Comprehensive Study of Channel Estimation Techniques for OFDM based Communication Systems

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Abstract**—** Orthogonal frequency division multiplexing (OFDM) is a special case of multi-carrier transmission and it can accommodate high data rate requirement of multimedia based wireless systems. Since channel estimation is an integral part of OFDM systems, it is critical to understand the basis of channel estimation techniques for OFDM systems so that the most appropriate method can be applied. In this article, an extensive overview of channel estimation techniques employed in OFDM systems are presented.

Keywords**—**OFDM; channel estimation; LMS; MMSE; Weiner filter.

I. INTRODUCTION

Driven by multimedia based applications, the future wireless systems will require high data rate and more reliable technologies. OFDM is a suitable candidate for systems employing such applications [1, 2]. OFDM divides the available spectrum into a number of orthogonal and overlapping narrowband sub-channels which converts a frequency selective fading channel into a flat fading channel [3]. Moreover, ISI is avoided by the use of cyclic prefix in which head part of OFDM symbol is attached to its tail [4]. Due to these advantages, OFDM has been adopted by many wireless standards such as DAB, DVB, WLAN, and WMAN [5, 6].
While evaluating OFDM system performance, the perfect knowledge of the channel is assumed usually for equalization. The perfect channel knowledge can be used to find the upper limit of OFDM system performance but such perfect channel knowledge is not available in real-life and needs to be estimated. Channel estimation can be done in various ways: with or with the help of a parametric model, with the use of frequency and/or time correlation properties of the wireless channel, blind or pilot based, adaptive or non-adaptive [7].

The quantities of interest are estimated without relying on a specific channel model in case of non-parametric methods whereas parametric estimation assumes a certain channel model and determines the parameters of this model to determine the quantities of interest [8].

A popular class of coherent demodulation for a wide class of digital modulation schemes has been proposed by Moher and Lodge [9], and is known as Pilot Symbol Assisted Modulation, PSAM. Aghamohammadi [10] et al. and Cavers [11] were among the first analyzing and optimizing PSAM given different interpolation filters. The main disadvantage of this scheme is the slight increase of the bandwidth. Channel estimation using superimposed pilot sequences is also a completely new area, idea for using superimposed pilot sequences has been proposed by various authors for different applications. In [12], superimposed pilot sequences are used for time and frequency synchronization. In [13], superimposed pilot sequences are introduced for the purpose of channel estimation, and main idea here is to linearly add a known pilot sequence to the transmitted data sequence and perform joint channel estimation and detection in the receiver.

The most commonly used method for channel estimation is pilot based estimation method which is applicable in systems where the sender emits some known signal to the receiver. Another type of estimation technique is blind estimation which uses some properties of the signal to estimate the channel. But blind estimation is rarely used in practical OFDM systems. Adaptive channel estimation methods are used for rapidly varying channel conditions [14].

II. CHANNEL MODEL

The relationship between transmitted signal X_k and received Y_k can be given as:

$$Y_k = H_k X_k + Z_k \qquad (1)$$

Where Z_k is the AWGN noise for the k^{th} sub-carrier, and H_k is the channel transfer function for the k^{th} sub-carrier. Pilot symbols are required to estimate the channel. We assume that every p^{th} sub-carrier contains known pilot symbols X_{pk} Raw channel estimation $\widehat{H}_{pk}$ can be done by using the known pilots symbols X_{pk} and the received symbols Y_{pk} at the pilot sub-carriers as:

$$\widehat{\mathrm{H}}\mathrm{pk} = \frac{Y_{pk}}{X_{pk}} + \frac{Z_{pk}}{X_{pk}} = H_{pk} Z'_{pk} \qquad (2)$$

Where Z pk is the noise contribution at the p_k-th sub-carrier, Z'_{pk} is a scaled noise contribution at that sub carrier. The channel has to be estimated over all sub-carrier frequencies and not just at pilot sub-carrier frequencies. Hence after FFT and zero-padding removal, the received signal at pilot locations is extracted from the signal. Raw channel is estimated at pilot sub-carrier frequencies using known pilots [15]. Different methods can then be applied to estimate channel at all frequencies Hˆpk from raw channel estimate.

A. Pilot Symbol Insertion

Pilots can be allocated by three different ways in the time-frequency domain of an OFDM system as discussed below:
An entire OFDM symbol may be allocated as pilot as shown in figure 1 a. highly frequency-dispersive channels usually use such an allocation for channel estimation [16].

- Pilots may be transmitted on individual sub-carriers during the entire transmission period as shown in Figure 1b. Such a strategy will be advantageous in high Doppler spread affected channels [16].

- Pilots may be allocated in spaced intervals in time and frequency as illustrated in Figure 1c and Figure 1d. Depending upon the time-frequency pilot spacing and channel properties, such an allocation strategy will work well in both high frequency-selective and high Doppler spread channels [16].

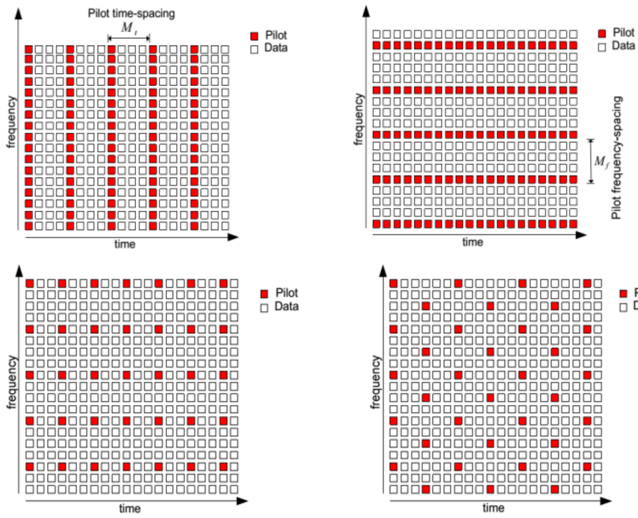

Fig. 1: Different pilot allocation possibilities [17]

III. CHANNEL ESTIMATION METHODS

Channel needs to be estimated in time as well as frequency domain in OFDM. Hence it is a two-dimensional problem. Therefore 2-D methods should be applied to estimate the channel from pilots. However, the scope of channel estimators can be limited to one-dimensional due to the computational complexity of 2-D estimators. The idea behind 1-D estimators is to estimate the channel in one dimension and later estimate the channel in the second dimension [18]. In this way they obtain a 2-D channel estimate. Channel estimation methods can be characterized into three types:

- Linear Interpolation
- Least-Squares Method
- Wiener filtering using second-order statistics of the channel (LMMSE method)

Linear Interpolation

Linear interpolation is the simplest method to estimate channel from raw channel estimates at pilot frequencies. This is done by linearly interpolating the raw channel estimates at the two nearest pilot sub-carriers. Although linear interpolation provides some limited noise reduction of the channel estimates at data locations, it is the simplicity of the solution that is attractive. Some additional gain may be obtained by averaging the interpolated channel estimates using a sliding window.it is noted that averaging window length (number of used pilots in window) is inversely proportional to the coherence bandwidth of the channel. For a channel with high frequency coherence bandwidth, a large number of interpolated channel coefficients may be used for averaging and vice versa [11].

General Linear Model (Least Squares Method)

The generalized linear model [16] uses a set basis function to estimate the channel transfer function. The basis functions could either be 2-D or 1-D. The channel estimation problem can be solved using 1-D generalized linear model framework [12]. The channel transfer function H_k can be modeled as a linear weighed sum of some basis function evaluated at the k^{th} sub-carrier frequency as:

$$H_k = \sum_{i=0}^{N-1} \theta_i \emptyset_i (f_k) \qquad (3)$$

Where

θ_i is the weighing factor of the basis function

$\emptyset_i (f_k)$ is the i_{th} basis function evaluated at that k_{th} sub-carrier frequency f_k

N is the number of basis functions used in the linear model.

The generalized linear model can be used to re-write the raw channel estimates, H_{pk} , in Equation (4) as

$$\hat{H}_{pk} = H_{pk} + Z_{pk} = \sum_{i=0}^{N-1} \theta_i \emptyset_i (f_{pk}) + Z_{pk} \qquad (4)$$

Where every p^{th} sub-carrier is a pilot,

H_{pk} is the actual channel transfer function at p_k-th carrier, Z_{pk} is the noise at the p_k-th carrier, and $\emptyset_i (f_{pk})$ is the i^{th} basis function evaluated at that p_k-th sub-carrier frequency f_k. The raw channel observations can be collected using matrix notation as:

$$\hat{H} = G.\,\theta + Z \qquad (5)$$

A least-squares estimate of the weighing matrix is given as

$$\hat{\theta}_{LS} = (G^H G)^{-1} G^H . \hat{H} \qquad (6)$$

The least-square estimate can then be used to estimate the channel at regular sub-carrier frequencies f_k as

$$\hat{H}_{LS,k} = \sum_{i=0}^{N-1} \hat{\theta}_{LS_i} \emptyset_i (f_{pk}) \qquad (7)$$

Wiener Filtering (LMMSE method)

Wiener filtering method uses knowledge of channel properties to estimate the unknown channel transfer function at non-pilot sub-carriers. These properties are assumed to be known at the receiver for the estimator to perform optimally. The Wiener filtering or Linear Minimum Mean Squares (LMMSE) estimator tries to minimize the expected mean-squared error between the actual and estimated channel. The theoretical framework for 1-D Wiener filtering is presented in [15].

The LMMSE $H_{LMMSE,k}$ estimate at k^{th} sub-carrier is calculated by filtering the of raw channel estimate vector by a Wiener filter C_{LMMSE} as follows

$$\hat{H}_{LMMSE,k} = C_{LMMSE}^H \hat{H} \qquad (8)$$

Where c_i is the i^{th} filter coefficients, and $\hat{H}_i$ is the i-th raw channel estimate. The Wiener filter coefficients are calculated as [10]

$$C_{LMMSE} = (R_h + \sigma^2 I)^{-1} r \qquad (9)$$

Where R_h is the autocorrelation matrix of the channel at pilot locations, σ^2 is the noise-variance per sub-carrier, and r is the cross-correlation vector of the channel at k^{th} sub-carrier and the channel at pilot locations [15].

IV. RESULTS

We now provide the simulation results using raw bit-error-rate (BER), as a measure of performance for the methods explained above. Raw BER is measured by counting the number of errors in estimated bits after equalization in the OFDM system. Performance of an OFDM system is shown with different estimation techniques. The plot of channel mean square error with EsN0 is shown in the graphs below:

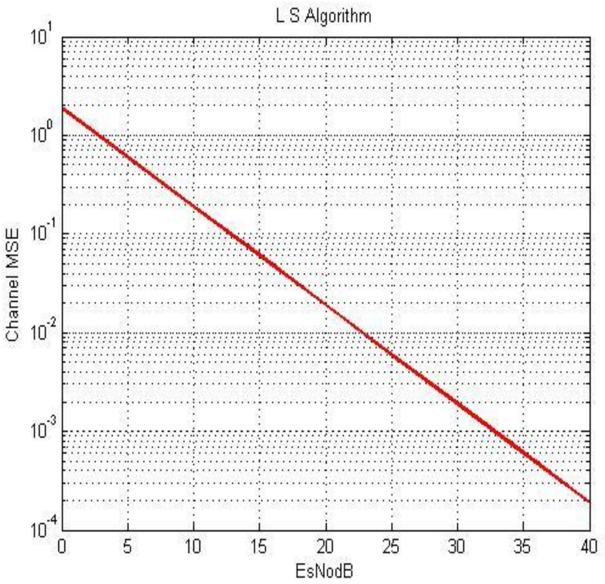

Fig. 2: Channel MSE v/s EsNodB for LS algorithm

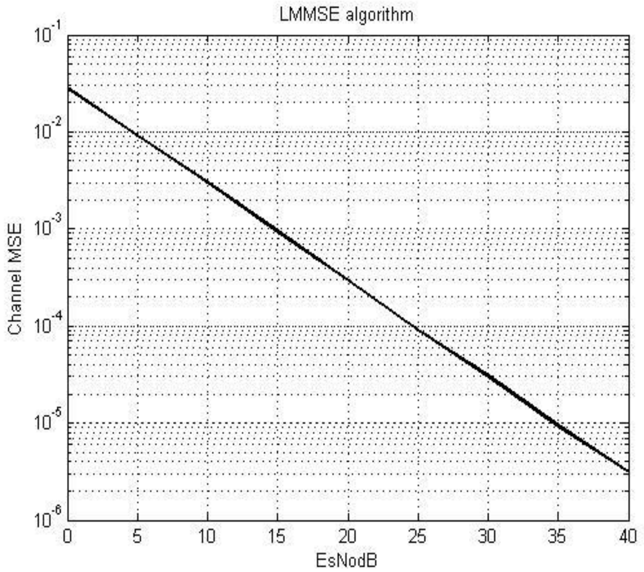

Fig. 2: Channel MSE v/s EsNodB for LMMSE algorithm

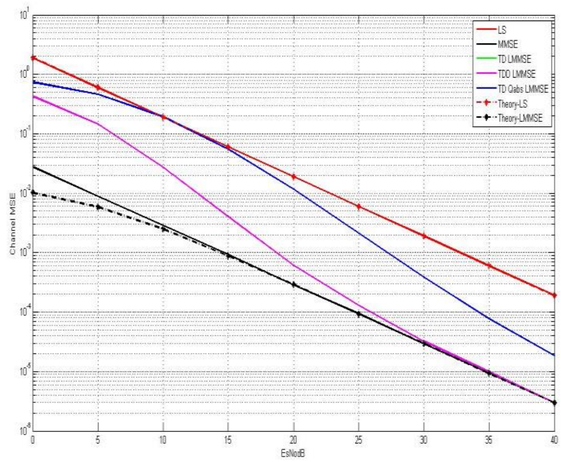

Fig. 2: Channel MSE v/s EsNodB for other different algorithm

V. CONCLUSION

In this paper, we have investigated three different frequency-based methods (interpolation, polynomial based model, and Weiner filtering) for channel estimation for an OFDM based system. The performance of these methods was simulated assuming perfect frequency synchronization and without receiver impairments. Raw BER was used a measure of performance evaluation. Interpolation was used for channel estimation in temporal-domain.

It was observed that the linear interpolation-based channel estimator works well for a channel with high coherence bandwidth (PA), but fails for a channel with low coherence bandwidth (PB). We also observe that averaging after linear interpolation helps to improve performance, as long as the averaging window length does not exceed the coherence bandwidth. The polynomial-based generalized linear model works well as long as the right selection of polynomial order and number of pilots (for calculating the channel estimator) is done. The polynomial order of 2 seems to be the optimal choice, and the number of pilots seems to depend upon the coherence bandwidth of the channel.

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