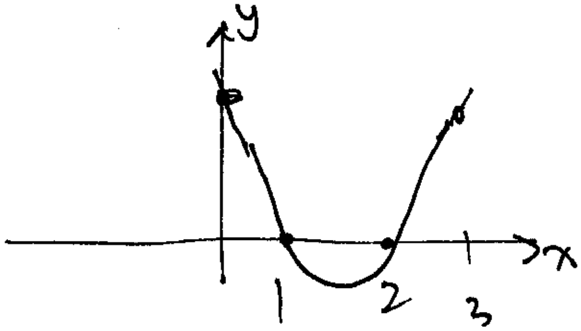
Math 1496 - Calc I

Consider $f(x) = x^2 - 3x + 2 = (x-1)(x-2)$



$f(0) = 2$

$f(3) = 9 - 9 + 2 = 2$

we say that there is a pt where $f' = 0$

<u>Rolle's Th^m</u> Let $f(x)$ be cont^s on $[a,b]$ and diff^ble on (a,b). If $f(a) = f(b)$ then there is at least 1 c in (a,b) such that $f'(c) = 0$.

In the previous example when

$f(0) = f(3)$

then $f'(x) = 2x - 3$

& $f'(3/2) = 2(3/2) - 3 = 0$ ✓

$c =$

$\downarrow$

$0 < 3/2 < 3$

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$f(x) = (x-4)(x+2)^2$ on $[-2, 4]$

1st – Can we use Rolle's Thm? – Yes, why

(i) f is cont's on $[-2, 4]$ diff'ble on $(-2, 4)$

(ii) $f(-2) = 0$, $f(4) = 0$ ✓

So there is at least 1 c in $(-2, 4)$

$f' = 1(x+2)^2 + (x-4)\,2(x+2)$

$= (x+2)(x+2+2(x-4))$

$= (x+2)(3x-6)$

$= 3(x+2)(x-2)$

Here's the one

So $f'(x) = 0$ when $x = -2, +2$

Note: Even though $f'(-2) = 0$

$x = -2$ is not in $(-2, 4)$

ex $f(x) = -x^4 + 2x^2 + 1$ on $[-2,2]$

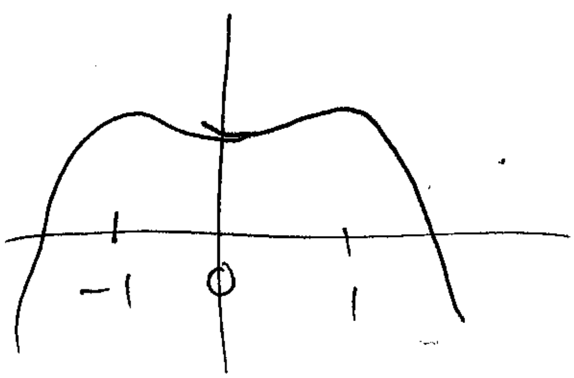
f is cont^s on $[-2,2]$ & diff on $(-2,2)$

$f(-2) = -7$ $f(2) = -7$ so Rolle's Th^m applies

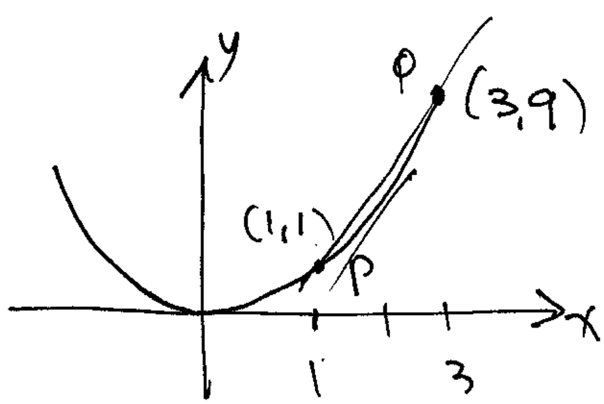
$f'(x) = -4x^3 + 4x$

$= -4x(x^2-1) = -4x(x-1)(x+1)$

$f' = 0$ when $x = 0, \pm 1$ so 3 places



Consider $f(x) = x^2$ on $[1,3]$



Consider the slope of line through P & Q

$m = \frac{\Delta y}{\Delta x} = \frac{9-1}{3-1} = \frac{8}{2} = 4$

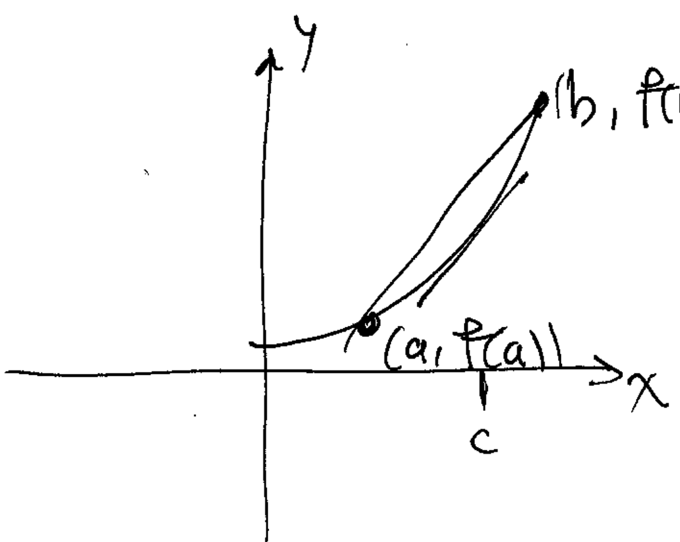
there is a pt ($x=c$) on $(1,3)$ such that the slope of the tangent to $f(x)=x^2$ is 4

$f'(x)=2x \quad 2c=4 \quad c=2$ in $(1,3)$ ✓

Mean Value Th^{m}

If $f(x)$ is cont^{s} on $[a,b]$ and diff^{ble} on (a,b) then there exists a # c in (a,b) such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$



Proof: Construct the eq^{n} of the line

$$m=\frac{f(b)-f(a)}{b-a} \qquad y-f(a)=\frac{f(b)-f(a)}{b-a}(x-a)$$

Define $g(x) = f(x) - y$

$$= f(x) - \frac{f(b)-f(a)}{b-a}(x-a) \cancel{+} f(a)$$

clearly $g(a) = 0$ $g(b) = 0$

$\because$ f is cont^s on $[a,b]$ & diff (a,b)

Then so is $g(x)$ (we are just ~~adding~~ subtracting a linear term)

So by Rolle's Thm there exists a c in (a,b)

$g'(c) = 0$

Now $g'(x) = f'(x) - \dfrac{f(b)-f(a)}{b-a}$

$g'(c) = 0 \Rightarrow f'(c) - \dfrac{f(b)-f(a)}{b-a} = 0$

$\Rightarrow$ $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ ▯