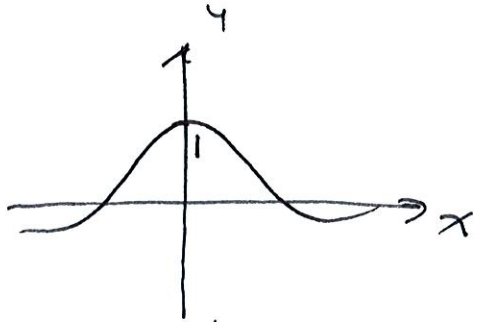
# Math 1496 – Calc 1

In a HW problem (and in class)

we considered

$$\lim_{x \to 0} \frac{\sin x}{x} \qquad \lim_{x \to 0} \frac{\sin 2x}{x}$$




| $x$ | $-.1$ | $-.01$ | $-.001$ | $0$ | $.001$ | $.01$ | $.1$ |
|---|---|---|---|---|---|---|---|
| $y$ | $.998$ | | $.9\dot{9}$ | $?$ | $.9\dot{9}$ | $.$ | $.998$ |

$.99998\dot{3}$

<u>Graphically</u>

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

<u>Numerically</u>

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \quad \text{(which agrees)}$$

Now we create an analytical technique

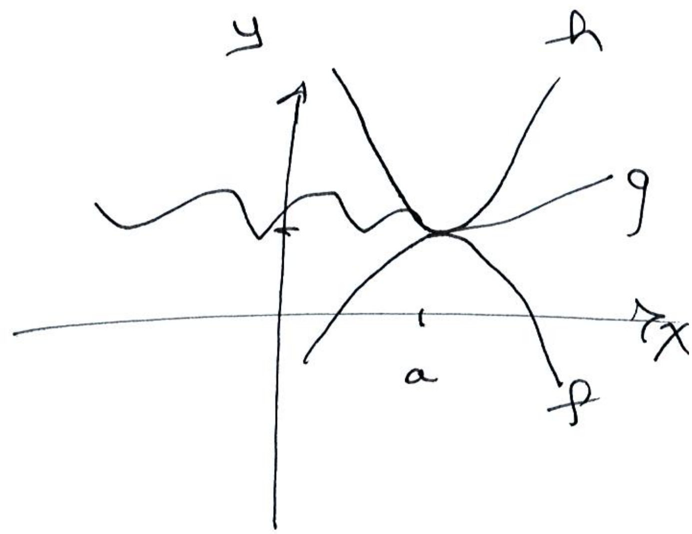
<u>Squeeze Thm</u>

If $f(x) \le g(x) \le h(x)$

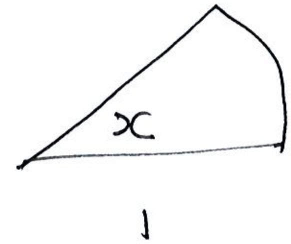
near $x = a$

$$\lim_{x \to a} f(x) = L \quad \lim_{x \to a} h(x) = L$$

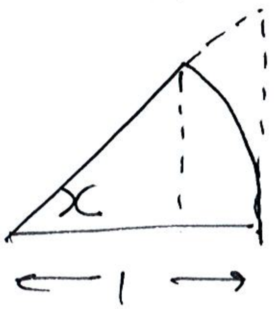
then $\displaystyle \lim_{x \to a} g(x) = L$



Consider the area of a sector of radius 1 & angle $x$

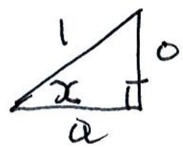



we create 2 triangle - one smaller with hypotenuse 1 and one large with base of 1




area small triangle $\leq$ area sector $\leq$ area large triangle

(1) <u>small triangle</u>



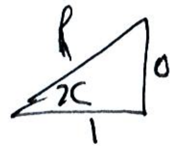

$\sin x = \frac{o}{1}$, $\cos x = \frac{a}{1}$

$$A = \frac{1}{2}bh = \frac{1}{2}\sin x \cos x$$

(2) <u>sector</u> $\frac{A_S}{A_C} = \frac{x}{2\pi} \Rightarrow A_S = \frac{x}{2\pi}\pi 1^2 = \frac{xr^2}{2}$

with $r=1$ $A_S = \frac{x}{2}$

(3) large triangle




$\tan x = \frac{o}{1}$ $a=1$

$$A = \frac{1}{2}bh = \frac{1}{2}\tan x$$

so $\frac{1}{2}\sin x \cos x \leq \frac{1}{2}x \leq \frac{1}{2}\frac{\sin x}{\cos x}$

$$\Rightarrow \cos x \leq \frac{\sin x}{x} \leq \frac{1}{\cos x}$$

$\because \lim_{x\to 0}\cos x = 1$ $\lim_{x\to 0}\frac{1}{\cos x} = 1$ by sq thm $\lim_{x\to 0}\frac{\sin x}{x} =$

Now consider

$$\lim_{x\to 0} \frac{\sin 2x}{x} = 2 \quad \text{from HW}$$

Now $\sin 2x \neq 2\sin x$

but $\sin 2x = 2\sin x \cos x$

so $$\lim_{x\to 0} \frac{\sin 2x}{x} = \lim_{x\to 0} \frac{2\sin x \cos x}{x}$$

$$= 2 \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \cos x = 2(1)(1) = 2 \checkmark$$

$$\lim_{x\to 0} \frac{\sin 3x}{x} \quad \text{Don't expand}$$

instead $$\lim_{x\to 0} \frac{\sin 3x}{3x} \cdot 3$$

let $t = 3x \quad x \to 0 \quad t \to 0$

& switch limit

$$\lim_{t\to 0} \frac{\sin t}{t} \cdot 3 = 3 \lim_{t\to 0} \frac{\sin t}{t} = 3$$

In general

$$\lim_{x\to 0} \frac{\sin kx}{x} = k$$

Do $\lim_{x\to 0} \frac{\sin 3x}{\sin 5x}$

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = \text{"}\frac{0}{0}\text{"}$$

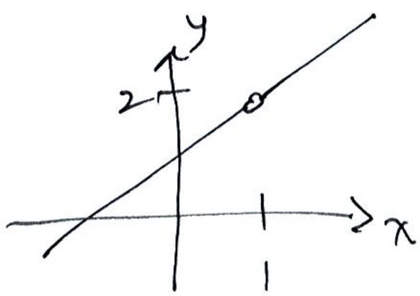
$$\lim_{x \to 0} \frac{\cos x - 1}{x} \cdot \frac{\cos x + 1}{\cos x + 1} = \lim_{x \to 0} \frac{\cos^2 x - 1}{x(\cos x + 1)}$$

$$= \lim_{x \to 0} \frac{-\sin^2 x}{x(\cos x + 1)} = -\lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{\sin x}{\cos x + 1}$$

$$= -(1)\left(\frac{0}{2}\right) = 0$$

It's important to realize not all limits exists.

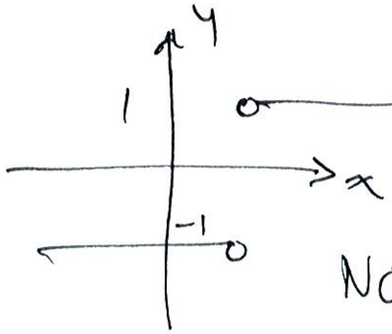
Ex




$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$$

even though there's no graph at $x = 1$

Ex




$f = \frac{|x-2|}{x-2}$ (HW)

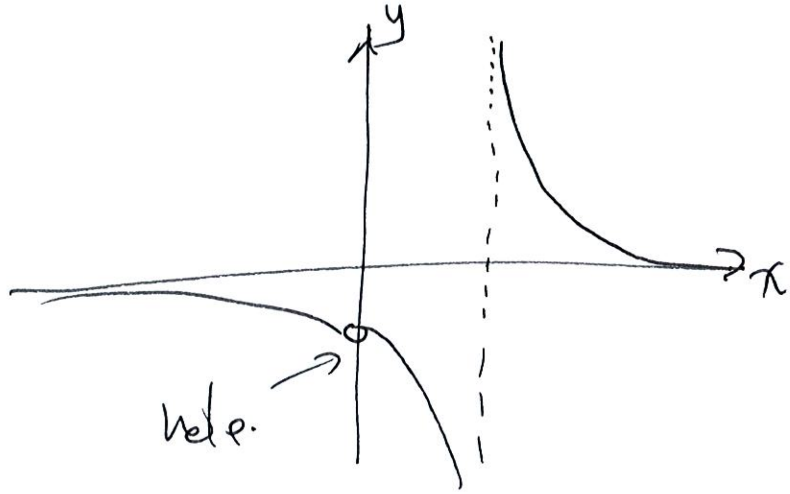
Now it dependent on how we approach $x = 2$ from the left

$$\lim_{x \to 2^-} f = -1, \quad \lim_{x \to 2^+} f = 1$$

these are different

Ex $f(x) = \frac{x}{x^2 - x} = \frac{x}{x(x-1)}$

what happens near $x = 0, 1$




So $\lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(x-1)} = \lim_{x \to 0} \frac{1}{x-1} = -1$ (so limit does exist)

but $\lim_{x \to 1} \frac{x}{x(x-1)}$ DNE.

and in fact it really depends on which side of $x = 1$ we approach - left or right

more ..... tomorrow! :)