Math 2471-Sample Test 1

1. Find the unit tangent and unit normal vector for the following vector functions

$$(i) \quad \vec{r}(t) = < 2t, t^2 >$$
$$(ii) \quad \vec{r}(t) = < e^t \cos t, e^t \sin t >$$

2. Prove the limits either exist or do not exist. In the former case use the squeeze theorem.

$$(i) \lim_{(x,y)->(0,0)} \frac{x^2 + xy + y^2}{x^2 + y^2} \qquad (ii) \lim_{(x,y)->(0,0)} \frac{y - x^3}{y + x^3}$$
$$(iii) \lim_{(x,y)->(0,0)} \frac{x^3 + y^3}{x^2 + y^2} \qquad (iv) \lim_{(x,y)->(0,0)} \frac{x^4 + 2y^4}{x^2 + y^2}$$

3. Sketch and name the following surfaces

$$(i) \quad -x^2 + y^2 + z^2 = 1 \qquad (ii) \quad -x^2 + y^2 - z^2 = 1$$
$$(iii) \quad x^2 + y^2 - z = 0 \qquad (iv) \quad -y^2 + z = 0$$

4. Find the domain and range of the following functions. Sketch the domain in the xy plane.

$$(i) \quad z = \sqrt{xy} \qquad (ii) \quad z = \frac{y}{x}$$

5. Find the first and second order partial derivatives (z_x, z_y, z_{xx}, z_{xy} and z_{yy}) for

$$z = \ln(x^2 + y^2)$$

and show

$$z_{xx} + z_{yy} = 0.$$

6. Find the equation of the tangent plane to the given surface at the specified point

$$x^2y + xz + yz^2 = 3, \quad P(1, 2, -1)$$