Math 3331 – ODE's

Applications

Population Growth

If a population grows according to its size, one model is

$$\frac{dP}{dt} \propto P \quad \text{or} \quad \frac{dP}{dt} = kP$$

Suppose we have a population of say 10,000 at $t=0$ and after 1 yr it was say 10,020 what would the population of 10 yrs

so $\frac{dP}{dt} = kP \quad P(0) = 10,000$
$\qquad\qquad\qquad P(1) = 10,020$

Separable ODE

$$\frac{dP}{P} = k\,dt \rightarrow \ln|P| = kt + \ln c$$

$P = ce^{kt}$ the conditions will give c & k

$P(0) = 10{,}000$

$\Rightarrow \quad 10{,}000 = ce^{k(0)} \Rightarrow \quad c = 10{,}000$

so $\quad P = 10{,}000\, e^{kt}$

Now $\quad P(1) = 10{,}020$

$\Rightarrow \quad 10{,}020 = 10{,}000\, e^{k}$

$$e^{k} = \frac{10{,}020}{10{,}000} = 1.002$$

and $\quad k = \ln 1.002 = .001998$

$$P(t) = 10{,}000\, e^{.001998t}$$

and $\quad P(10) = 10{,}000\, e^{.001998(10)}$

$= 10{,}201.81 \quad$ or $\quad 10{,}201$

Problem with model – pop. can grow unbounded

Suppose we have a campus of 1,000 student at 1 person comes in sick. After 4 days there are 7 people sick. Find a formula for the # sick people in days. Let $P(t)$ be # sick people

$$\frac{dP}{dt} \propto P \quad \& \quad \frac{dP}{dt} \propto 1000-P$$

so $$\frac{dP}{dt} = kP(1000-P)$$

$P(0) = 1$, $\quad P(4) = 7$

Separate ODE

$$\frac{dP}{P(1000-P)} = k\,dt$$

using Partial Fractions

$$\frac{1}{1000}\left(\frac{1}{P} + \frac{1}{1000-P}\right)dP = k\,dt$$

integrate giving

$$\ln|p| - \ln|1000-p| = 1000kt + c$$

let $\bar{k} = 1000k$

so $\ln\left|\dfrac{p}{1000-p}\right| = \bar{k}t + c$

Now the conditions

$P(0) = 1 \Rightarrow \quad c = \ln \dfrac{1}{999} = -6.9068$

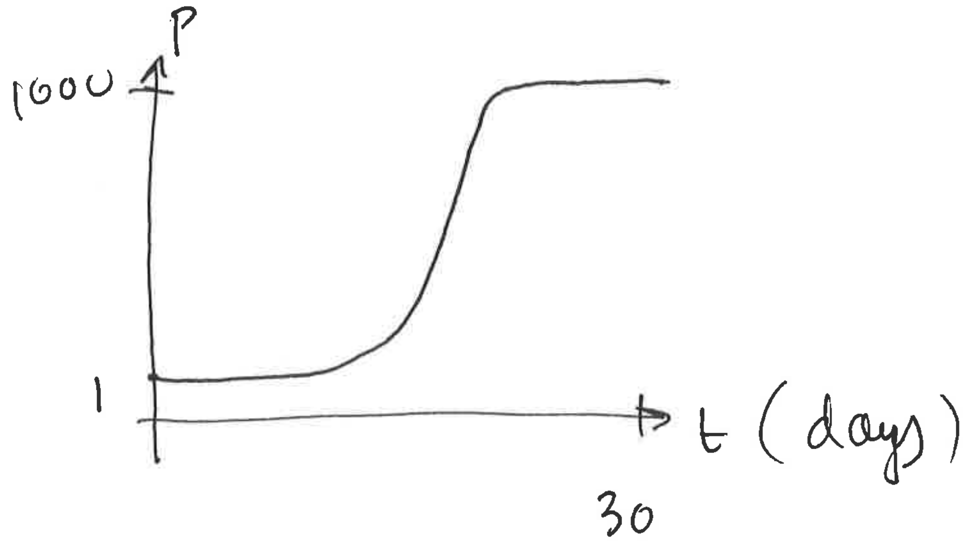
$P(4) = 7 \Rightarrow \ln\left|\dfrac{7}{993}\right| = \bar{k}\,4 - 6.9068$

so $\bar{k} = .4880$

$$\ln\left|\frac{p}{1000-p}\right| = .4880t - 6.9068$$

$$\frac{p}{1000-p} = e^{.4880t} \cdot e^{-6.9068} = .001\, e^{.4880t}$$

solving for p $\quad P = \dfrac{1000\, e^{.4880t}}{1000 + e^{.4880t}}$ (some round off error)


P
1000
1
t (days)
30