An Improved Wideband Covariance Matrix Sparse Representation (W-CMSR) Method for Wideband Direction of Arrival Estimation Using Simulated Annealing

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Abstract - In this paper, we optimize the size of aperture and number of sensors for Wideband Covariance Matrix Sparse representation (W-CMSR) method for Wideband Direction of Arrival estimation using Simulated Annealing. The performance of W-CMSR is obtained with initial number of sensors and aperture size. Then, we find the performance of W-CMSR with optimized number of sensors and aperture size using simulated annealing. We observe that same performance of W-CMSR is obtained with reduced number of sensors and aperture size by using simulated annealing.

Keywords - *Wideband Direction of Arrival estimation; Wideband Covariance Matrix Sparse representation; Simulated Annealing; sensor number; aperture size.*

I. INTRODUCTION

In a number of Sonar and Radar systems, Wideband signals are used frequently. Direction of Arrival (DOA) estimation has been proposed by many methods like ISSM [1] and CSSM [2]. We find Wideband DOA estimation by decomposing the incident Wideband signals into narrowband components with coherent or incoherent methods. The focusing matrices for wideband array processing has been explained in [3]. Test of orthogonality of projected subspaces (TOPS) and Weighted average of signal subspaces (WAVES) for wideband direction finding are dealt in [4] & [5] . Several methods have been considered for sparsity based DOA estimation [6], [7], [8]. The global matched filter (GMF) invented by Fuchs [6] is one of them. The technique of L1-SVD [7] invented by Malioutov considers the singular decomposition to reduce the dimension of observation. A family of convex functions is used to approach the L0-norm constraint and JLZA-DOA method [8] explains this approach.

In W-CMSR, the spacing restriction of half-wavelength in avoiding ambiguity is relaxed from highest to lowest signal frequency [9]. W-CMSR depends less on a priori information of incident signal number than subspace based methods. W-CMSR is immune to imperfect DOA pre-estimates such that no spectral decomposition or focusing is introduced. To improve the performance of DOA estimation, the a priori information of signal spectrum can be utilized.

In this paper, we optimize the size of aperture and number of sensors for Wideband Covariance Matrix Sparse representation (W-CMSR) method for Wideband Direction of Arrival estimation using Simulated Annealing. The performance of W-CMSR is obtained with initial number of sensors and aperture size. Then, we find the performance of W-CMSR with optimized number of sensors and aperture size using simulated annealing. We observe that same performance of W-CMSR is obtained with reduced number of sensors and aperture size by using simulated annealing.

In this paper, Section I contains the introduction, Section II explains the data model for W-CMSR, Section III explains the optimized number of sensors and aperture size for W-CMSR by using Simulated Annealing (SA) algorithm, Section IV contains the simulations and Section V discusses the conclusion.

II. DATA MODEL FOR W-CMSR

Consider M element array and K wideband signals encroaching on it from directions $\theta_1, \ldots \ldots, \theta_k$. We obtain N snapshots as shown in Fig1.
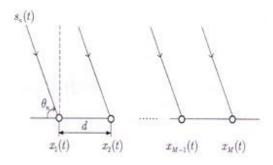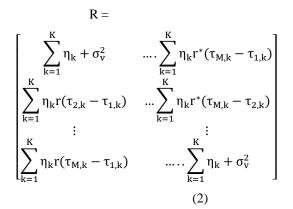

Fig.1: Uniform linear array of M sensors with interval d.

At time t, the snapshot is obtained as,

$$x(t) = [\textstyle\sum_{k=1}^{K} s_k(t + \tau_{1,k}) + v_1(t), \ldots \ldots \ldots, \sum_{k=1}^{K} s_k(t + \tau_{M,k}) + v_M(t)]^T \quad t = 1, ., N \qquad (1)$$

The propagation delay of the kth waveform is $\tau_{m,k}$, kth signal is $s_k(t)$ and the additive noise at the mth sensor is $v_m(t)$ [9] .We take the covariance matrix sparse representation for Wideband DOA estimation. The Wideband array output has the perturbation-free covariance matrix shown below and the variance of the additive noise is given as σ_v^2.

$$R =$$

$$\begin{bmatrix} \sum_{k=1}^{K} \eta_k + \sigma_v^2 & .... & \sum_{k=1}^{K} \eta_k r^*(\tau_{M,k} - \tau_{1,k}) \\ \sum_{k=1}^{K} \eta_k r(\tau_{2,k} - \tau_{1,k}) & ... & \sum_{k=1}^{K} \eta_k r^*(\tau_{M,k} - \tau_{2,k}) \\ \vdots & & \vdots \\ \sum_{k=1}^{K} \eta_k r(\tau_{M,k} - \tau_{1,k}) & ..... & \sum_{k=1}^{K} \eta_k + \sigma_v^2 \end{bmatrix}$$

$$(2)$$

Where $\eta_k(k = 1, ..., K)$ is the power of the kth signal, $r(\tau)$ is its unified correlation function [9]. The unknown noise variance contaminate the diagonal covariance elements. The array output's covariance matrix is conjugate symmetric. The lower left triangular elements are denoted by upper right triangular ones. The unknown noise variance contaminate the main diagonal elements and we slide over them and the lower left triangular elements are aligned column by column to get the measurement vector given below,where $R_{m1,m2}$denotes the (m_1 , m_2)th element of R [9].

y=[$R_{2,1}$,......$R_{M,1}$, $R_{3,2}$,.......,$R_{M,2}$,......,$R_{M-1,M-2}$,$R_{M,M-2}$,$R_{M,M-1}$]^T (3)

K components are obtained by splitting this vector,

$$y = \sum_{k=1}^{K} \eta_k y_k \qquad (4)$$

with,

$$y_k = [r(\tau_{2,k} - \tau_{1,k}), ..., r(\tau_{M,k} - \tau_{1,k}), ..., r(\tau_{M,k} - \tau_{M-1,k})]^T \qquad (5)$$

If the waveform components are separated then the waveform directions can be determined. At direction θ, the waveforms propagation delay from the reference point to the mth sensor is given by r_m^θ , then the unit power signal component is obtained as,

$$y^\theta = [r(\tau_2^{(\theta)} - \tau_1^{(\theta)}), ..., r(\tau_M^{(\theta)} - \tau_1^{(\theta)}), ..., r(\tau_M^{(\theta)} - \tau_{M-1}^{(\theta)})]^T \qquad (6)$$

The incident signals potential spatial scope is sampled to obtain a direction set Θ. If Δθ is taken as interval and we discretize the [-90^0,90^0] scope, then,

$$y = y^{(\Theta)} \eta \qquad (7)$$

where η is a sparse vector with non–zero values $\eta_k(k = 1, ..., K), y^\Theta = \{y^\theta | \theta \in \Theta\}$ [9]

For Wideband DOA estimation, we consider the above model and obtain the constrained sparsity-enforcing objective function,

$$\eta^\wedge = \arg \min_\eta ||\eta||_0, \text{subject to } y = y^\Theta \eta$$

$$(8)$$

where the spatial distribution of the incident waveforms is denoted by $\eta^\wedge$ and $L0 - norm$ is represented by $||.||_0$,

$$\eta^\wedge = \arg \min_\eta ||\eta||_1 \text{subject to } ||y^\wedge - y^\Theta \eta||_2 \leq \beta$$
(9)   where the fitting error threshold is β and the estimate of y is y$^\wedge$ [9] .

$$\beta = \mu \times \{\frac{M(M-1)}{2N}[\Xi(\sum_{k=1}^K \eta_k)^2 + 2\sigma_v^2(\sum_{k=1}^K \eta_k) + \sigma_v^4]\}^{1/2}$$
(10)

where μ is a weighting factor [9].

III. OPTIMIZED NUMBER OF SENSORS AND APERTURE SIZE FOR W-CMSR USING SIMULATED ANNEALING

The arrival direction of waveforms is determined by the method of Wideband direction of arrival. In the case of wireless communication for greater data rates, sound finding, radio telescopes, beam forming and smart antennas, the waveform direction is of much importance. The number of sensors, the array size of aperture, the SNR and the number of snapshots are the parameters on which the resolution of waveforms striking an array is dependent upon. The number of sensors and the aperture size of the array governs the design of the best array [10].

We take an equally spaced array where the size of aperture and number of sensors have to be determined using Simulated Annealing (SA) approach.

By keeping high resolution, the linear array with smallest aperture size and least number of sensors is optimized using SA method.

By using W-CMSR, the inaccuracy rate of one estimationis calculated. Previous_difference is the name given to this rate. One of the cost function can be considered as the difference between the previous_difference and the current_difference. Whether the previous sizes of aperture are bigger than the current ones and whether the previous sensor numbers are bigger than the current ones determines the other cost function. To get the design with decent probability, loops are applied to the whole process.

Algorithm for Simulated Annealing for Optimized number of sensors and aperture size for W-CMSR for wideband DOA estimation:

1. The total number of experiments, "the value of starting temperature, threshold, cooling rate", range of sensor numbers and aperture sizes are given.
2. If "iteration is less than threshold" is false, calculate optimum number of sensors and aperture sizes in all the experiments and end. If this condition is true, go to step 3.
3. If "iteration is less than threshold" is true, randomly select number of sensors and aperture sizes.
4. Estimate DOA by W-CMSR method.
5. Calculate error in estimation.
6. If "this error is less than the previous error and sensor numbers and aperture sizes are less than the previous ones" is true, then save the current value of sensor number and aperture size as optimum. Increase the value of iteration by 1 and go to step 2. Execute steps 2, 3, 4, 5, 6. Otherwise if this condition is false, go to step 7.
7. If "this error is less than previous error and sensor numbers and aperture sizes are less than the previous ones" is false, then check the condition "when exp(-diff/temp) > rand(1)". If this condition is true, save the current value of sensor number and aperture size as optimum. Increase the iteration by one and go to step 2. If the above condition is false, old no. of sensors and aperture sizes are retained as optimum. Increase the iteration by one and go to step 2.

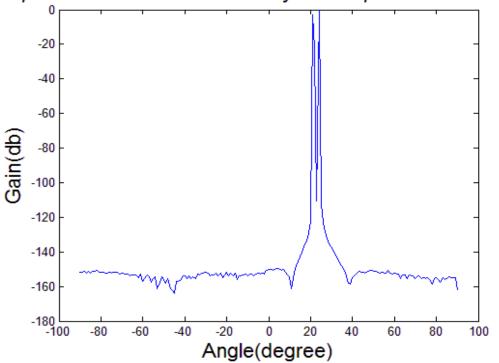The smallest size of aperture and the least number of sensors optimize the array design through simulation.
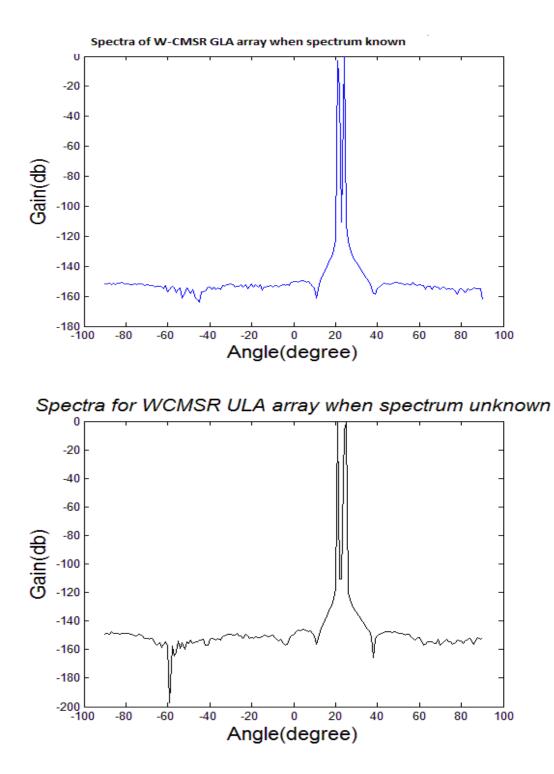
IV.   Simulations

For Wideband DOA estimation we study the performance of W-CMSR . A 7-ULA array is considered. The approaches of W-CMSR(GLA) an W-CMSR(ULA) are considered in 7-ULA. A priori information of source number is not required for W-CMSR. To look for solution of L1-norm based optimization problems, the optimization tool of SeDuMi [11] is used.

To illustrate the half-wavelength relaxation property of W-CMSR, we consider the relaxation of inter spacing of the array from half wavelength when signal frequency is highest

Case1: Consider two 0-dB BPSK waveforms with bandwidth of 40% and central frequency of 70 MHz encroach from directions 20° and 28° on a 7-ULA. We obtain 64 snapshots. The Direction of Arrival of Wideband signals is determined for W-CMSR with μ = 1.5. With respect to highest signal frequency, consider the interspacing of the array as half-wavelength. Fig.2 show that two sharp peaks are obtained in case of W-CMSR at 20° and 28°.

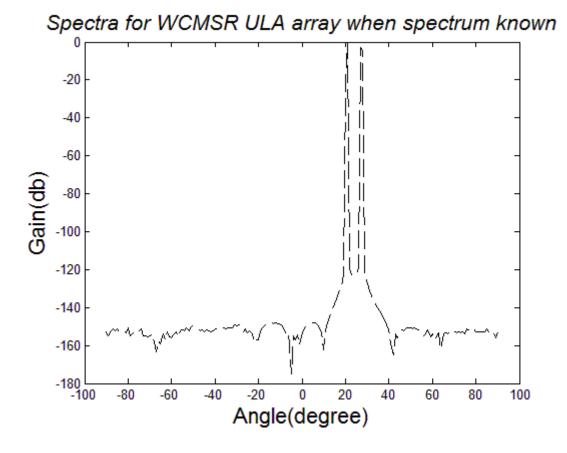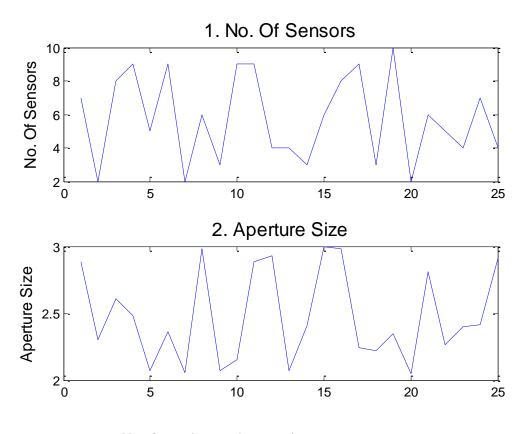Spectra for WCMSR GLA array when spectrum unknown

Spectra of W-CMSR GLA array when spectrum known

Spectra for WCMSR ULA array when spectrum unknown

Fig 2(a),(b),(c),(d)W-CMSR (Interspacing w.r.t High frequency): At DOA angles 20°,28°

Case 2 : During the experiment, two wideband BPSK signals with DOA in 20 degree and 28 degree acting as source signals. The chosen frequency is 70 MHz. The SNR is 0 db and 64 is the number of snapshots. For Simulated Annealing (SA) approach, the begining temperature is 200 degree, cooling rate is 0.75 and threshold is 50. The aperture size varies from 2 m to 3 m in steps of 0.01m and the sensors number is 2 to 10. 25 experiments are performed.

The smallest size of aperture and the least number of sensors optimize the array design through simulation. A best match of number of sensors and aperture size is obtained through this design and is considered to be accurate. The mean values of 25 trials by Simulated Annealing is 2.6 m for size of aperture and 4 for sensor number. Following figure 3 indicates the design of number of sensors and aperture size in 25 experiments.

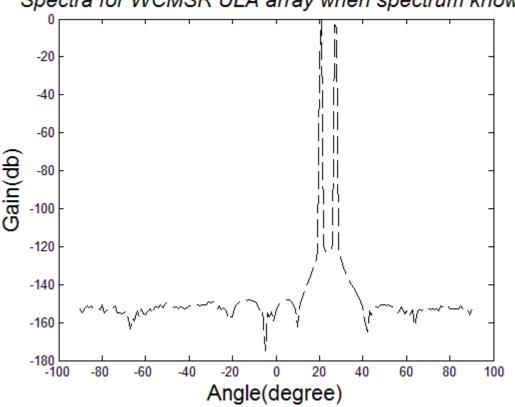**No. of experiments along x-axis.**

Fig 3   No. of sensors and Aperture size for DOA at 20 degree and 28 degree:

By Simulated Annealing (SA) approach, the number of sensors is reduced to 4 and the performance of W- CMSR method is still high as shown in the following figure 4.

Fig 4. The performance of W-CMSR is still high with reduced number of sensors as 4 and DOA of two sources at 20^0 and 28^0

Spectra for WCMSR ULA array when spectrum known

Fig 2 shows the performance of W-CMSR for number of sensors as 7 and Fig 4 shows the performance of W-CMSR for reduced number of sensors as 4 by using the concept of Simulated Annealing (SA). In Fig 4, similar performance of W-CMSR is obtained as in Fig 2 but with reduced number of sensors by using SA.

V. CONCLUSION

With respect to highest signal frequency, and by keeping half-wavelength as the interspacing of the array, we get two sharp peaks for W-CMSR at 20° and 28°. The aperture size and number of sensors are determined for an equally spaced array using the Simulated Annealing approach. The smallest aperture size and least number of sensors with high resolution are optimized using SA technique. A suitable match of number of sensors and aperture size is obtained through this design which is considered to be accurate. The mean values of 25 trials by Simulated Annealing is 2.6 m for the aperture size and 4 for the sensor number. By Simulated Annealing (SA) approach, the number of sensors is reduced to 4 and the performance of W-CMSR method is still high.

VI. REFERENCES

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