Lumen artis mathematicae

Up to Year 13

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Algebra ...the way to do it

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Complete Book

‘Algebra…the way to do it’

The Author

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SECTION 1

What is Algebra?

The word **'algebra'** comes from the Arabic word '**algorithm**', meaning **'a step-by-step process for performing calculations'**.
Algebra is a **special kind of arithmetic that uses letters (or symbols)** instead of numbers to represent quantities.
The only difference is that ***x***, for example, can stand for **any quantity**, whereas a **number** like **3**, for example, **stands only** for a **set of three things**.

In calculations, ***x*** is used in exactly the same way as **3**, or **any** other number.

The Four Basic Rules

The ***four basic rules***, namely:

addition, subtraction, multiplication and **division**,

are applied in **algebra** in the **same way** as they are in **arithmetic**.

(i) Addition

x* + 3** means **3** is **added to *x.
x* + 3*x means **1 of *x*** is added to **3 of *x***, giving **4 of *x*** altogether.
We call this **4*x***.
Note the difference between ***x* + 3** and ***x* + 3*x***.
x* + *y means a quantity ***x*** is added to a quantity ***y***.
Note that ***x* + *y*** is the **same** as ***y* + *x*.**

Since the **order is not important** we say that quantities are **commutative under addition.**

(ii)`Subtraction

3 - *x* means ***x* is subtracted from 3**.

3*x* - *x* means **1 of *x*** is subtracted from **3 of *x*,**
leaving **2 of *x*.** We call this **2*x***.

N.B Note the **difference** between **3 - *x*** and **3*x* - *x***.

Also: ***x* - *y*** is **not the same** as ***y* – *x*.**

Since the **order is important**, we say that quantities
are **not commutative under substraction.**

(ii) Multiplication

There is **no need to use a multiplication sign (×)** in algebra:

2*x* means **2** × ***x*** or ***x* + *x*** and
5*x* means **5** × ***x*** or ***x* + *x* + *x* + *x* + *x***

As in arithmetic, **multiplication is**
a short method of addition, where we have:

2 × 9 short for **9 + 9** and

5 × 9 short for **9 + 9 + 9 + 9 + 9.**

xy means a quantity ***x*** is multiplied by another quantity ***y***.
Note that ***xy*** is the **same** as ***yx***.

Since the **order is not important**, we say that quantities
are **commutative under multiplication**.

(iii) Division

$\frac{x}{3}$ means ***x*** is divided by **3**, giving $\frac{1}{3}$ of ***x*.**

$\frac{x+1}{3}$ means **1 is added to *x*** and this **result** is **divided by *y* 3**.

$\frac{x}{y}$ means ***x*** is divided by ***y*** and

$\frac{y}{x}$ means ***y*** is divided by ***x***.

Since $\frac{x}{y}$ is **not the same** as $\frac{y}{x}$, as, for instance, **4 ÷ 2** is **not equal** to **2 ÷ 4**, we say that quantities are **not commutative under division**.

Brackets

Brackets are used like a **pocket** to hold things safely together. The **contents** of brackets must be treated as a **single quantity** and **worked out on their own**:

Eg. (i) **3(*x* + 1)** means **1 is added to *x*** and this **result** is **multiplied by *y* 3**.

Eg. (ii) **2(3*x* - 1)** means **1 is subtracted from 3 times *x*** and **the result** is **doubled**.

It is possible to **remove brackets** by **multiplying each term inside** the brackets by the '**multiplier' outside** the **brackets**.

Removing brackets:

Eg. (i) **3(*x* + 1)** is **equal** to **3*x* + 3**

and **(ii)** **2(3*x* – 1)** is equal to **6*x* – 2**.

Powers

x can be raised to a **power** which means it is **multiplied by itself** the number of times that the power states:

x^2 means $x \times x$.

Note the **difference** between x^2 and $2x$:

$$x^2 = x \times x \quad \text{and} \quad 2x = x + x.$$

If $x = 4$, then $x^2 = 4 \times 4 = 16$

and $2x = 2 \times 4 = 8$.

Again, note the **difference** between x^3 and $3x$:

$$x^3 = x \times x \times x \text{ and } 3x = 3 \times x.$$

If $x = 4$, then $x^3 = 4 \times 4 \times 4 = 64$

and $3x = 3 \times 4 = 12$.

Like Terms

Since all terms containing ***x*,** for example**,** in any algebraic expression are **the same**, they can be **collected together** to form a **single term** in ***x***, by **adding, subtracting, multiplying** and **dividing** as required. The **terms** that are **alike** are called **like terms** and adding and subtracting them to form a **single term** in ***x*** is **collating like terms**.

To **express** an **algebraic** expression in its **simplest** form, **like terms must be collated**.

Eg.(i) **Simplify** $2x - 3x + 5x - 4x + x + 1 - 5 + 2$.

Collating like terms, we have:

$+ 2x - 3x + 5x - 4x + 1x = + 1x = x$ and

$+ 1 - 5 + 2 = -2$,

giving $x - 2$ as the **simplest form**.

Like Terms (contd.)

Eg.(ii) **Simplify** $2x + 1 + x - 3 - 4x + 5 - x - 10$.

Collating like terms, we have:

$+2x + 1x - 4x - 1x = -2x$

and $+1 - 3 + 5 = +3$,

giving $-2x + 3$ as the **simplest form**.

Eg.(iii) **Simplify** $-xy + x - 2xy + 3 - 4x - 7 + z - 5z + p$.

Collating like terms, we have:

$-1xy - 2xy = -3xy$,

$+1x - 4x = -3x$,

$+3 - 7 = -4$

and $+1p = +p$,

giving $-3xy - 3x - 4 + p$ as the **simplest form**.

Eg. (iv) Write the following in its **simplest form**, by **firstly removing** the **brackets**, and then **collating like terms**:

$2x - 7 - (3x + 5) + 3y - 2 + 5(2y - 2) + (y - 4) + 10$

N.B. $-(3x + 5)$ is $-\underline{1}(3x + 5)$ and $+(y - 4)$ is $+\underline{1}(y - 4)$.

(Worked Answer on next page).

Eg. (iv): **Removing brackets** gives:
$2x - 7 - 3x - 5 + 3y - 2 + 10y - 10 + y - 4 + 10$

Collating x terms, we have:
$2x - 3x = -1x = -x$.

Collating y terms we have:
$+ 3y + 10y + 1y = + 14y$.

Collating numbers, we have:
$- 7 - 5 - 2 - 10 - 4 + 10 = - 18$.

The **simplest form** of the **whole expression** is, therefore:
$-x + 14y - 18$.

Exercise 1 - The four rules, use of symbols, removing brackets and simplifying.

1. Write the following **English** in the "language" of **algebra**:

   (i) A quantity x is **added** to **two**.
   (ii) A quantity y is **subtracted** from **five**.
   (iii) **Twice** a quantity x is **added** to **three**.
   (iv) **Thirteen** is **subtracted** from **four times** x.

2. Write the following **algebra** in the language of **English**:

   (i) $4 + x$.
   (ii) $6 - x$.
   (iii) $5x - 1$.
   (iv) $12 - 7x$.

Exercise 1 (contd.)

3. Write the following **English** in **algebra**:

(i) A quantity x is divided by **two**.
(ii) **Two** is divided by a quantity y.
(iii) **One** is added to x and the **result** is **doubled**.
(iv) **One** is added to x and the **result** is **halved**.

4. Write the following **algebra** in **English**:

(i) $\frac{x}{2}$
(ii) $\frac{2}{x}$
(iii) $\frac{x+1}{2}$
(iv) $\frac{x+y}{x-y}$

5. **Remove the brackets** from each of the following:

(i) $2(x-4)$.
(ii) $3(2x+6)$.
(iii) $-4(5x+4)$.
(iv) $-(2x+1)$. **N.B**. This means $-1(2x+1)$.

6. **Simplify** each of the following, by **removing brackets** and then **collating like terms**:

(i) $2(x+4)-3x-9$.
(ii) $6x-3(2x+1)+6$.
(iii) $-(2x+1)+2x+1$.
(iv) $3(2x-2)+2(3x-2)$.
(v) $2(x-1)-3x+(3x-1)$.
(vi) $x(2x-3)+x^2(x+y)-7x+3$.