Math 2471 – Calc 3

Last class – tangent plane

$$f_x|_P (x-x_0) + f_y|_P (y-y_0) - (z-z_0) = 0$$

Ex Find the TP at $P(2,-1,1)$ for $z = e^{x+2y}$

$z_x = e^{x+2y}(1)$ $\quad z_x|_P = e^{2-2}(1) = e^0 \cdot 1 = 1$

$z_y = e^{x+2y}(2)$ $\quad z_y|_P = e^{2-2}(2) = e^0 \cdot 2 = 2$

$$1(x-2) + 2(y+1) - (z-1) = 0$$

Suppose that the surface is given implicitly

$$x^2 z + y z^3 + xy = z^2$$

we really can't solve for z so now what

Recall from Calc 1 implicit Differentiation

$$x^2 + xy + y^2 = 1$$

$$2x + xy' + y + 2yy' = 0$$

$$xy' + 2yy' = -(2x+y)$$

$$y' = \frac{-(2x+y)}{x+2y}$$

So with partial derivative.

Consider $x^2 + y^2 + z^2 = 9$

$\frac{\partial}{\partial x}$: $\quad 2x + 2z\frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{x}{z}$

$\frac{\partial}{\partial y}$: $\quad 2y + 2z\frac{\partial z}{\partial y} = 0 \Rightarrow \frac{\partial z}{\partial y} = -\frac{x}{z}$

Given $P(2,2,1)$ these derv. are easy to evaluate

Consider $x^2 z + y z^3 + xy = z^2$

$\frac{\partial}{\partial x}$: $2xz + x^2 z_x + 3yz^2 z_x + y = 2z z_x$

$$x^2 z_x + 3y z^2 z_x - 2z z_x = -y - 2xz$$

$$(x^2 + 3yz^2 - 2z) z_x = -y - 2xz$$

$$z_x = \frac{-y - 2xz}{x^2 + 3yz^2 - 2z}$$

similar for z_y

so How about in general

$$F(x, y, z) = 0$$

$\frac{\partial}{\partial x}$: $F_x + F_z z_x = 0$ or $z_x = -\frac{F_x}{F_z}$

$\frac{\partial}{\partial y}$: $F_y + F_z z_y = 0$ or $z_y = -\frac{F_y}{F_z}$

Previous Ex

$$F = x^2 z + y z^3 + xy - z^2$$

$$F_x = 2xz + y$$

$$F_y = z^3 + x$$

$$F_z = x^2 + 3yz^2 - 2z$$

$$z_x = -\frac{F_x}{F_z} = -\frac{(y+2xz)}{x^2+3yz^2-2z}$$ same

Now let's return to the TP prom

$$f_x(x-x_0) + f_y(y-y_0) - (z-z_0) = 0$$

$$-\frac{F_x}{F_z}(x-x_0)-\frac{F_y}{F_z}(y-y_0)-(z-z_0)=0$$

or $\quad F_x\big|_P(x-x_0)+F_y\big|_P(y-y_0)+F_z\big|_P(z-z_0)=0$

Ex Find eq^n of TP for $x^3z+2yz^2=x+y$

at $P(2,-6,+1)$

$f=x^3z+2yz^2-x-y$

$F_x=3x^2z-1$ $\qquad F_x=3(2)^2(1)-1=11$

$F_y=2z^2-1$ $\quad$ at P $\quad F_y=2(1)^2-1=1$

$F_z=x^3+4yz$ $\qquad F_z=8+4(-6)(1)=8-24=-16$

$11(x-2)+1(y+6)+4(z-1)=0$

HW pg 935-6 # 10, 13, 14, 15