Write your name here

Surname	Other names

Core Mathematics C12

SWANASH

A☆ Practice Paper

Time: 2 hours 30 minutes	Paper - J	Year: 2017-2018

The formulae that you may need to answer some questions are found at the end of this A star practice paper.

A student may use any basic scientific calculator except: facility for symbolic algebra manipulation, differentiation, integration, retrievable mathematical formulae.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more/less space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answer if you have time at the end.
- Practicing many **Swanash A-star** papers will enhance your final exam grades.

Turn over

1. Given that

$$(k - x)^7 \equiv a - bx + bx^2 + \ldots,$$

find the values of the positive integers a, b and k.

(5)

(Total 5 marks)

2. (a) Find the range of values of x for which

(i) $4(2 - x) \leqslant 2x + 1$ **(2)**

(ii) $(x - 5)(x - 5) > 0$ **(1)**

(iii) **both** $4(2 - x) \leqslant 2x + 1$ **and** $(x - 5)(x - 5) > 0$ **(3)**

(b) Solve $(x + 4)(x - 5)(x + 6)(x - 7) = 504$ **(6)**

Question 2 continued

(Total 12 marks)

3. Answer this question without a calculator.

(a) Solve the equation

$$(5^{4x+1})(2^{4x+1}) = 100^{3x}$$

(2)

(b) Given that, $\sqrt{(16 + 2\sqrt{55})} = \sqrt{a} + \sqrt{b}$

Find the positive integers a and b $(a < b)$.

(4)

Question 3 continued

(Total 6 marks)

4.

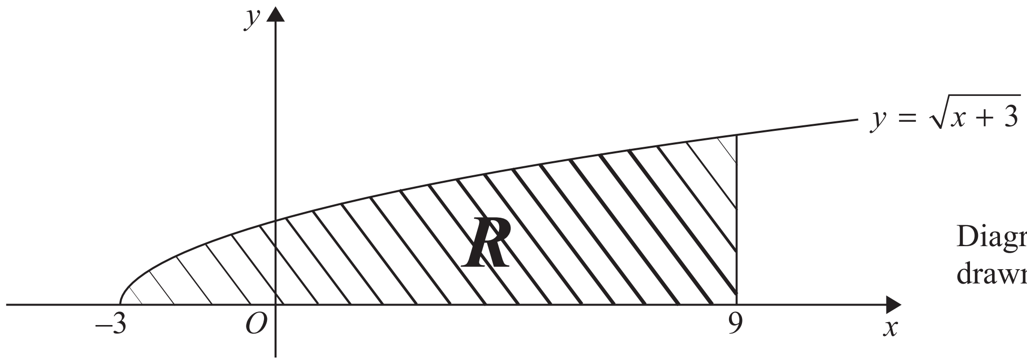


Diagram **NOT** drawn to scale

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{x+3}$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 9$

The table below shows corresponding values of x and y for $y = \sqrt{x+3}$

x	–3	0	3	6	9
y	0	1.732		3	

(a) Complete the table above, giving the missing values of y to 3 decimal places.
(2)

(b) Use the trapezium rule, with all of the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.
(3)

Use your answer to part (b) to find approximate values of

(c) (i) $\displaystyle\int_{-3}^{9} \left(2 + \sqrt{x+3}\right) \mathrm{d}x$

(ii) $\displaystyle\int_{-2}^{10} \frac{\sqrt{x+2}}{3} \mathrm{d}x$
(4)

Question 4 continued

(Total 9 marks)

5. A sequence $U_1, U_2, U_3, \ldots$ satisfies

$$U_{n+1} = \frac{2}{5}U_n + 30, \text{ for each natural number, } \quad n \geqslant 1$$

Given that $U_2 = 5$,

(a) Find the exact values of U_3, U_4, and U_5

(3)

(b) Find the value of U_∞

(2)

(c) Find $\sum_{n=2}^{27}\left(\frac{2}{5}n + 30\right)$

(3)

Question 5 continued

(Total 8 marks)

6. Given that $\int_0^1 3x^{3k}\,dx = 1$, where k is a constant.

(a) Find the value of k

(4)

Given that $(m-1)\int_0^m (6x^5 + 5x^4 + 4x^3)\,dx = 26m^4$,

(b) Find the positive value of m

(5)

Question 6 continued

(Total 9 marks)

7. Given $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants.

When $f(x)$ is divided by $(x - 1)$ the remainder is 7. When $f(x)$ is divided by $(x - 2)$ the remainder is 5.

Find the remainder when $f(x)$ is divided by $(x - 1)(x - 2)$

(5)

Question 7 continued

(Total 5 marks)

8. (a) Using *only* the left-hand side (LHS), prove that

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} \equiv \frac{\sin\theta + 1}{\cos\theta}, \qquad 90° < \theta < 270°$$

(5)

(b) Hence solve, for $90° < \theta < 270°$, the equation

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = 3\cos\theta$$

Give your answer to one decimal place.

(4)

Question 8 continued

(Total 9 marks)

9. (a) **Mr.Swarna** drives north, east, south and west and then repeats these moves *indefinitely* as shown in the Figure 2. The first leg of the journey is 8 km, and each leg is half as long as the preceding one. If the starting point is C (3,2) then find the ultimate, final, destination coordinates.

(Assume **Mr.Swarna** and his vehicle are considered as a point)

(4)

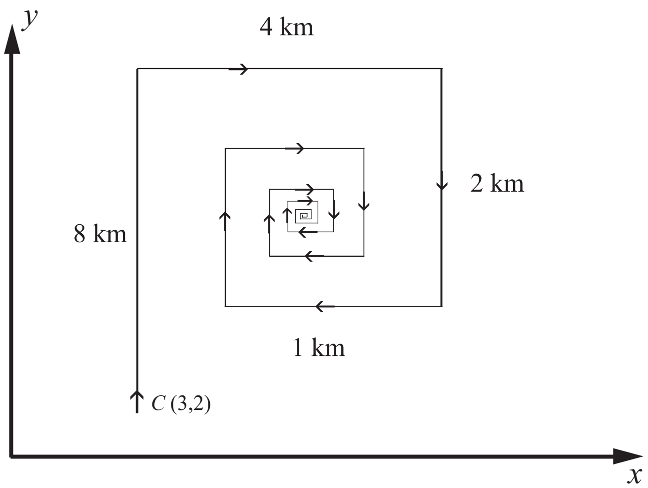


Diagram **NOT** drawn to scale

Figure 2

(b) The sum, S_n , of the first n terms of a series is given by

$$S_n = 5^n - 1.$$

(i) Show that the fourth term of the series is 500. **(3)**

(ii) Given that the nth term of the series is $k(5^n)$. Find k. **(1)**

(iii) Prove that the series is geometric. **(3)**

Question 9 continued

(Total 11 marks)

10. Match the functions P, Q, R, S, T and U to one of the graphs (a), (b), (c), (d), (e) and (f)

$P.\quad y = 2\sin\left(\frac{\pi}{2}x\right)$

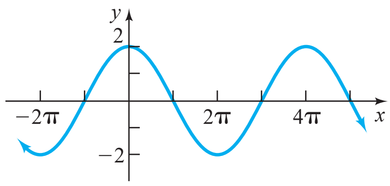
$S.\quad y = 2\cos\left(\frac{1}{2}x\right)$

$Q.\quad y = -2\cos\left(\frac{\pi}{2}x\right)$

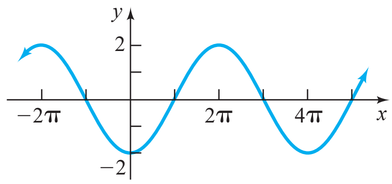
$T.\quad y = 3\sin(2x)$

$R.\quad y = -2\cos\left(\frac{1}{2}x\right)$

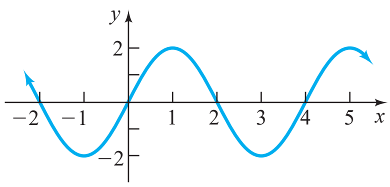
$U.\quad y = -2\sin\left(\frac{1}{2}x\right)$



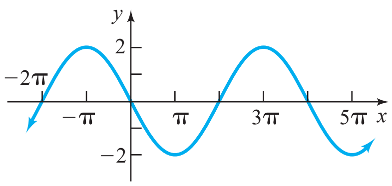
(a)



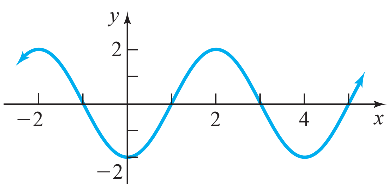
(b)



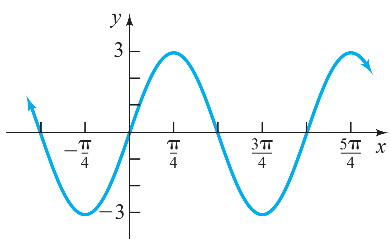
(c)



(d)



(e)



(f)

(5)

Question 10 continued

(Total 5 marks)

11.

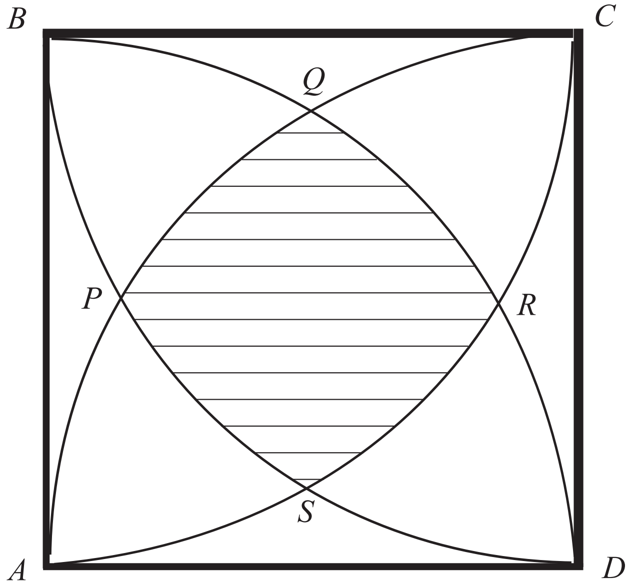


Diagram not drawn to scale

Figure 3

Figure 3 shows a square, *ABCD*, with side length 10 cm. The two arcs from *A* to *C* have centres at *B* and *D*, and the two arcs from *B* to *D* have centres at *A* and *C*.

(a) Find the perimeter of the shaded region, *PQRS*.

(2)

(b) Find the area of the shaded region, *PQRS*.

(4)

Question 11 continued

(Total 6 marks)

12.

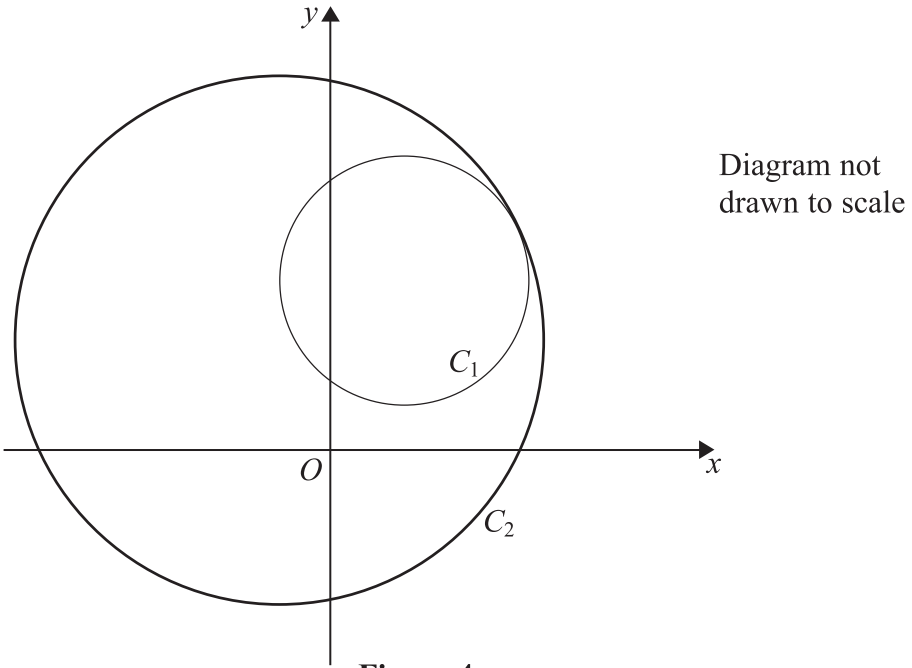


Figure 4

Figure 4 shows two circles C_1 and C_2. The equation of circle C_1 is $(x - 2)^2 + (y - 4)^2 = 9$

and the equation of circle C_2 is $(x + 2)^2 + (y - 1)^2 = 64$

(a) Without solving the simultaneous equation show that the above two circles C_1 and C_2 are *touching internally*.

(4)

The point P lies on C_1.

(b) Find the largest value of the length OP to 2 decimal places.

(3)

The point Q lies on C_2.

(c) Find the smallest value of the length OQ to 2 decimal places.

(3)

Question 12 continued

(Total 10 marks)

13. (a) Without using your calculator, solve

$$2x = \log_{\frac{1}{2}}16 + \log_{\frac{1}{3}}9$$

(2)

(b) Using your calculator, solve

$$\left(\log_x 4\right)\left(\log_x 10\right) = 5$$

(4)

Question 13 continued

(Total 6 marks)

14.

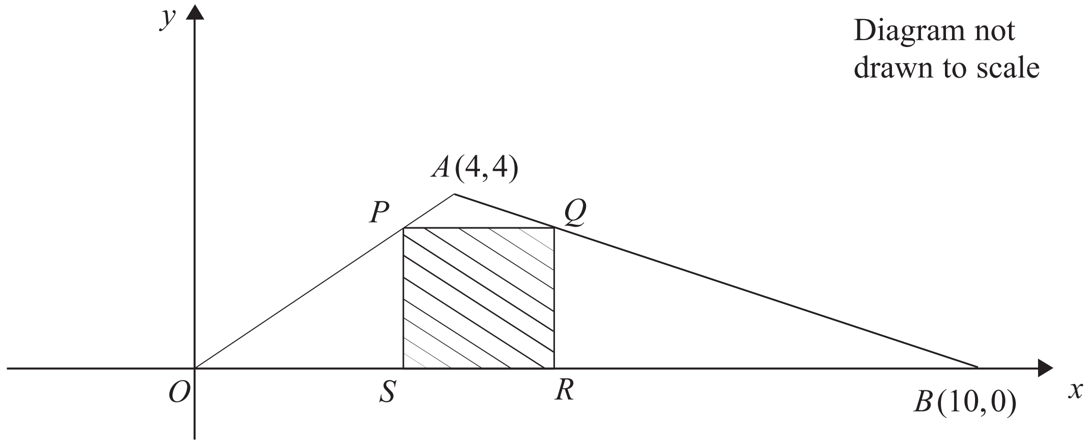


Figure 5

Figure 5 shows a sketch of the graph of $y = \mathrm{g}(x)$, $0 \leqslant x \leqslant 10$ and a square $PQRS$

The graph of $y = \mathrm{g}(x)$ consists of two line segments, from $O(0,0)$ to $A(4,4)$ and from $A(4,4)$ to $B(10,0)$.

The square $PQRS$ touches $y = \mathrm{g}(x)$ at the points P, Q and the square touches the x-axis at the points S and R as shown in Figure 5.

(a) Find the shaded area of the square $PQRS$.

Show each step of your working and give your answer as a fraction in its simplest form.

(4)

(b) Sketch the graph with equation

$$y = -\frac{3}{2}\mathrm{g}(x), \quad 0 \leqslant x \leqslant 10$$

On your sketch show the coordinates of the points to which O, A and B are transformed.

(2)

Question 14 continued

Question 14 continued

(Total 6 marks)

15.

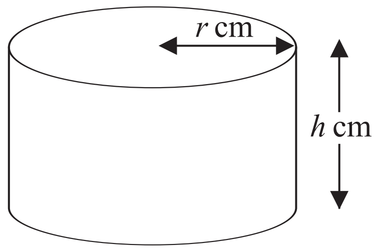


Diagram not drawn to scale

Figure 6

Figure 6 shows a design for an ancient 'Thamilan Odiyal Kool' cup.

It is in the shape of a right circular cylinder with height *h* cm and radius *r* cm.

The cup has a base but has *no lid*, is open at the top and assume that the thickness of the material is negligible.

The cup is designed to hold 300 cm^3 of 'Odiyal Kool' when full.

(a) The total external surface area, S cm^2, of the cup is given by the formula

$$S = \pi r^2 + 2\pi rh$$

Using calculus **or** using the special dimension of this ancient 'Thamilan Odiyal Kool' cup, find the minimum value of S, giving your answer to 2 decimal places.

(3)

(b) Write down the relationship between radius, r, and height, h, for any ancient 'Thamilan Odiyal Kool' cup with the lid when the area, S, is minimum.

(1)

Question 15 continued

(Total 4 marks)

16.

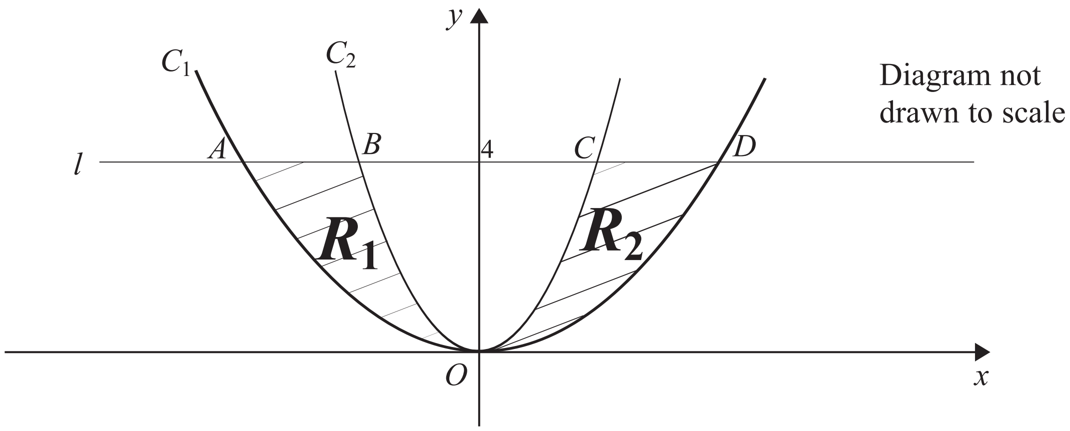


Figure 7

Figure 7 shows a sketch of part of the curves C_1, C_2 and a line l with equations $y = x^2$, $y = 4x^2$ and $y = 4$ respectively.

The y coordinate of all the points A, B, C and D is 4.

(a) Find $\frac{dy}{dx}$, at B **(3)**

(b) Find an equation of tangent to curve C_2 at B in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

(c) The equation of tangent to curve C_1 at A intersect the y-axis at M. Write down the coordinates of the point M. **(1)**

The finite regions R_1 and R_2, ($R_1 = R_2$), shown shaded in Figure 7, is bounded by the curves C_1, C_2 and the line l.

(d) Use integration to find the exact area of the shaded region $R_1 + R_2$. **(6)**

Question 16 continued

(Total 14 marks)

TOTAL FOR PAPER: 125 MARKS

Formulae for Core Mathematics C12

Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Binomial series

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots (|x| < 1,\ n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

The trapezium rule $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$