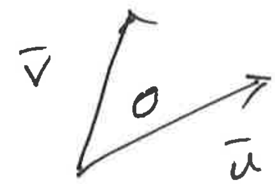
Math 1497 Calc 2

Last class we considered multiplying vectors and introduced the dot product & cross product

If $\bar{u} = \langle u_1, u_2 \rangle$, $\bar{v} = \langle v_1, v_2 \rangle$

then $\bar{u} \cdot \bar{v} = u_1 v_1 + u_2 v_2$

$= \|\bar{u}\| \, \|\bar{v}\| \cos\theta$



Two definitions ↑ for dot product

and cross product

$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

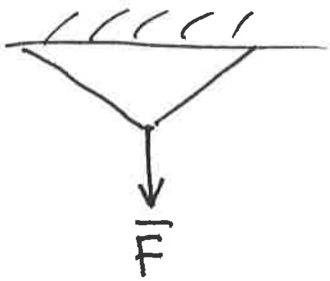
$$= \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \vec{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \bar{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \bar{k}$$

Important to note

$\bar{u} \cdot \bar{v} = \#$ scalar $\qquad \bar{u} \cdot \bar{v} = 0 \Leftrightarrow \bar{u} \perp \bar{v}$

$\bar{u} \times \bar{v} =$ vector

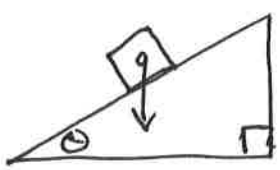
Projections



in this example their are 2 Forces along the cables and overall resultant $\downarrow$ is F and overall $\longleftrightarrow$ is zero

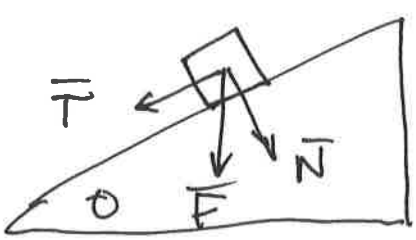
So this is the idea of projections

Another, we have a force due to gravity



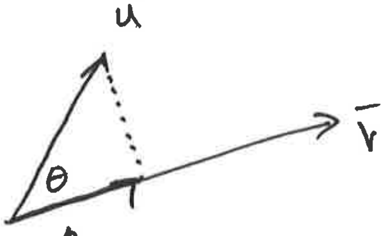
on the block. This force is a combination of 2 forces

(i) normal to surface
(ii) tangent to surface



So given a vector $\bar{u}$, we wish to project it onto a vector $\bar{v}$.

Called $\text{proj}_{\bar{v}}\,\bar{u}$



this vector here is the projection of $\bar u$ onto $\bar v$ and it is this vector we wish to calculate. First, I think it is clear that it is some scalar multiple of $\bar v$ as it pts in the same direction as $\bar v$

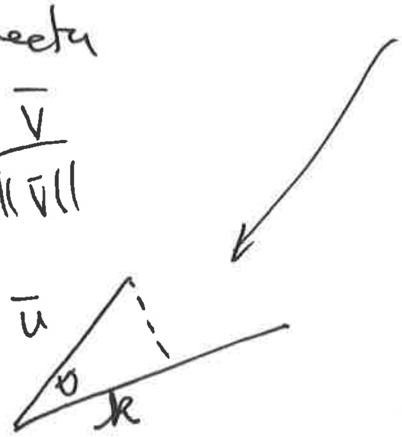
So 1st, let us create a unit vector in $\bar v$'s direction i.e. $\dfrac{\bar v}{\|\bar v\|}$

unit vector $\dfrac{\bar v}{\|\bar v\|}$

so $\text{proj}_{\bar v}\bar u = \dfrac{k\,\bar v}{\|\bar v\|}$

from triangle

$$\cos\theta = \frac{k}{\|\bar u\|} \Rightarrow k = \|\bar u\|\cos\theta$$



so $\text{proj}_{\bar v}\bar u = \left(\dfrac{\|\bar u\|\cos\theta}{\|\bar v\|}\right)\bar v$

$$= \frac{\|\bar u\|\|\bar v\|\cos\theta}{\|\bar v\|^2}\,\bar v = \left(\frac{\bar u\cdot\bar v}{\bar v\cdot\bar v}\right)\bar v$$

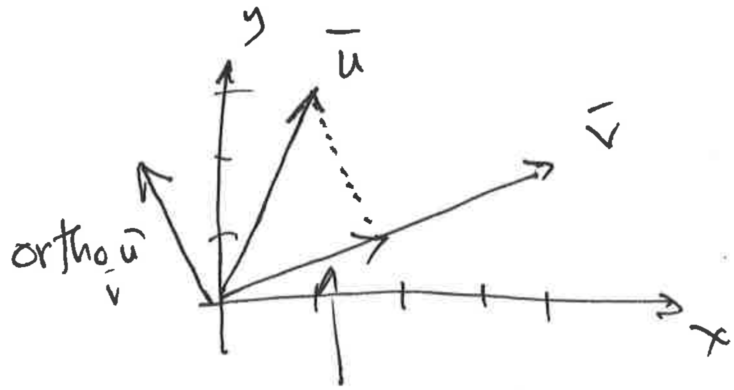
so we have

$$\operatorname{proj}_{\bar{v}} \bar{u} = \left(\frac{\bar{u} \cdot \bar{v}}{\bar{v} \cdot \bar{v}}\right) \bar{v}$$

Ex If $\bar{u} = \langle 1, 3 \rangle$ & $\bar{v} = \langle 4, 2 \rangle$

find $\operatorname{proj}_{\bar{v}} \bar{u}$

$$\bar{u} \cdot \bar{v} = \langle 1, 3 \rangle \cdot \langle 4, 2 \rangle$$
$$= 4 + 6 = 10$$
$$\bar{v} \cdot \bar{v} = \langle 4, 2 \rangle \cdot \langle 4, 2 \rangle$$
$$= 16 + 4 = 20$$



$\operatorname{proj}_{\bar{v}} \bar{u}$ looks like about $\frac{1}{2} \bar{v}$

$$\operatorname{proj}_{\bar{v}} \bar{u} = \left(\frac{\bar{u} \cdot \bar{v}}{\bar{v} \cdot \bar{v}}\right) \bar{v}$$
$$= \frac{10}{20} \bar{v}$$
$$= \frac{1}{2} \langle 4, 2 \rangle = \langle 2, 1 \rangle$$

there's another vector that we want to identify

called the orthogonal complement

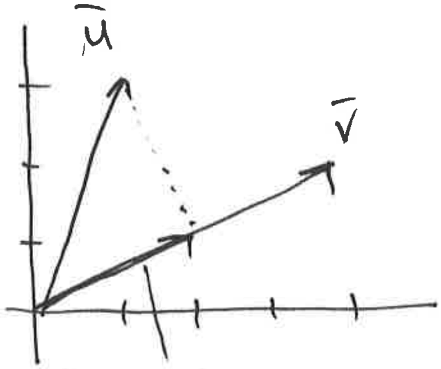
$$\bar{u} - \operatorname{proj}_{\bar{v}} \bar{u} = \operatorname{ortho}_{\bar{v}} \bar{u} = \langle 1, 3 \rangle - \langle 2, 1 \rangle$$
$$= \langle -1, 2 \rangle$$

These 2 compose to give $\bar{u}$

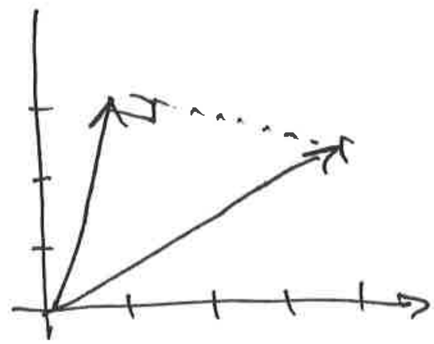
It's important to realize that

$$\text{proj}_{\bar{v}}\bar{u} \neq \text{proj}_{\bar{u}}\bar{v}$$

As we will show via the previous ex



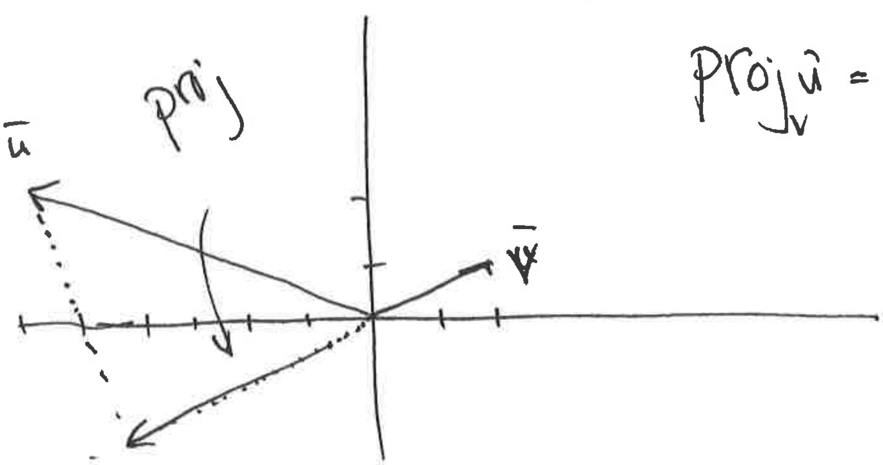
$\text{proj}_{\bar{v}}\bar{u}$



$$\text{proj}_{\bar{u}}\bar{v} = \left(\frac{\bar{u}\cdot\bar{v}}{\bar{u}\cdot\bar{u}}\right)\bar{u} = \frac{10}{10}\bar{u} = \langle 1, 3\rangle$$

$\bar{u}\cdot\bar{u} = 1^2+3^2 = 10$ so we see they are different

<u>Ex2</u> Find $\text{proj}_{\bar{v}}\bar{u}$ when $\bar{u} = \langle -6, 2\rangle$ $\bar{v} = \langle 2, 1\rangle$



$$\text{proj}_{\bar{v}}\bar{u} = \frac{-12+2}{4+1}\bar{v} = \frac{-10}{5}\bar{v}$$

$$= -2\bar{v}$$

$$= \langle -4, -2\rangle$$

$$\text{ortho}_{\bar{v}}\,\bar{u} = \bar{u} - \text{proj}_{\bar{v}}\,\bar{u}$$

$$= \langle -6, 2 \rangle - \langle -4, -2 \rangle$$

$$= \langle -6+4, 2+2 \rangle = \langle -2, 4 \rangle$$