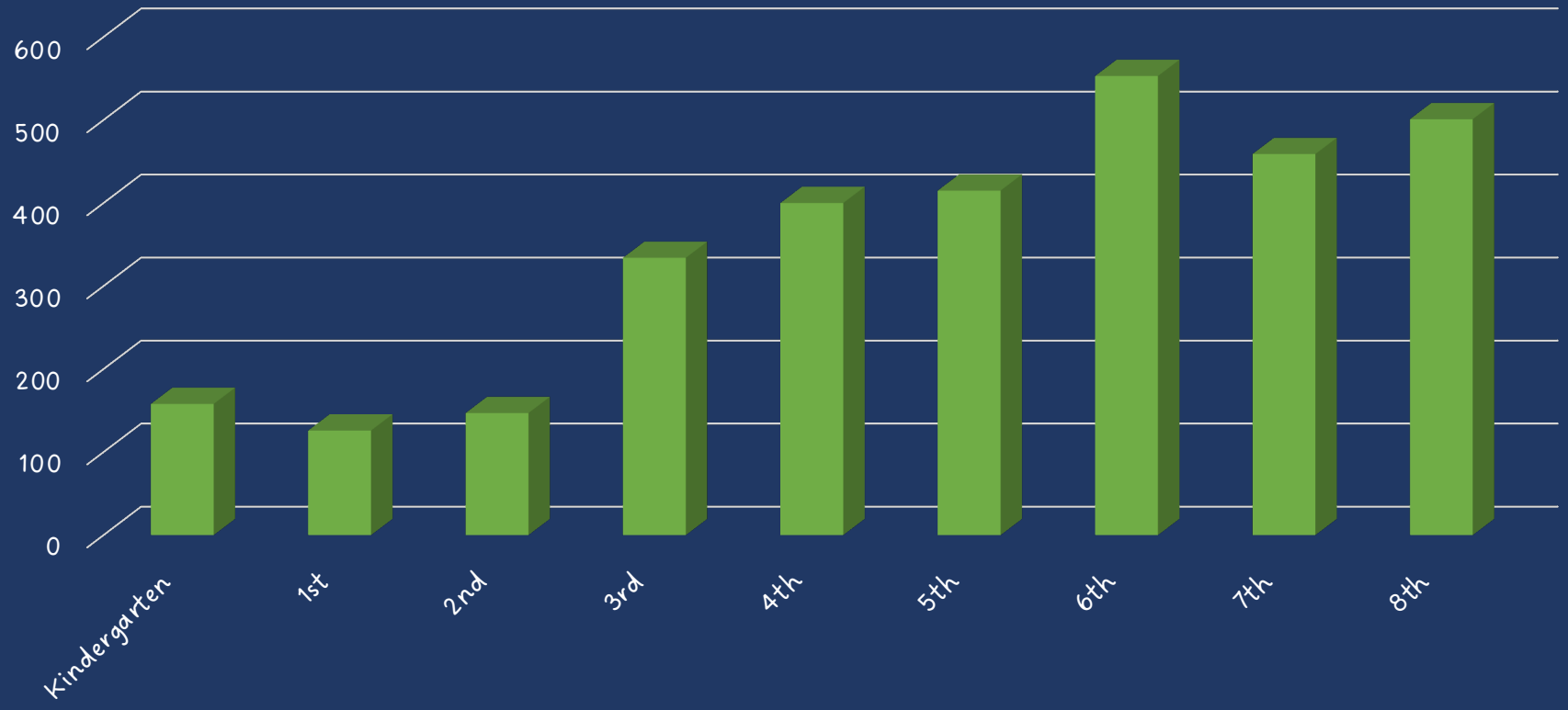
(Lin et al., 2021)

Early mathematics vocabulary related to mathematics and literacy.

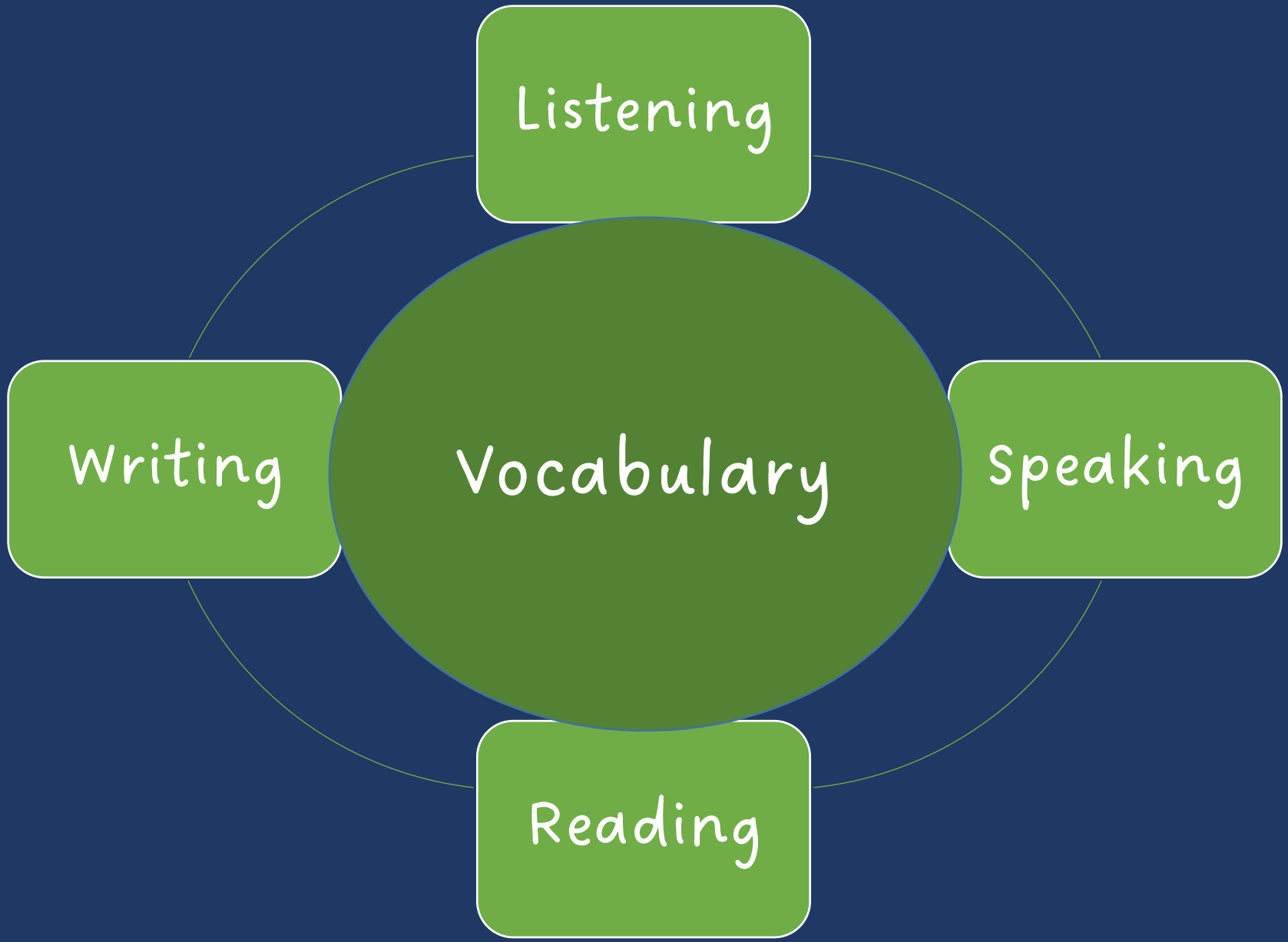
(Hornburg et al., 2018; Purpura et al., 2017)

Students who experience difficulty with mathematics demonstrate lower mathematics vocabulary performance.

(Hughes et al., 2020; Powell & Nelson, 2017; Powell et al., 2017; Unal et al., 2021)



Powell, Bos, & Lin (2019)



1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

round

square

second

base

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

trapezoid

numerator

parallelogram

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

6. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

6. Some math terms are related but have distinct meanings

7. Some math concepts are verbalized in more than one way

skip count vs. multiples

one-fourth vs. one quarter

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms have more than one meaning

4. Some math terms are only used in math

5. Some math terms are homophones

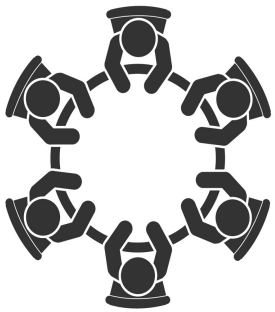
6. Some math terms are related but have distinct meanings

7. Some math concepts are verbalized in more than one way

8 Some informal terms may be used for formal math terms

vertex vs. corner

rhombus vs. diamond



Why might your students have difficulty with math vocabulary?

Use formal math language

Use terms precisely

What number is in the tens place?

What digit is in the tens place?
What is the value of the digit in the tens place?

135

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.

The alligator eats the bigger number

is less than
OR
is greater than

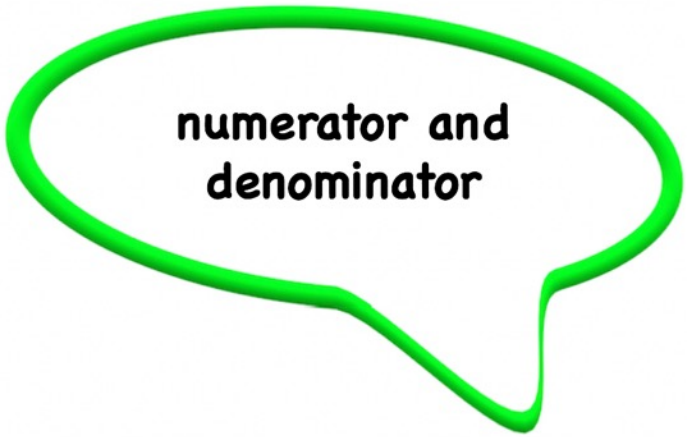
Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.

$$\begin{array}{r} 167 \\ +\ 294 \\ \hline \end{array}$$

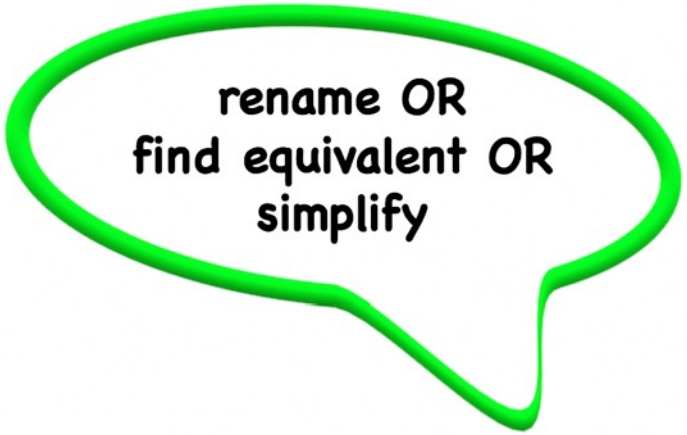
Why this is important...

- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.



Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

Four point seven
Four point oh seven

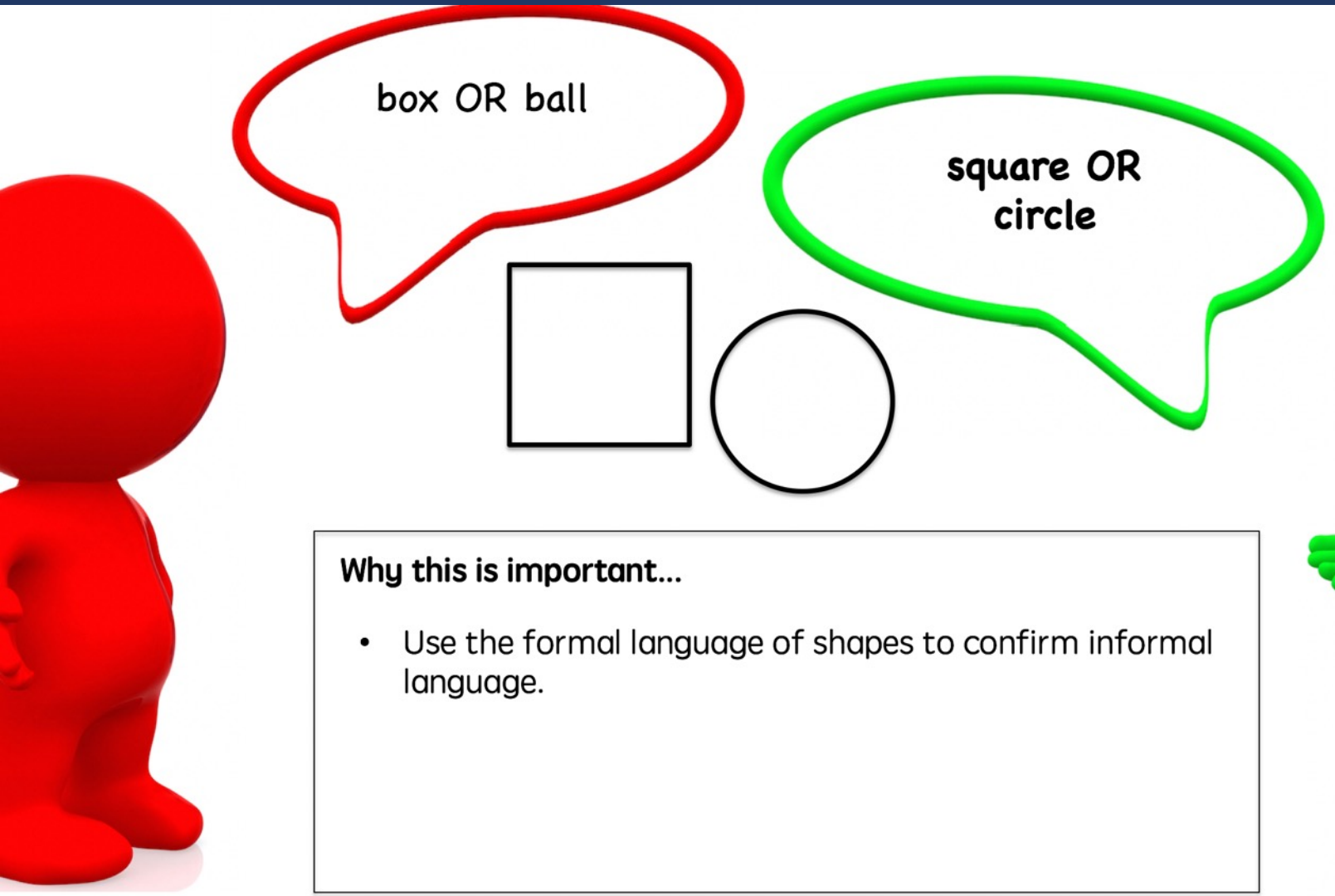
Four and seven tenths
Four and seven hundredths

4.7

4.07

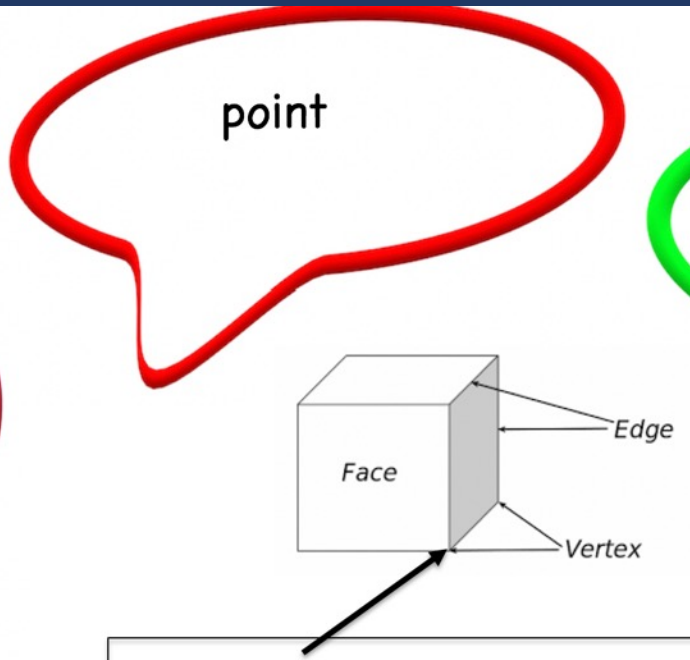
Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



Why this is important...

- Use the formal language of shapes to confirm informal language.

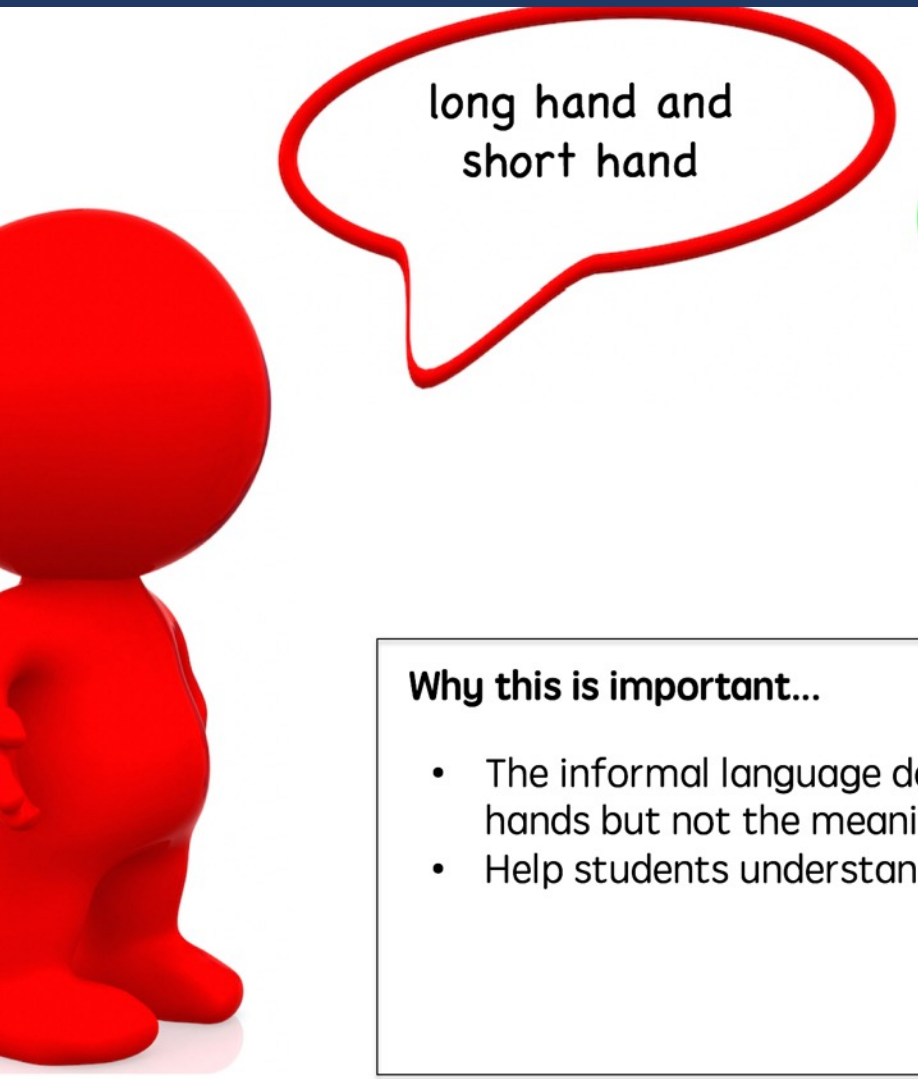


Why this is important...

- This is the endpoint where two or more line segments or rays meet.

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



Why this is important...

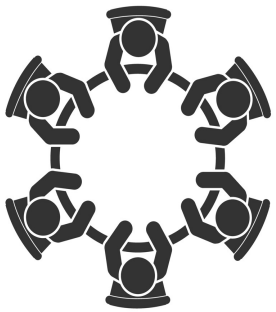
- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.

Vocabulary

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...



Identify examples of "Instead of ____, say ____."

Use formal math language

Use terms precisely

Math Vocabulary

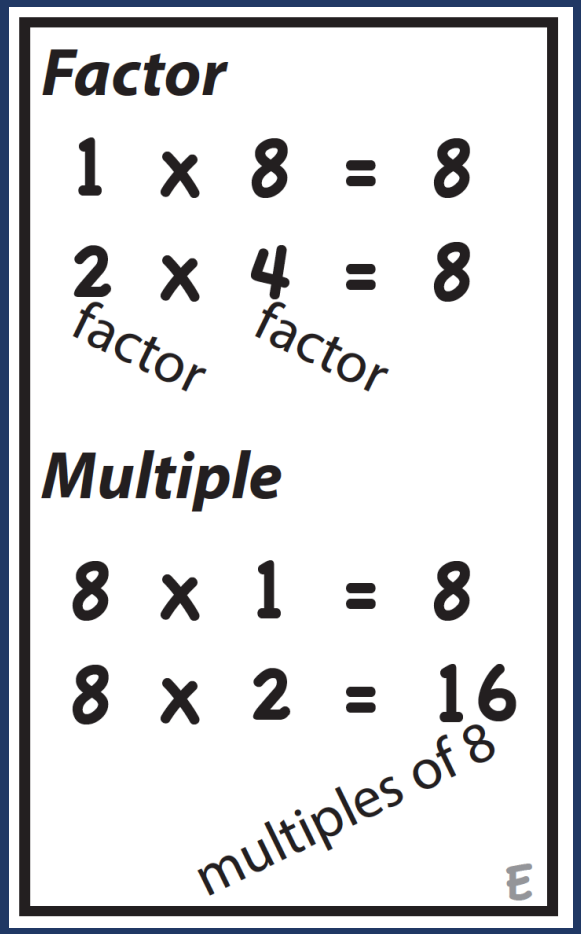
Use Terms With Precision

Strategies for Teaching Mathematics Language


What are your strengths?


What are your opportunities for growth?


What will you do on Monday? Next month? Next year?



Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

4:3

Proper fraction

$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

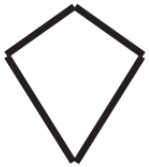
D

Equation	$9x - 4 = 7x$
Expression	$9x - 4$
Formula	$a^2 + b^2 = c^2$
Function	$f(x)$
Inequality	$9x - 4 > 6x$

C

Quadrilaterals

Kite



Rhombus

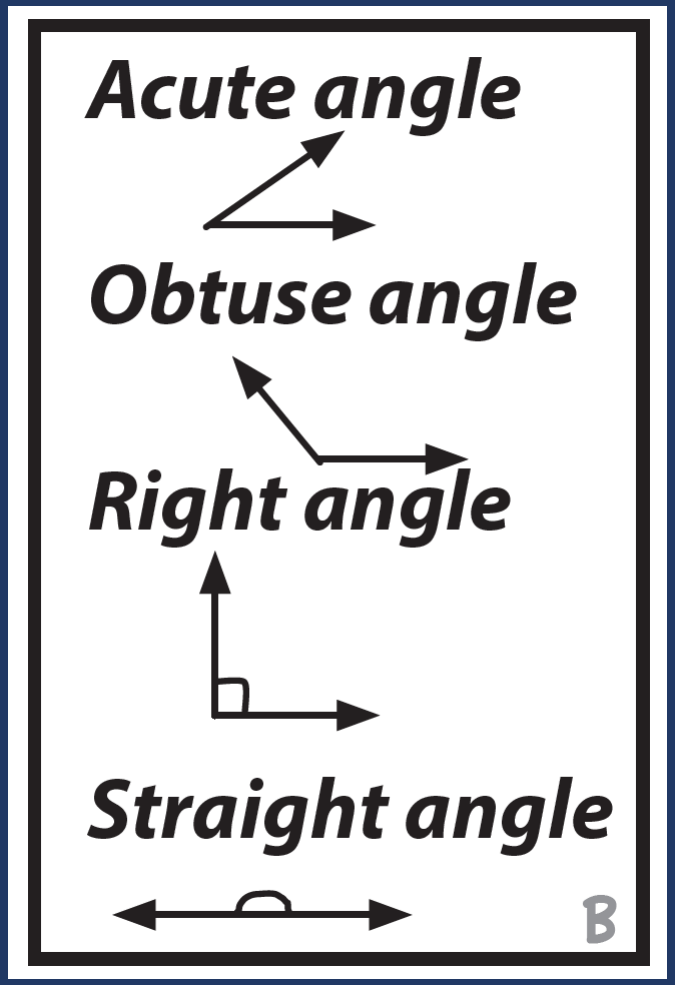
Parallelogram

Square

Rectangle

Trapezoid

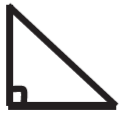
A



Acute triangle

Obtuse triangle

Right triangle



Equilateral triangle

Isosceles triangle

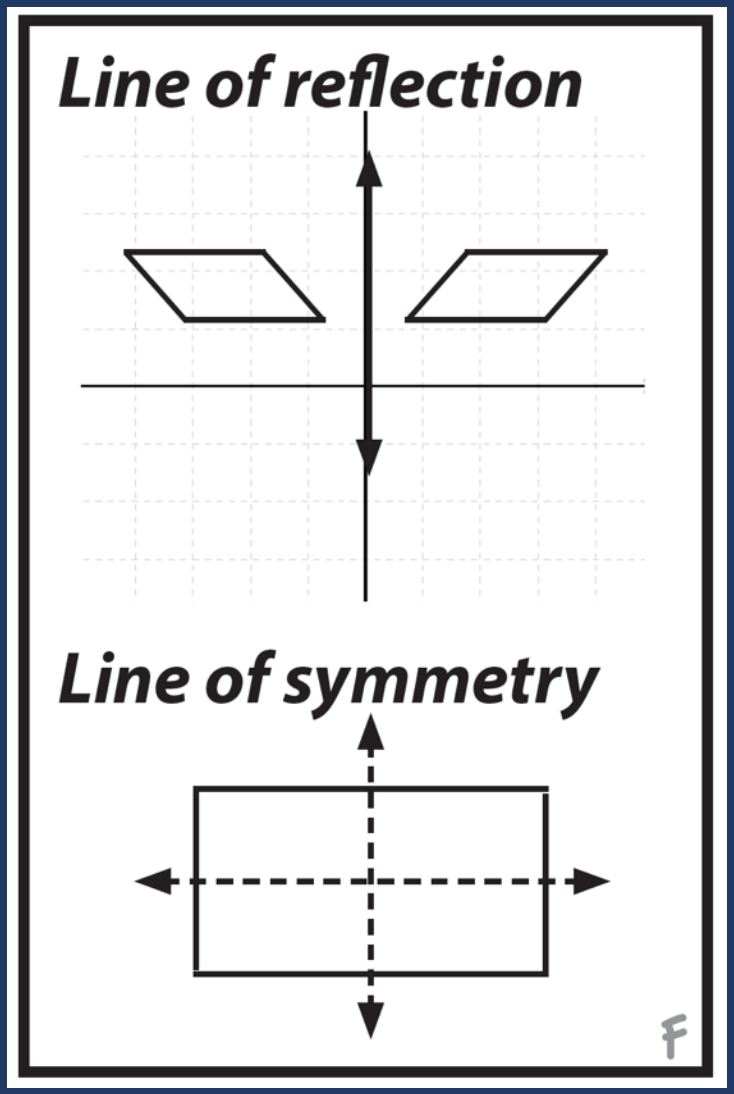
Scalene triangle

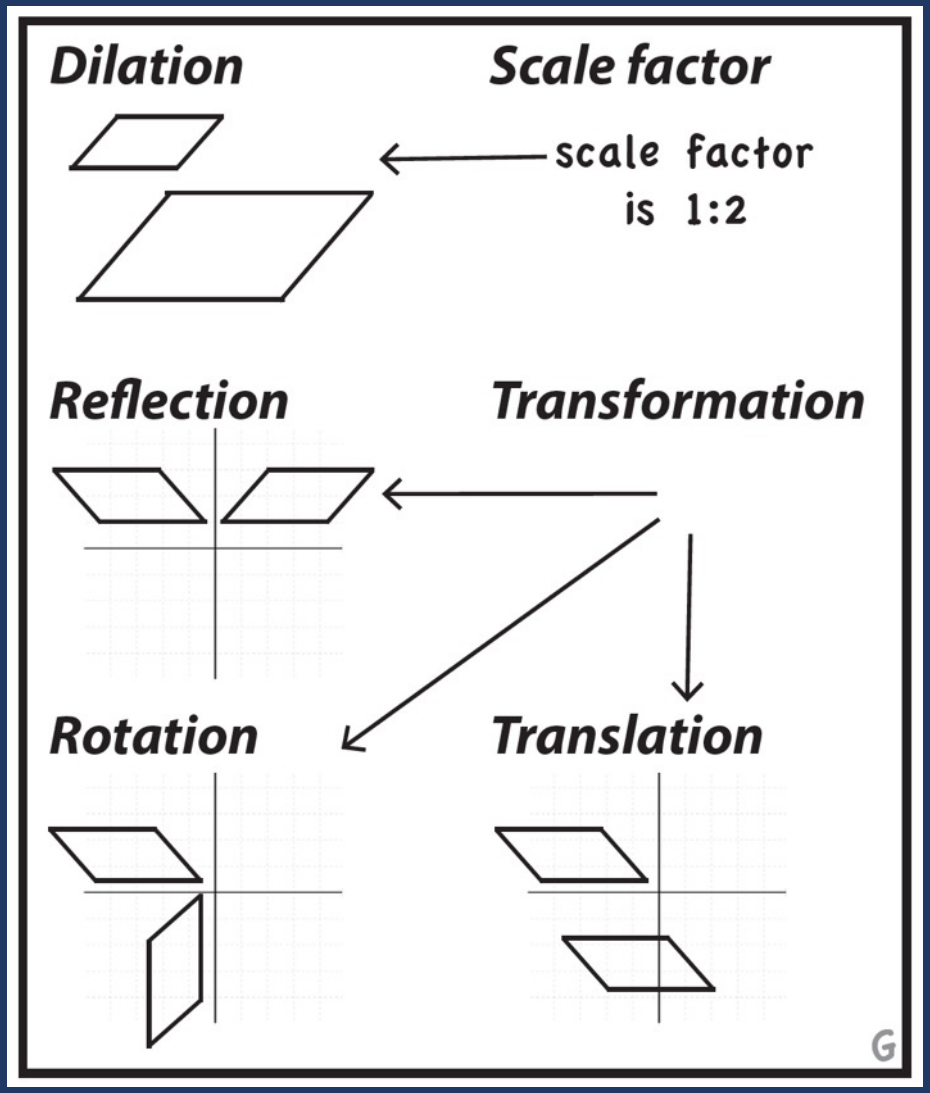
C

Congruent figures

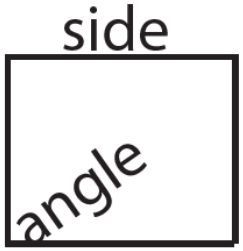
Similar figures

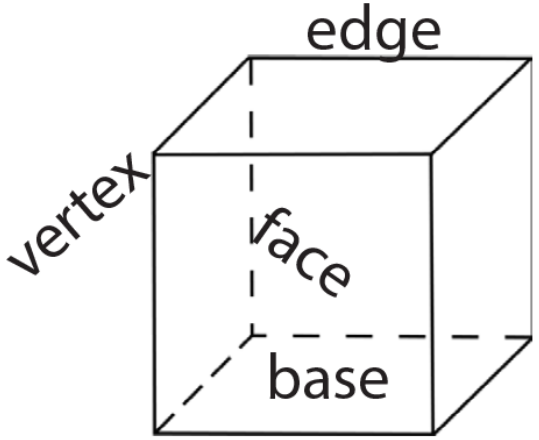
E

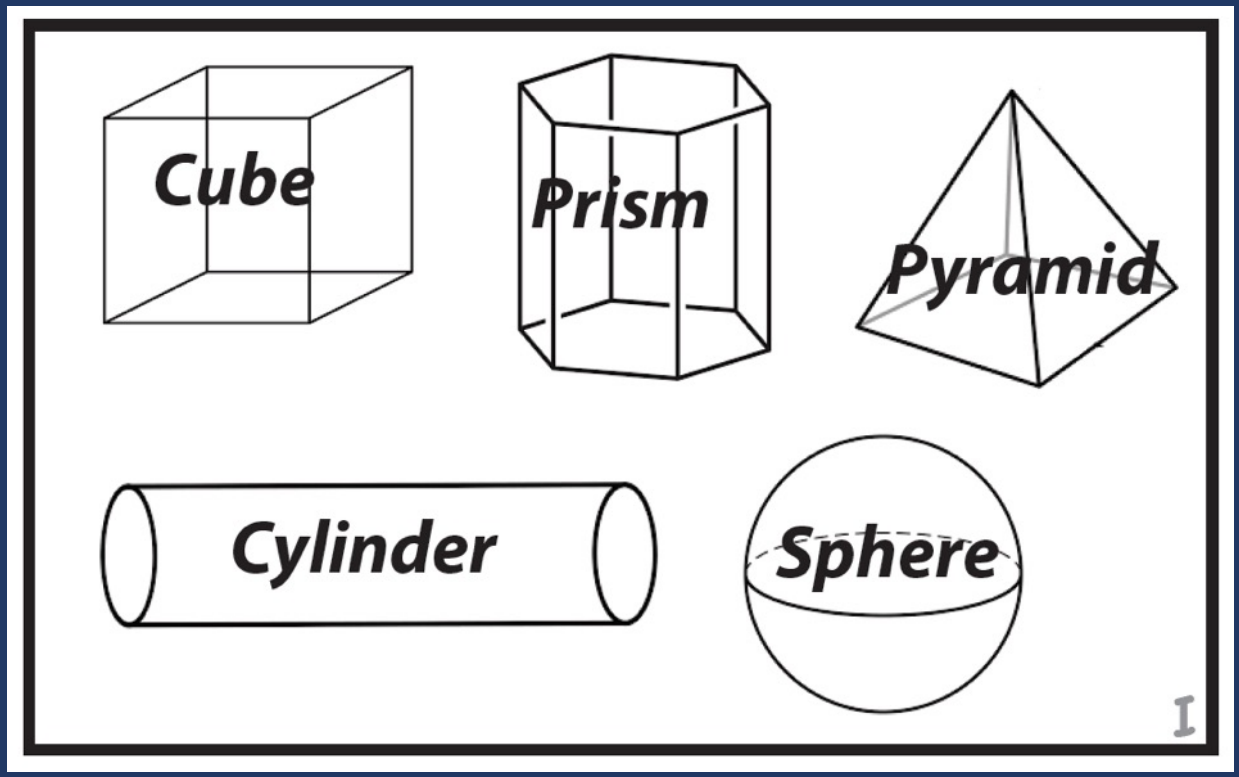




Angle
Base
Edge
Face
Side
Vertex







Math Vocabulary

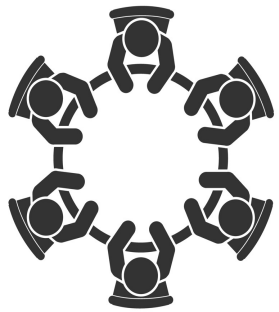
Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What will you do on Monday? Next month? Next year?



Discuss terms you want your students to use with precision.

Use formal math language

Use terms precisely

Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language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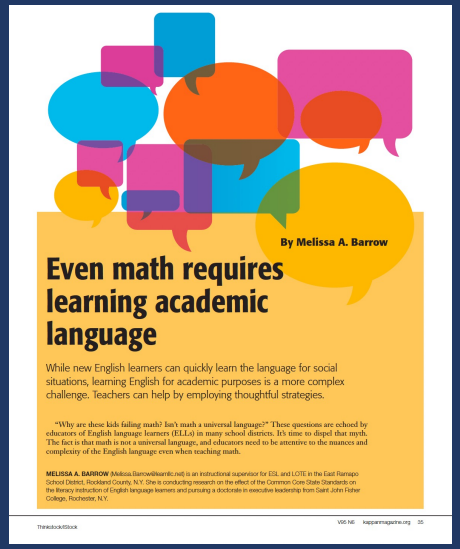

By Melissa A. Barrow

Even math requires learning academic language

While new English learners can quickly learn the language for social situations, learning English for academic purposes is a more complex challenge. Teachers can help by employing thoughtful strategies.

"Why are these kids failing math? Isn't math a universal language?" These questions are echoed by educators of English language learners (ELLs) in many school districts. It's time to dispel that myth. The fact is that math is not a universal language, and educators need to be attentive to the nuances and complexity of the English language even when teaching math.

MELISSA A. BARROW (Melissa.Barrow@learnltc.net) is an instructional supervisor for ESL and LOTE in the East Ramapo School District, Rockland County, N.Y. She is conducting research on the effect of the Common Core State Standards on the literacy instruction of English language learners and pursuing a doctorate in executive leadership from Saint John Fisher College, Rochester, N.Y.

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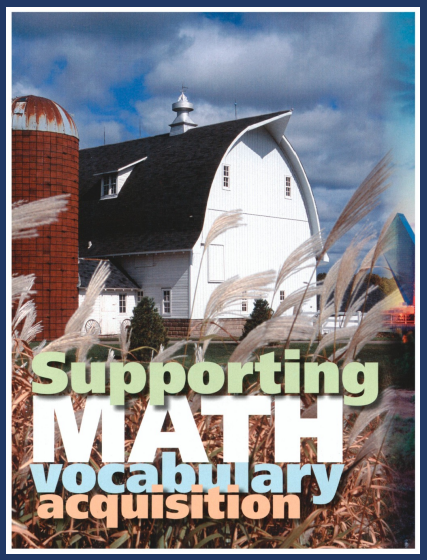
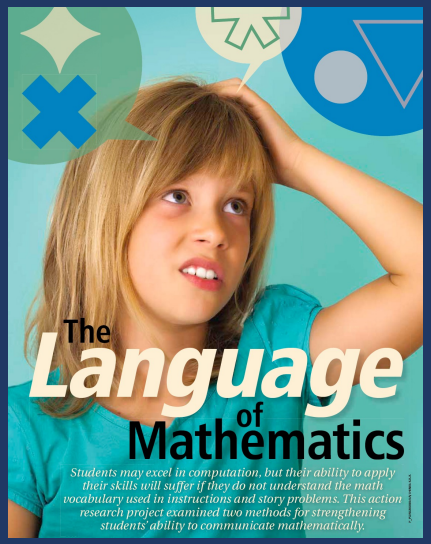
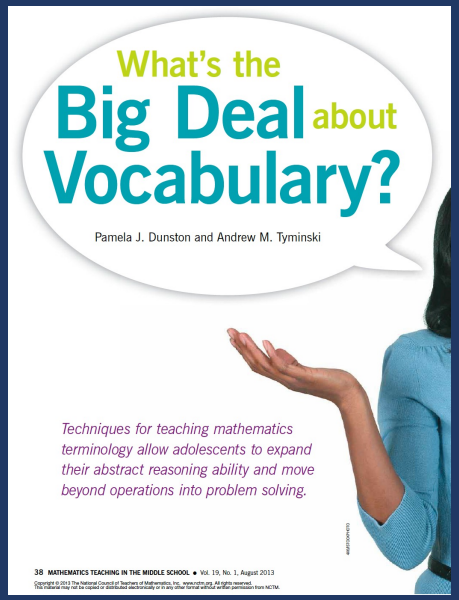


Supporting MATH vocabulary acquisition



The Language of Mathematics

Students may excel in computation, but their ability to apply their skills will suffer if they do not understand the math vocabulary used in instructions and story problems. This action research project examined two methods for strengthening students' ability to communicate mathematically.

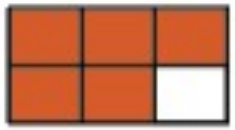
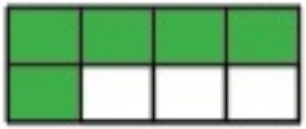
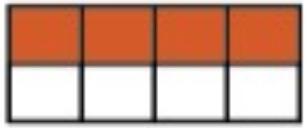


What's the Big Deal about Vocabulary?

Pamela J. Dunston and Andrew M. Tyminski

Techniques for teaching mathematics terminology allow adolescents to expand their abstract reasoning ability and move beyond operations into problem solving.

Use semantic maps

Term	Definition	Example	Nonexample
integer	(... -3, -2, -1, 0, 1, 2, 3...)	15	$\frac{1}{3}$
denominator	The equal parts of a whole or set.	$\frac{5}{8}$ 8 is the denominator 	$\frac{5}{8}$ 
numerator	The equal parts of a given fraction.	$\frac{5}{8}$ 5 is the numerator 	$\frac{5}{8}$ 

(Stevens et al., 2022)

Use word walls

difference

The result of subtracting or the result when comparing two numbers.

5 – 4 = 1

1 is the difference

horizontal line

A straight line that goes from left to right or right to left.

equivalent

Two numbers that have the same value.

$\frac{2}{4}$ **is equivalent to** $\frac{1}{2}$

total

The result or sum when adding numbers.

5 + 7 = 12

(mathspiral.com)

Use flash cards

addend

quotient

divisor

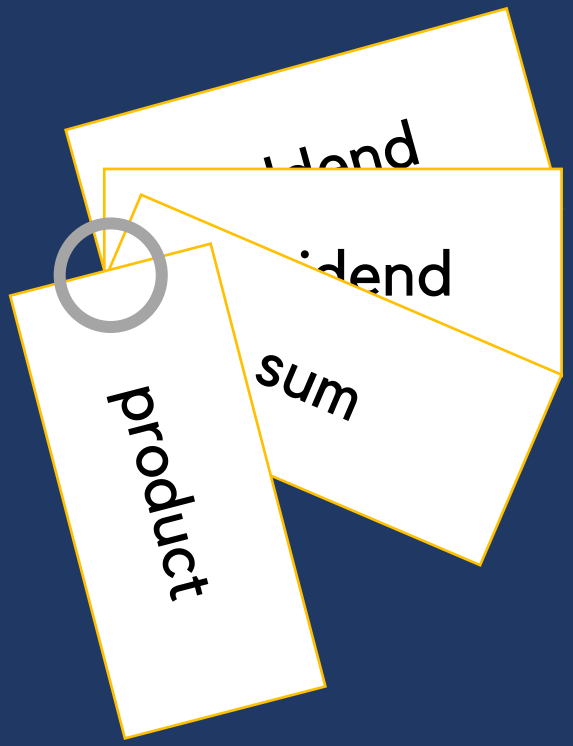
dividend

factor

sum

product

difference

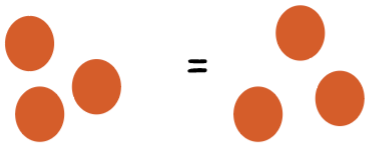


(Petersen-Brown et al., 2019)

Use glossaries

E

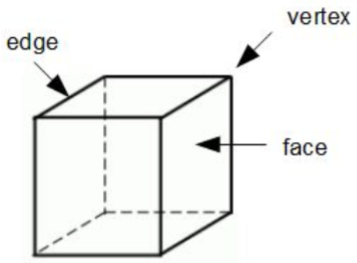
equal - with the same value



equation - expressions with an equal sign

$$5 = 2 + 3$$

edge - line segment between faces



Glosario de Matemáticas

Término	Definición
adición *(add)*	Juntar o agregar.
algoritmo *(algorithm)*	Es una serie de pasos organizados que describe el proceso que se debe seguir, para dar solución a un problema específico.
alinear *(array)*	Un conjunto de objetos, imágenes o números alineados en columnas y filas.
ancho *(width)*	La medida de un lado de un objeto, generalmente el lado más corto.
ángulo *(angle)*	Dos semirrectas o segmentos de línea recta que comparten un punto final.
ángulo agudo *(acute angle)*	Un ángulo que mide menos de 90°.
ángulo obtuso *(obtuse angle)*	Un ángulo que mide entre 90° y 180°.
ángulo recto *(right angle)*	Un ángulo que mide exactamente 90°.
área *(area)*	La cantidad de unidades cuadradas que cubre una figura geométrica cerrada.
balance presupuestario *(balance the budget)*	Un presupuesto es cuando la cantidad total de dinero gastado, ahorrado y compartido es igual al ingreso total.
báscula *(balance scale)*	Instrumento de medición que se utiliza para medir el peso o la carga.
base (computación) *(base (computation))*	Un número que se multiplica por un exponente.
base (geometría) *(base (geometry))*	La forma inferior de una figura tridimensional.

SPIRAL
Specialized Math Intervention to Reach All Learners

(mathspiral.com)

Use anchor charts

Addition Computation

1
17 ← addend
+ 59 ← addend
76 ← sum

Quadrilaterals

Kite

Rhombus

Parallelogram

Square

Rectangle

Trapezoid

Use graphic organizers

horizontal

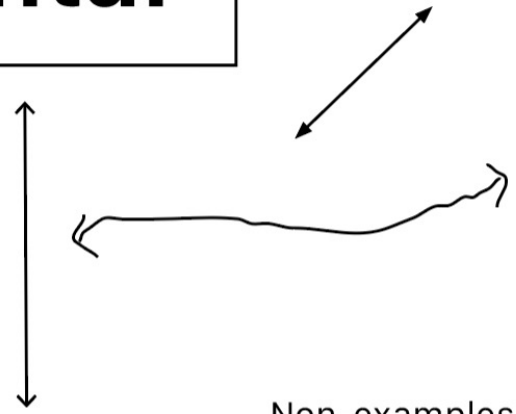
Definition

a straight line that goes left to right or right to left

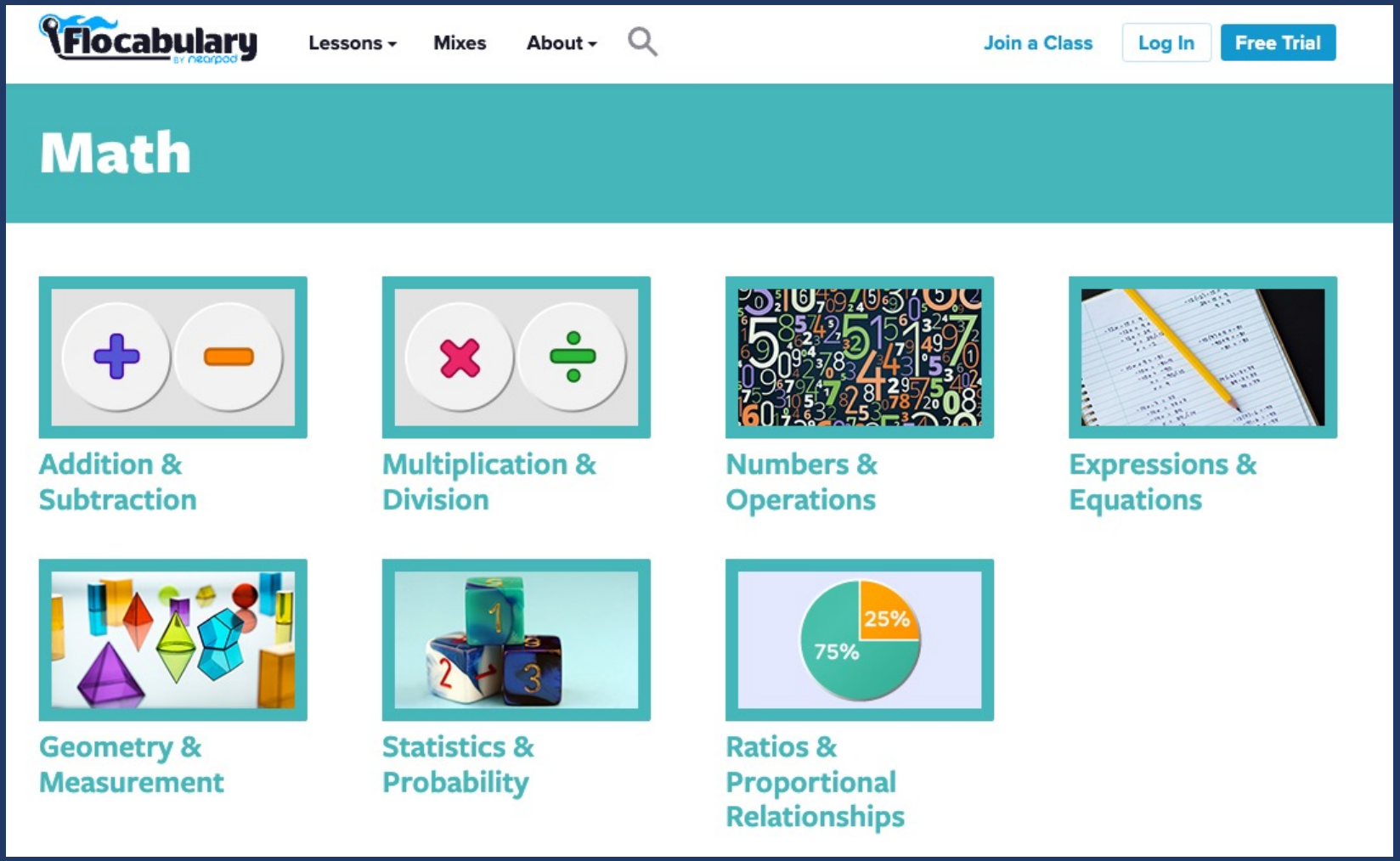
Picture

Examples

Non-examples



Use technology



Use games

Name: _____ Date: _____

3RD GRADE MATH

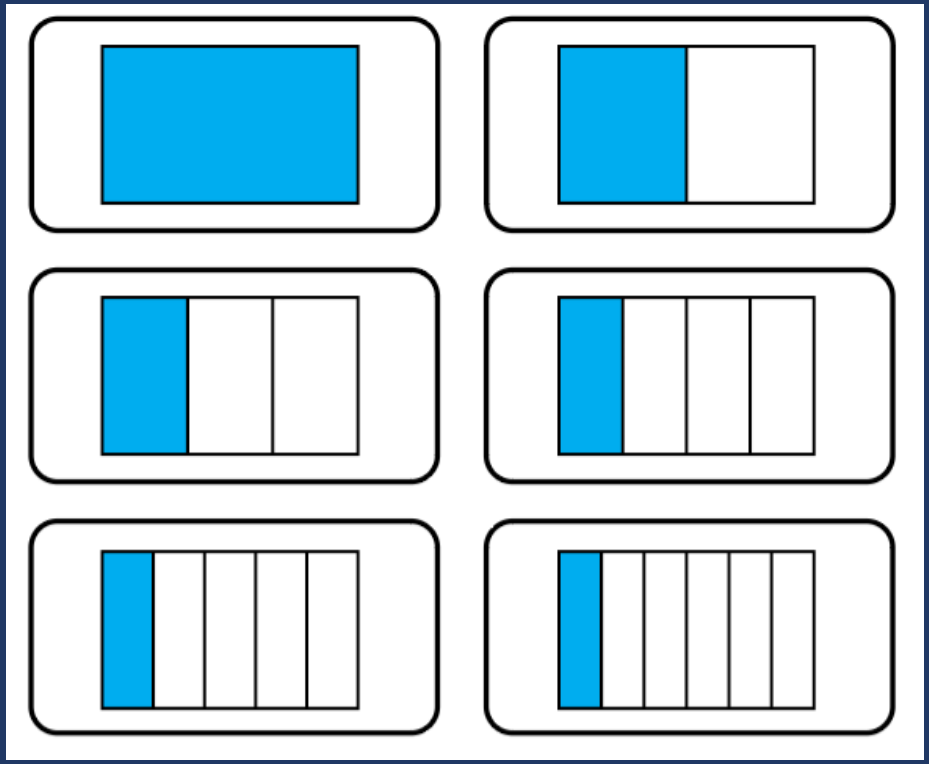
T W D H P T R I Y T S F T O Y T S P X G U C I C
M S G C E N T I M E T E R S Y S C Y Z D R G D J
I M I E U J R L H F O A W Z X U A L F J O Q F T
U P F N Z L R V C N N P Z D D H L P H F A V J Z
Y R U P M I A C O R D P I V J T E I H Y M F M S
E K X H R T T Y U J E S V X S V A T Z W M S E T
W Z Z W G E Q G M P C V C Z E H E L W L T T A A
N U V V B R J F H Y N E O Z H T K U K F A Z S W
Q Y J P S H A C D X A M Y G C U K M Z L J E U P
T Z W B P N R Z Y F L I Y D N A J J Z M W D R S
O W W U W T G I T I A T F F I S Q G X K K I E Z
E W R Q C S U U C U B J L E Z P B H G B C V M Q
F R A C T I O N S K Z O W M D X B M M C O I E C
N Z C A T L I J B J M U F O X X A D O Y L D N K
D E V P T U C M Y S A H K Z S U O W R S C V T G
M X N L N S O L V E R R I Y I B Z N Q O X C P W
M V W Z C D Q Z Z U G X L U K P Y O B S W I M N
K F L J U R J G J R O Q E M X C U O H N M S F D
S U B T R A C T G U L Y Y A K W U N U S O L H H
E G F D D A I O I A I Y R R J I C A P A C I T Y
I L W B A T R D O X K C J G H T D X E G C S M Z
X J Q Y A H L T B J R A I W J B C K I U S S I U
N U O D N R L V T I T L C U P C L W C B F S Z Q
J F N P R R K P Y M R E L U R Q H M A E Y Y Z H

ADD
BALANCED
CAPACITY
CENTIMETERS
CLOCK
DIVIDE
FRACTIONS
GRAM
INCHES
KILO GRAM
LITER
MEASUREMENT
MULTIPLY
NOON
RULER
SCALE
SOLVE
SUBTRACT
TIME

square	circle	decagon	triangular prism	cone
cylinder	cube	sphere	line	pyramid
parallelogram	octagon	FREE	trapezoid	oval
kite	pentagon	cylinder	rectangle	line segment
hexagon	rhombus	triangle	quadrilateral	rectangular prism

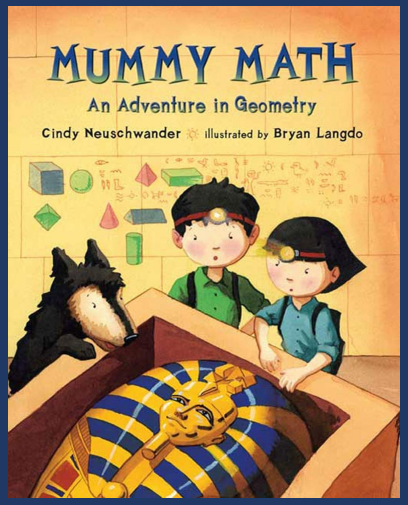
https://wordmint.com/public_puzzles/13055

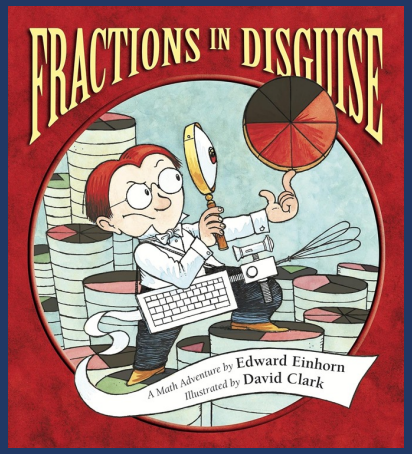
Use games

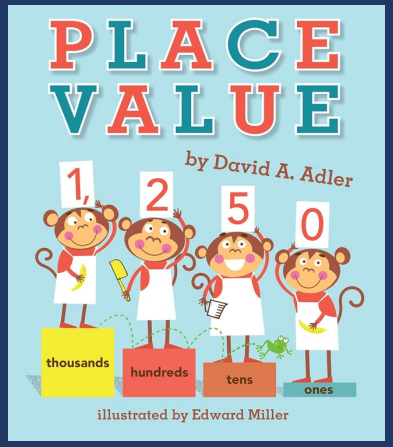


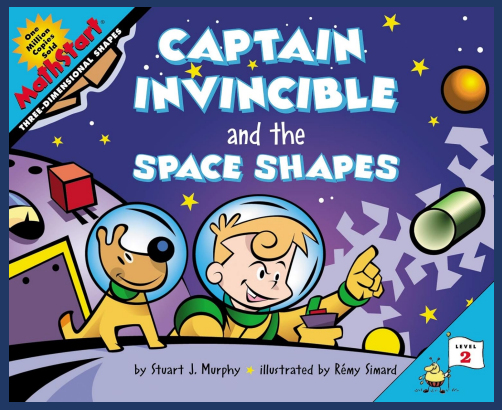
one	one-half
one-third	one-fourth
one-fifth	one-sixth

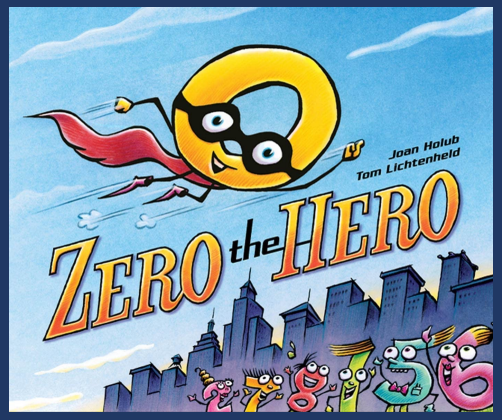
Use read-alouds

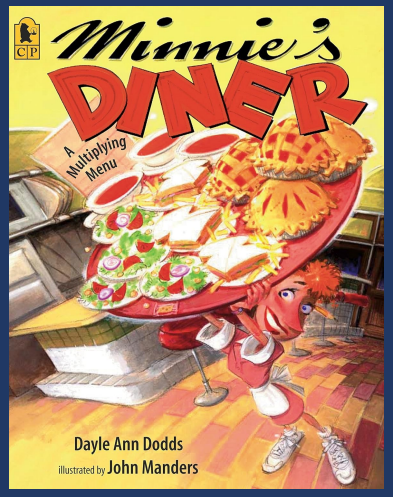












(Purpura et al., 2017)

Model and practice

Teacher	**Let's work on addition. Today, let's think about addition as combining. What does it mean to combine?**
Students	Put together.
Teacher	**When we combine, we put things together. When you cook, you put ingredients together. For example, to make macaroni and cheese, you combine what?**
Students	Macaroni noodles and cheese!
Teacher	**That's right. You combine macaroni and cheese! Now, let's think about combining numbers. Look at this problem.** (Show problem.)

(Powell & Driver, 2015; Stevens et al., 2022)

Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language

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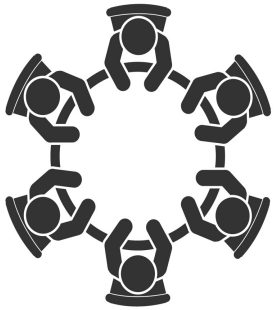
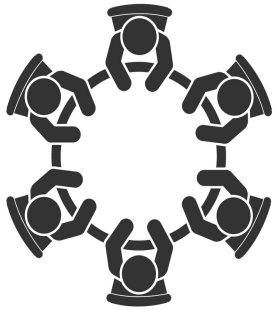
What are your opportunities for growth?

What will you do on Monday? Next month? Next year?



Discuss your strategy for focusing on mathematical vocabulary in your teaching.



What are your strengths with vocabulary?

What are the opportunities for growth?

What will you do on Monday? Next month? Next year?

Introductions

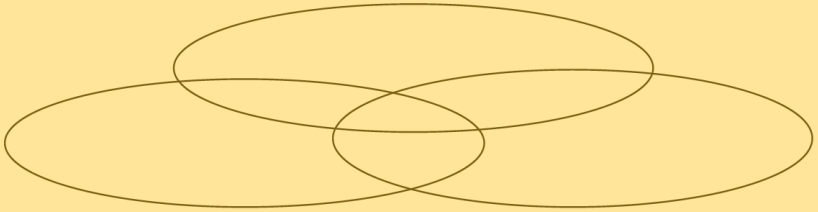
Vertical Planning

Vocabulary

Representations

Representations

Research and Information



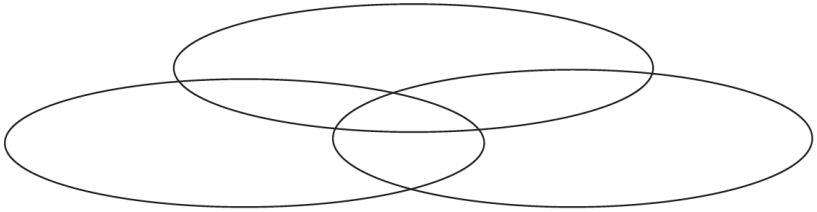
Hands-on manipulatives contribute to increases in mathematics performance.
(Bouck & Park, 2018; Carbonneau et al., 2013; Namkung & Bricko, 2021; Sherman & Bisanz, 2009; Strickland & Maccini, 2012)

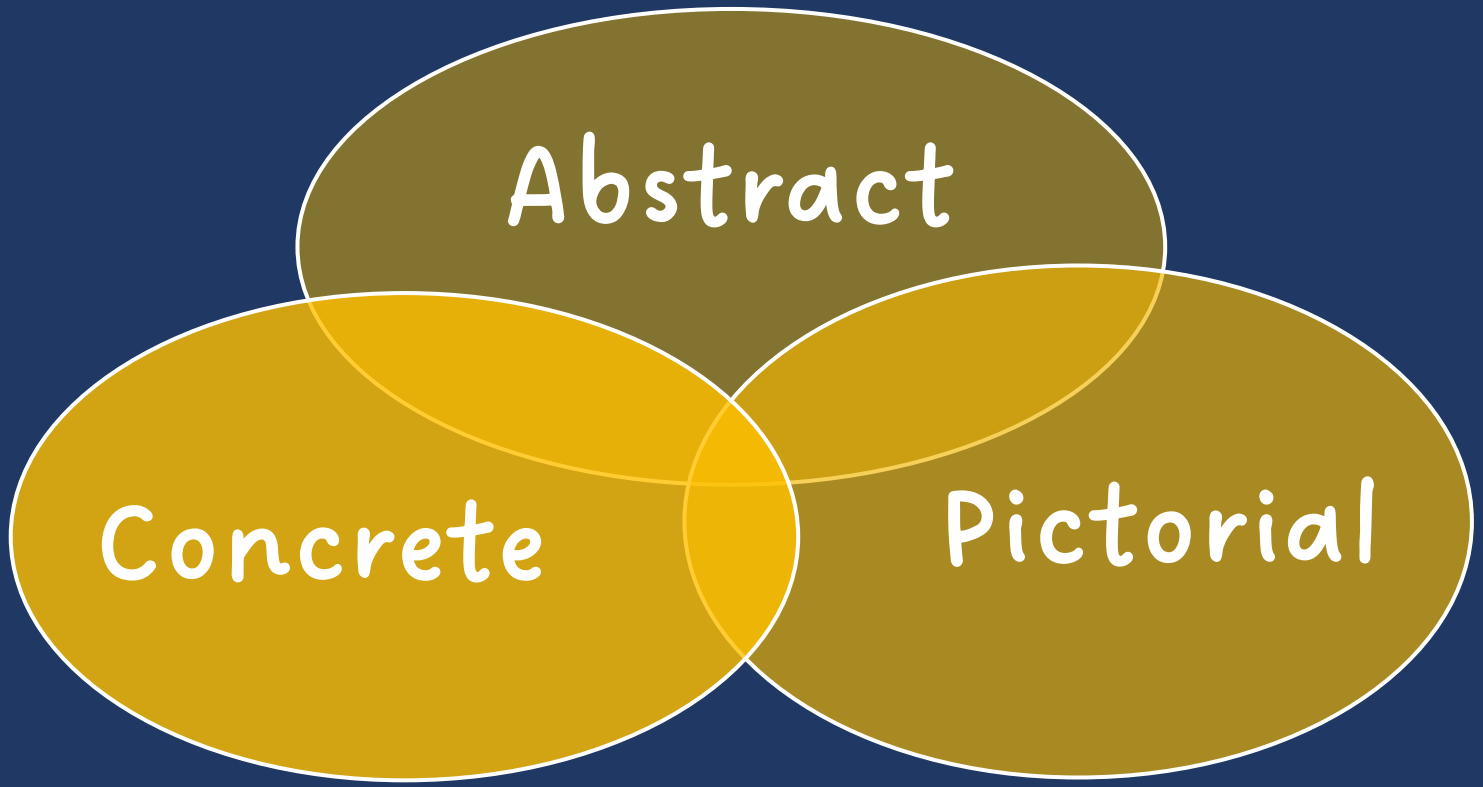
Virtual manipulatives contribute to increases in mathematics performance.
(Bouck et al., 2020; Satsangi et al., 2016)

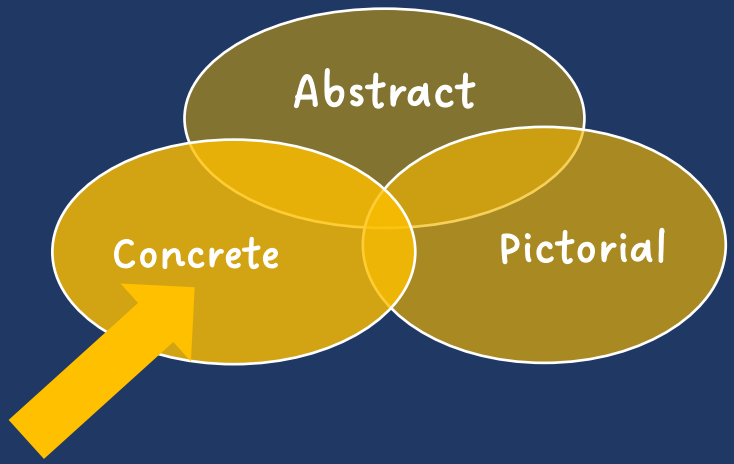
Other visuals (e.g., graphic organizers) contribute to increases in mathematics performance.
(Jitendra et al., 2009; Sharp & Dennis, 2017; van Garderen, 2007; Xin, 2008)

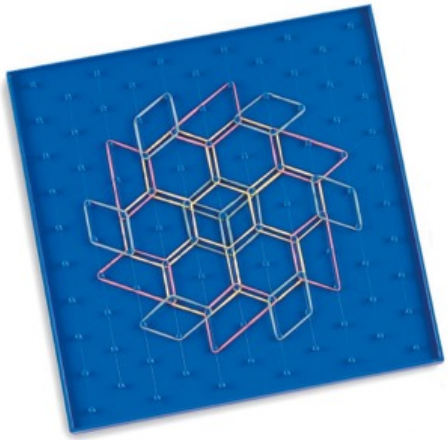
Representations

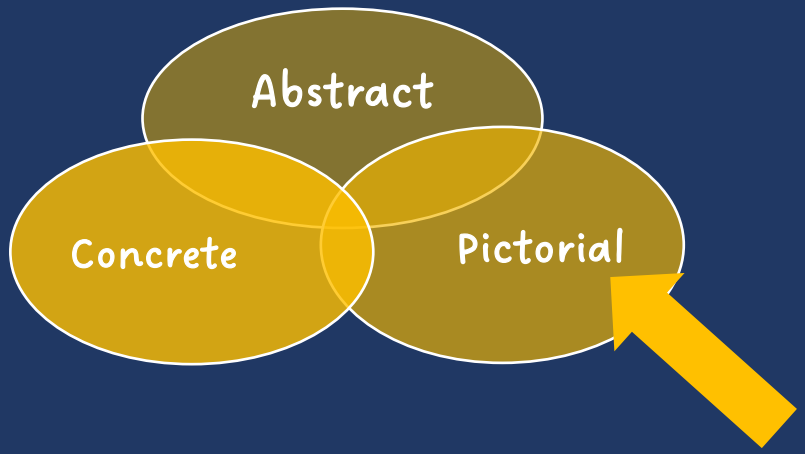
Research and Information

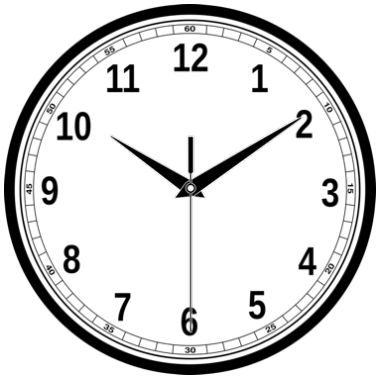


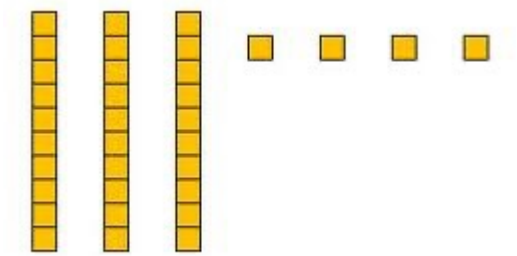


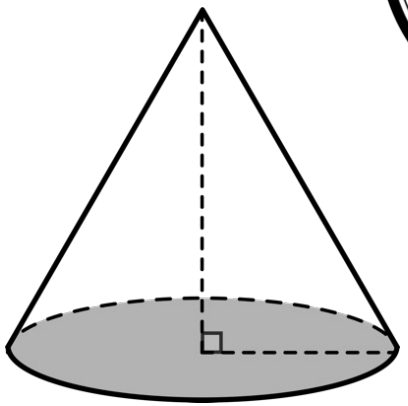


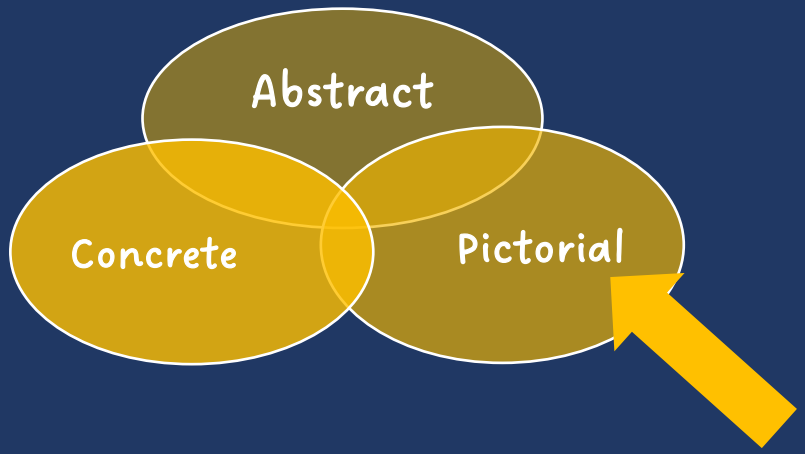


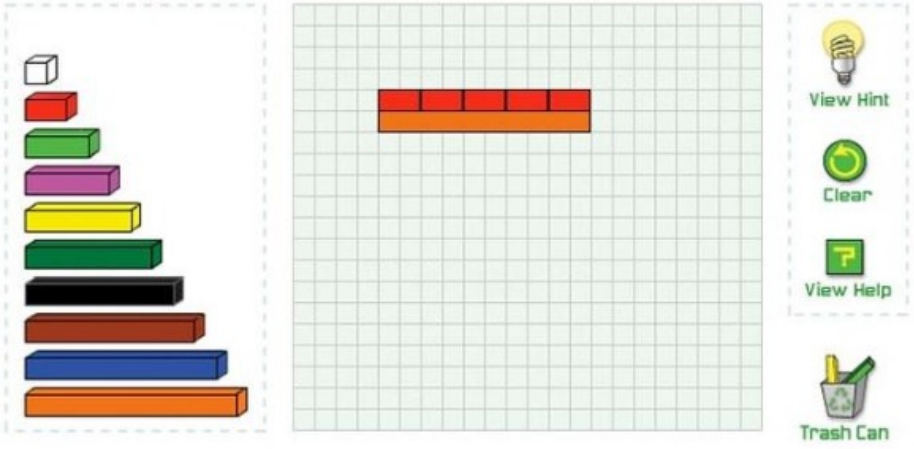


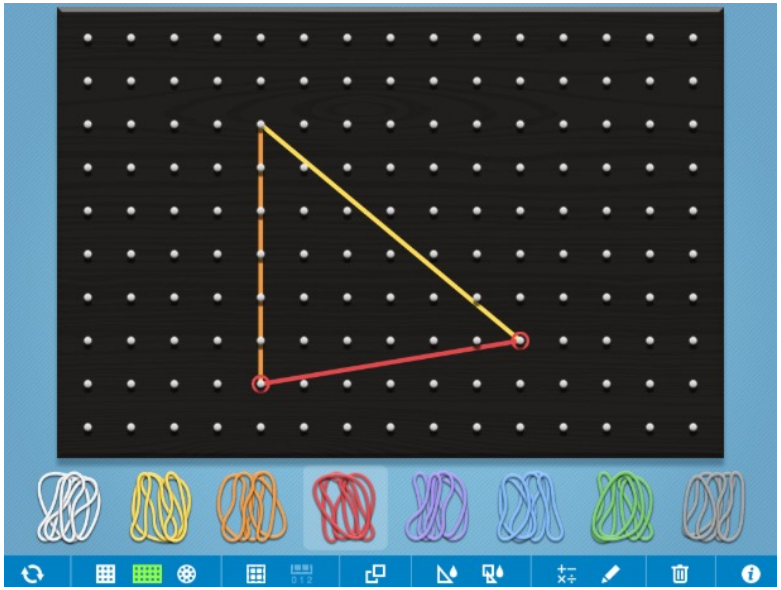


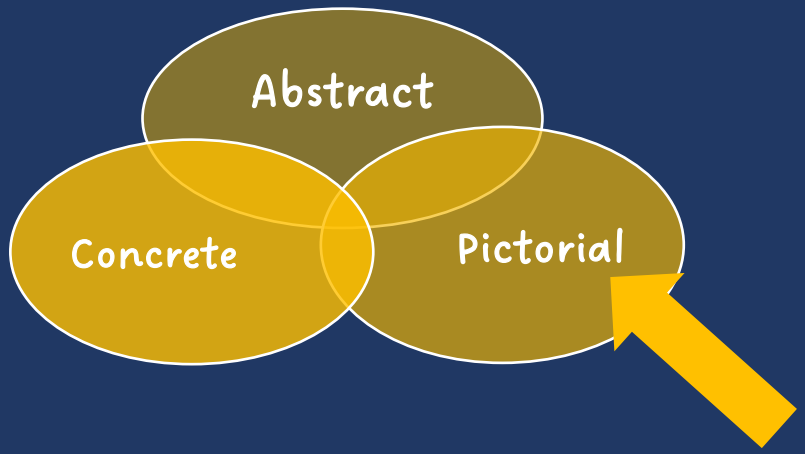


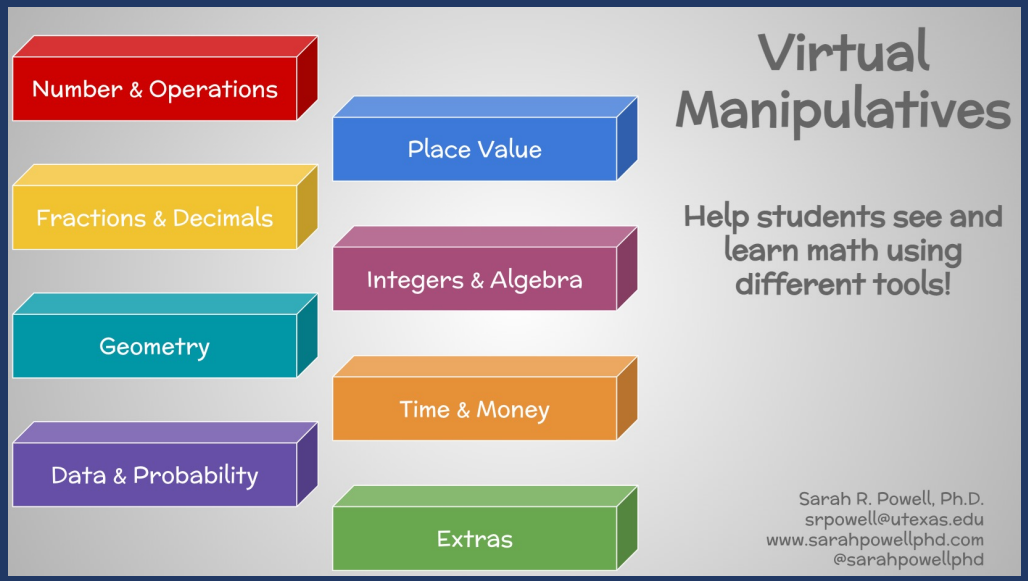












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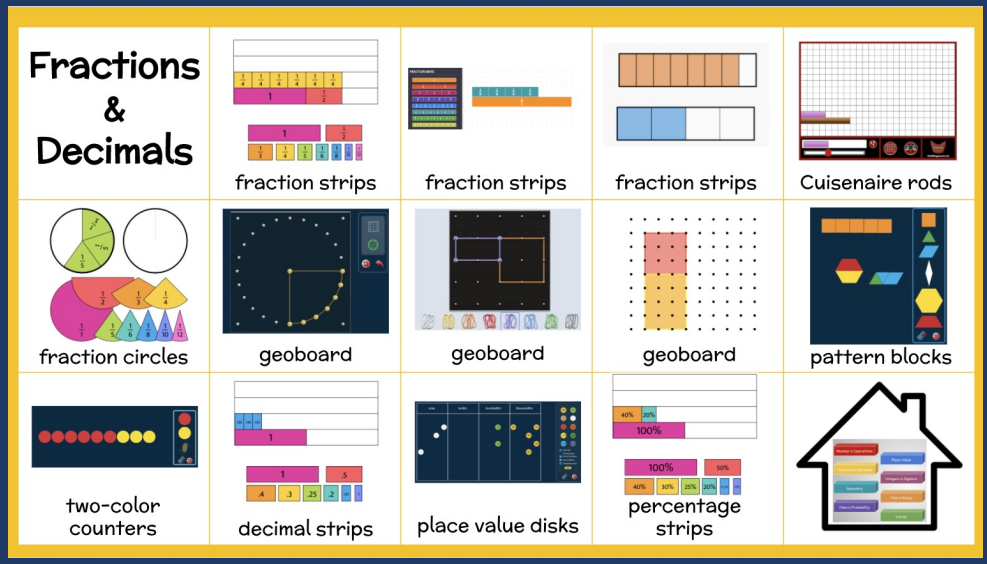
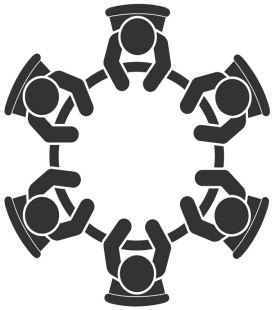
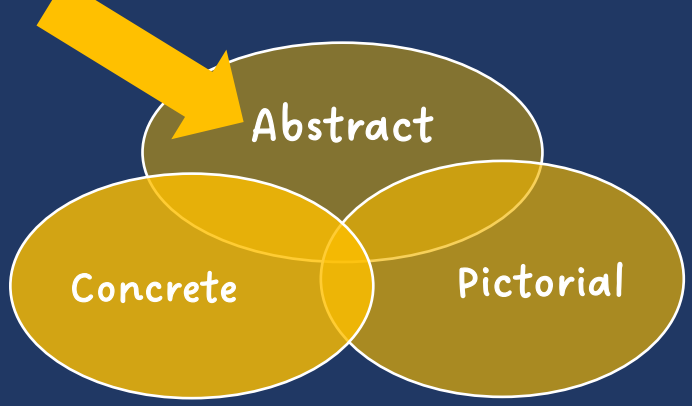




Explore 3 virtual manipulatives.

Share with a partner.



$2 + 8 = 10$

34 = 3 tens and 4 ones

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

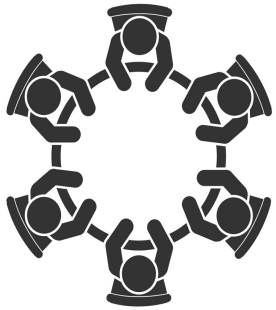
$$\begin{array}{r} 4{,}179 \\ +\ \ 569 \\ \hline \end{array}$$

Representations

 What are your strengths?

 What are your opportunities for growth?

 What will you do on Monday? Next month? Next year?



What are your strengths with representations?

What are the opportunities for growth?

What will you do on Monday? Next month? Next year?

Introductions

Vertical Planning

Vocabulary

Representations

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