



Lumen artis mathematicae

Up to Year 13

Author: R. M. O'Toole

Algebra ...the way to do it

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Complete Book

‘Algebra...the way to do it’

The Author

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SECTION 1

What is Algebra?

The word ‘**algebra**’ comes from the Arabic word ‘**algorithm**’, meaning ‘**a step-by-step process for performing calculations**’.

Algebra is a **special kind of arithmetic that uses letters (or symbols)** instead of numbers to represent quantities.

The only difference is that x , for example, can stand for **any quantity**, whereas a **number** like **3**, for example, **stands only** for a **set of three things**.

In calculations, x is used in exactly the same way as **3**, or **any** other number.

The Four Basic Rules

The *four basic rules*, namely:

addition, subtraction, multiplication and division,

are applied in **algebra** in the **same way** as they are in **arithmetic**.

(i) **Addition**

$x + 3$ means **3** is **added to** x .

$x + 3x$ means **1 of** x is added to **3 of** x , giving **4 of** x altogether.

We call this **4x**.

Note the difference between $x + 3$ and $x + 3x$.

$x + y$ means a quantity x is added to a quantity y .

Note that $x + y$ is the **same** as $y + x$.

Since the **order is not important** we say that quantities are **commutative under addition**.

(ii) Subtraction

$3 - x$ means x is subtracted from 3.

$3x - x$ means 1 of x is subtracted from 3 of x , leaving 2 of x . We call this $2x$.

N.B Note the **difference** between $3 - x$ and $3x - x$.

Also: $x - y$ is **not the same** as $y - x$.

Since the **order is important**, we say that quantities are **not commutative under subtraction**.

(ii) Multiplication

There is **no need to use a multiplication sign** (\times) in algebra:

$2x$ means $2 \times x$ or $x + x$ and

$5x$ means $5 \times x$ or $x + x + x + x + x$

As in arithmetic, **multiplication is a short method of addition**, where we have:

2×9 short for $9 + 9$ and

5×9 short for $9 + 9 + 9 + 9 + 9$.

xy means a quantity x is multiplied by another quantity y .

Note that xy is the **same** as yx .

Since the **order is not important**, we say that quantities are **commutative under multiplication**.

(iii) Division

$\frac{x}{3}$ means x is divided by **3**, giving $\frac{1}{3}$ of x .

$\frac{x+1}{3}$ means **1 is added to x** and this **result is divided by 3**.

$\frac{x}{y}$ means x is divided by y and

$\frac{y}{x}$ means y is divided by x .

Since $\frac{x}{y}$ is **not the same** as $\frac{y}{x}$, as, for instance, $4 \div 2$ is **not equal** to $2 \div 4$, we say that quantities are **not commutative under division**.

Brackets

Brackets are used like a **pocket** to hold things safely together. The **contents** of brackets must be treated as a **single quantity** and **worked out on their own**:

Eg. (i) $3(x + 1)$ means **1 is added to x** and this **result is multiplied by 3**.

Eg. (ii) $2(3x - 1)$ means **1 is subtracted from 3 times x** and **the result is doubled**.

It is possible to **remove brackets** by **multiplying each term inside** the brackets by the **'multiplier' outside the brackets**.

Removing brackets:

Eg. (i) $3(x + 1)$ is equal to $3x + 3$

and **(ii)** $2(3x - 1)$ is equal to $6x - 2$.

Powers

x can be raised to a **power** which means it is **multiplied by itself** the number of times that the power states:

x^2 means $x \times x$.

Note the **difference** between x^2 and $2x$:

$$x^2 = x \times x \quad \text{and} \quad 2x = x + x.$$

If $x = 4$, then $x^2 = 4 \times 4 = 16$

$$\text{and} \quad 2x = 2 \times 4 = 8.$$

Again, note the **difference** between x^3 and $3x$:

$$x^3 = x \times x \times x \quad \text{and} \quad 3x = 3 \times x.$$

If $x = 4$, then $x^3 = 4 \times 4 \times 4 = 64$

$$\text{and} \quad 3x = 3 \times 4 = 12.$$

Like Terms

Since all terms containing x , for example, in any algebraic expression are **the same**, they can be **collected together** to form a **single term** in x , by **adding, subtracting, multiplying** and **dividing** as required. The **terms** that are **alike** are called **like terms** and adding and subtracting them to form a **single term** in x is **collating like terms**.

To **express an algebraic** expression in its **simplest** form, **like terms must be collated**.

Eg.(i) **Simplify** $2x - 3x + 5x - 4x + x + 1 - 5 + 2$.

Collating like terms, we have:

$$\begin{aligned} + 2x - 3x + 5x - 4x + 1x &= + 1x = x \text{ and} \\ + 1 - 5 + 2 &= -2, \end{aligned}$$

giving $x - 2$ as the **simplest form**.

Like Terms (contd.)

Eg.(ii) Simplify $2x + 1 + x - 3 - 4x + 5 - x - 10$.

Collating like terms, we have:

$$+2x + 1x - 4x - 1x = -2x$$

$$\text{and } +1 - 3 + 5 = +3,$$

giving $-2x + 3$ as the **simplest form**.

Eg.(iii) Simplify $-xy + x - 2xy + 3 - 4x - 7 + z - 5z + p$.

Collating like terms, we have:

$$-1xy - 2xy = -3xy,$$

$$+1x - 4x = -3x,$$

$$+ 3 - 7 = -4$$

$$\text{and } + 1p = + p,$$

giving $-3xy - 3x - 4 + p$ as the **simplest form**.

Eg. (iv) Write the following in its **simplest form**, by **firstly removing the brackets**, and then **collating like terms**:

$$2x - 7 - (3x + 5) + 3y - 2 + 5(2y - 2) + (y - 4) + 10$$

N.B. $-(3x + 5)$ is $-\underline{1(3x + 5)}$ and $+(y - 4)$ is $+\underline{1(y - 4)}$.

(Worked Answer on next page).

Eg. (iv): Removing brackets gives:

$$2x - 7 - 3x - 5 + 3y - 2 + 10y - 10 + y - 4 + 10$$

Collating x terms, we have:

$$2x - 3x = -1x = -x.$$

Collating y terms we have:

$$+ 3y + 10y + 1y = + 14y.$$

Collating numbers, we have:

$$- 7 - 5 - 2 - 10 - 4 + 10 = - 18.$$

The **simplest form** of the **whole expression** is, therefore:

$$- x + 14y - 18.$$

Exercise 1 - The four rules, use of symbols, removing brackets and simplifying.

1. Write the following **English** in the “language” of **algebra**:

- (i) A quantity x is **added** to **two**.
- (ii) A quantity y is **subtracted** from **five**.
- (iii) **Twice** a quantity x is **added** to **three**.
- (iv) **Thirteen** is **subtracted** from **four times** x .

2. Write the following **algebra** in the language of **English**:

- (i) $4 + x$.
- (ii) $6 - x$.
- (iii) $5x - 1$.
- (iv) $12 - 7x$.

Exercise 1 (contd.)

3. Write the following **English** in **algebra**:

- (i) A quantity x is divided by **two**.
- (ii) **Two** is divided by a quantity y .
- (iii) **One** is added to x and the **result** is **doubled**.
- (iv) **One** is added to x and the **result** is **halved**.

4. Write the following **algebra** in **English**:

- (i) $\frac{x}{2}$
- (ii) $\frac{2}{x}$
- (iii) $\frac{x+1}{2}$
- (iv) $\frac{x+y}{x-y}$

5. **Remove the brackets** from each of the following:

- (i) $2(x - 4)$.
- (ii) $3(2x + 6)$.
- (iii) $-4(5x + 4)$.
- (iv) $-(2x + 1)$. **N.B.** This means $-1(2x + 1)$.

6. **Simplify** each of the following, by **removing brackets** and then **collating like terms**:

- (i) $2(x + 4) - 3x - 9$.
- (ii) $6x - 3(2x + 1) + 6$.
- (iii) $-(2x + 1) + 2x + 1$.
- (iv) $3(2x - 2) + 2(3x - 2)$.
- (v) $2(x - 1) - 3x + (3x - 1)$.
- (vi) $x(2x - 3) + x^2(x + y) - 7x + 3$.