Math 4315 – PDE's

From the proceeding class to find p on the boundary we differentiated the actual boundary condition. So if

$$u(x,1) = x^2 + x$$

then $\quad u_x(x,1) = 2x+1$ so if $x = r$ then $p = 2r+1$

What do we do when the boundary condition is

$$u(x,x) = x \quad \text{or} \quad u(x,1-x) = 1$$

How do we differentiate these? Recall from Calc 3 the chain rule

if $\quad u = F(x,y)$ & $x = f(r),\ y = g(r)$

then $\quad u = F(f(r), g(r))$

so $\quad \dfrac{du}{dr} = \dfrac{\partial u}{\partial x}\dfrac{dx}{dr} + \dfrac{\partial u}{\partial y}\cdot\dfrac{dy}{dr}$

As we don't know u yet we have

$$\frac{du}{dr} = p\,\frac{dx}{dr} + q\,\frac{dy}{dr}$$

we can find these as we know x & y

Here are a few examples:

(i) $u(x, x) = x$ so if $x = r$ $u(r, r) = r$

so $\frac{d}{dr} u(r, r) = \frac{d}{dr}(r)$

$$\frac{\partial u}{\partial x}\frac{dx}{dr} + \frac{\partial u}{\partial y}\frac{dy}{dr} = 1 \qquad x = r,\ y = r$$

so $\boxed{p + q = 1}$

(ii) $u(x, 1-x) = x^2 + 2x$

$x = r,\ y = 1 - r$ $\quad u = x^2 + 2x = r^2 + 2r$

$$\frac{d}{dr} u(r, 1-r) = \frac{d}{dr}(r^2 + 2r)$$

$\Rightarrow p \cdot 1 + q(-1) = 2r + 2$

so $\boxed{p - q = 2r + 2}$

(iii) $u(x, 1) = \sin x$

$x = r,\ y = 1$ $\quad u = \sin r$

$$\frac{d}{dr}(u(r, 1)) = \frac{d}{dr}\sin r \Rightarrow p \cdot 1 + q \cdot 0 = \cos r$$

$\boxed{p = \cos r}$