Calculus II Sample Final Questions

1. Integrate the following

(i) $\int \sin^2 x \, \cos^3 x \, dx$ (ii) $\int x \ln x \, dx$ (iv) $\int x \sin 2x \, dx$

(iv) $\int \dfrac{dx}{x^2+3x+2}$ (v) $\int \dfrac{x}{\sqrt{1-x^2}}\, dx$ (vi) $\int_0^\infty x e^{-x^2}\, dx$

$(viii)$ $\int \dfrac{dx}{(x^2+1)^{3/2}}$ $(viii)$ $\int \dfrac{x\, dx}{(x-1)(x-2)^2}$ (ix) $\int \dfrac{dx}{x\,(x^2+1)}$

(x) $\int x\, e^{-3x}\, dx$ (xi) $\int_0^1 \dfrac{dx}{\sqrt{1-x^2}}$ (xii) $\int \dfrac{dx}{x^2\sqrt{x^2-4}}$

2. Do the following

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ (iii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

(iv) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$ (v) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ (vi) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n \ln n}$

(vii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$ $(viii)$ $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$ (ix) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2}+\frac{1}{n}\right)^n$

3. Calculate the 4^{th} degree Taylor polynomial with remainder for the following. Expand about the point $x = c$ given.

$$(i) \quad f(x) = \sin x \quad \text{about} \quad x = \pi$$

$$(ii) \quad f(x) = \frac{1}{x+2} \quad \text{about} \quad x = 1$$

4. Determine the radius and interval of convergence of the following.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(4x)^n}{(n+1)!}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n\,3^n}$

5. Polar Areas

(i) Find the area inside both $r = \sin\theta$ and $r = \cos\theta$.

(ii) Find the area inside $r = 2\cos\theta$ and outside $r = 1/2 + \cos\theta$.

6. Planes and Lines

(i) Find the equation of the plane contains the lines

$$\begin{aligned} x &= -1 + t, \quad & x &= 2 - s \\ y &= 1 + t, \quad & y &= s \\ z &= 2t, \quad & z &= 2 \end{aligned}$$

(ii) Find the equation of the plane that contains the points $P(1,1,3)$, $Q(-2,4,-3)$ and $R(3,-4,4)$

(iii) Find the equation of the line perpendicular to the plane in part (i) through the point P.

(iv) Find the equation of the line through P and Q in part (ii).

7. Vector Projections

Find the projection of $\vec{u}$ onto $\vec{v}$ and its orthogonal complement for the following:

$$\begin{aligned} (i)\ \ \vec{u} &= <-1,3>, \quad \vec{v} = <2,2>, \\ (ii)\ \ \vec{u} &= <5,5>, \quad \vec{v} = <1,2>. \end{aligned}$$