Math 4315 - PDEs Home Work 2

1. Solve the following PDEs using a change of coordinates $(x, y) \to (r, s)$

$$
\begin{aligned}
(i) \quad & u_x + u_y = 6y, \\
(ii) \quad & u_x - 2u_y = -u, \quad u(x,0) = x \\
(iii) \quad & yu_x - x\,u_y = x, \\
(iv) \quad & 2xu_x + 3y\,u_y = x, \quad u(x,x) = 1, \\
(v) \quad & (u+x)u_x + (u+y)\,u_y = u, \\
(vi) \quad & u_x + 2u\,u_y = 1, \quad u(x,x) = 0
\end{aligned}
$$

2. Show that under the change of variables

$$r = R(x+y), \quad s = s(x,y),$$

the PDE

$$u_x - u_y = 0,$$

becomes

$$u_s = 0.$$

For the following boundary conditions, show that it is possible to choose $R(x+y)$ and $s(x,y)$ such that the boundary in the (r,s) plane is $s = 0$ and the two boundaries can be connected via $x = r$.

$$
\begin{aligned}
(i) \quad & u(x,0) = f(x) \\
(ii) \quad & u(x,1) = f(x) \\
(iii) \quad & u(x,x) = f(x) \\
(iv) \quad & u\left(x, x^2\right) = f(x)
\end{aligned}
$$

Due: Sept. 21, 2018.