

**Corporate Finance, Berk, Jonathan and Demarzo, Peter, 4<sup>th</sup> Edition  
Chapter 18: Capital Budgeting and Valuation with Leverage**

**Key Points:**

- Formulas for the Weighted Average Cost of Capital
- Discuss the WACC method, APV method and FTE method
- Discuss project based WACC
- Discuss which method to use for different leverage policies

**18.1 Overview of Key Concepts**

- There are three methods discussed in this chapter to value a firm:
- The WACC Method, the APV Method and the FTE Method.
- You should become familiar with the three methods and in what circumstances it is best to use each method.

**18.2 The Weighted Average Cost of Capital Method (WACC)**

The WACC method takes the interest tax shield concept into account by using the after tax cost of capital as the discount rate.

$$r_{WACC} = \frac{E}{E + D} r_E + \frac{D}{E + D} r_D (1 - T_c)$$

$E$  = Market Value of Equity

$r_E$  = Equity Cost of Capital

$D$  = Market Value of Debt (net of cash)

$r_D$  = Debt Cost of Capital

$T_c$  = Marginal corporate tax rate

If we assume a constant Debt-to-Equity ratio and that the WACC remains constant, we can calculate the Levered Value of a firm.

**Memorization: Definition**

**Levered Value of an Investment:** Present value of a firm's future free cash flow using the firm's WACC.

The investment's initial levered value  $V_0^L$  is:

$$V_0^L = \frac{FCF_1}{1 + r_{WACC}} + \frac{FCF_2}{(1 + r_{WACC})^2} + \frac{FCF_3}{(1 + r_{WACC})^3} + \dots$$

Using the WACC to Value a Project:

One can use the Free Cash Flows and the WACC to calculate the Levered Value of a Firm.

**Example:**

A firm has a project that requires a \$24M upfront investment and \$6.67M of upfront marketing expenses. The project produces the following Incremental Earnings forecast for the next 4 years and resulting balance sheet:

Year	0	1	2	3	4
Sales	-	60.00	60.00	60.00	60.00
Cost of Goods Sold	-	(25.00)	(25.00)	(25.00)	(25.00)
<b>Gross Profit</b>	-	35.00	35.00	35.00	35.00
Operating Expenses	(6.67)	(9.00)	(9.00)	(9.00)	(9.00)
Depreciation	-	(6.00)	(6.00)	(6.00)	(6.00)
<b>EBIT</b>	(6.67)	20.00	20.00	20.00	20.00
Income Tax (40%)	2.67	(8.00)	(8.00)	(8.00)	(8.00)
<b>Unlevered Net Income</b>	(4.00)	12.00	12.00	12.00	12.00
<b>Free Cash Flow</b>					
Plus: Depreciation		6.00	6.00	6.00	6.00
Less: Capital Expenditures	(24.00)				
Less: Increase in NWC	-	-	-	-	-
<b>Free Cash Flow</b>	<b>(24.00)</b>	<b>18.00</b>	<b>18.00</b>	<b>18.00</b>	<b>18.00</b>

Assets		Liabilities & Equity		Cost of Capital	
Cash	20	Debt	320	Debt	6%
Existing Assets	600	Equity	300	Equity	10%
Total Assets	620	Total	620		

Calculate the projects Levered Value and determine if the firm should take the project.

**Solution:**

Calculate the firms WACC using the balance sheet above. Note: first you must calculate the firm's net debt amount. Net Debt = Debt – Cash = 320 – 20 = 300

$$r_{WACC} = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D (1+T_C) = \frac{300}{300+300} 10\% + \frac{300}{300+300} 6\% (1-40\%) = 6.8\%$$

You can determine the projects Levered Value by discounting the projects Free Cash Flows with the calculated WACC.

$$V_0^L = \frac{18}{1.068} + \frac{18}{1.068^2} + \frac{18}{1.068^3} + \frac{18}{1.068^4} = \$61.25$$

The project costs \$24M in upfront investment plus \$6.67M in marketing expenses. However, we save taxes on the \$6.67M of \$2.67M. Therefore, the true cost at time 0 is \$24M + \$4M = \$28M. Therefore, since the project generates \$61.25M it has an NPV of \$33.25M (=61.25-28).

**Summary of the WACC Method:**

The most commonly used method in practice for capital budgeting purposes. The firm can use the WACC as the companywide cost of capital for new investments that are comparable to the rest of the firm and that will not alter the firm’s debt-to-equity ratio.

**Memorization: Concept**

**The WACC Method:**

1. Determine the Free Cash Flow of the investment
2. Computer the WACC
3. Computer the value of the investment, including the tax benefit of leverage, by discounting the Free Cash Flow of the investment using the WACC.

**Implementing a Constant Debt-Equity Ratio:**

By taking the project above, the firm has increased its assets by  $V_0^L = \$61.25M$ , in order to maintain the firm’s constant Debt-to-Value ratio of 50%, the firm must add  $50\% \times 61.25M = \$30.625M$  in new debt. Instead of borrowing the full 30.625M, the firm uses \$20M of cash and borrows the remaining \$10.625M. The firm’s balance sheet would look like this after the project:

Assets		Liabilities & Equity	
Cash	-	Debt	320
Current Assets	600	New Debt	10.625
RFX Project	61.25	Equity	300
		New Equity	30.625
Total	661.25	Total	661.25

The cost of the project is only \$28M but the firm has added \$30.625M in debt and cash. So the firm pays the difference to shareholders \$2.625M ( $30.625 - 28 = 2.625$ ) as a dividend.

So the total value that the shareholders get is the increase in equity of \$30.625M and the dividend of \$2.625M for a total of \$33.25M, which is the NPV we calculated previously.

**Memorization: Definition**

**Debt Capacity:** the amount of debt at date “t” that is required to maintain the firm’s target debt-to-value ratio, “d”.

$$D_t = d \times V_t^L$$

where  $V_t^L$  is the project’s levered value of Free Cash Flow after time “t”

$$V_t^L = \frac{FCF_{t+1} + V_{t+1}^L}{1 + r_{WACC}}$$

Example:

Assume we use the above Free Cash Flow and WACC (6.8%), we can determine the project's levered value at time "t" and the resulting Debt Capacity.

Year	0	1	2	3	4
Free Cash Flow	(28.00)	18.00	18.00	18.00	18.00
Levered Value	61.25	47.41	32.63	16.85	-
Debt Capacity	30.62	23.71	16.32	8.43	-

The Levered Value at time 0 is \$61.25M (as calculated above) and the Debt Capacity with a constant Debt-to-Value (d) ratio of 50% is \$30.62M. The WACC above was 6.8%.

To calculate the Levered Value at time 1 use the formula above:

$$V_0^L = \frac{FCF_1 + V_1^L}{1 + r_{WACC}} \Rightarrow 61.25 = \frac{18.00 + V_1^L}{1 + 6.8\%} \Rightarrow V_1^L = 61.25 \times 1.068 - 18 = 47.415$$

Then the Debt Capacity is:

$$D_1 = d \times V_1^L = 50\% \times 47.415 = 23.7075$$

### 18.3 Adjusted Present Value (APV)

The Adjusted Present Value (APV) method splits the levered value of a firm into the unlevered value of a firm and the present value of interest tax shields.

$$V^L = APV = V^U + PV(\text{Interest Tax Shield})$$

#### The Unlevered Value of a Project:

To calculate the unlevered value of a firm you first need to calculate the Pre-Tax WACC.

$$\text{Pre-Tax WACC} = r_U = \frac{E}{E + D} r_E + \frac{D}{E + D} r_D$$

Note: the tax shield will have the same risk as the firm if the firm maintains a target leverage ratio.

#### **Memorization: Definition**

**Target Leverage Ratio:** a firm adjusts its debt proportionally to the project's value or its cash flows.

For the firm above, we had a Debt-to-Value ratio of 50%, an Equity Cost of Capital of 10% and a Debt Cost of Capital of 6%. Therefore, the unlevered cost of capital  $r_U$  is:

$$r_U = 0.50 \times 10\% + 0.50 \times 6\% = 8.0\%$$

Then the unlevered value of the project is:

$$V^U = \frac{18}{1.08} + \frac{18}{1.08^2} + \frac{18}{1.08^3} + \frac{18}{1.08^4} = 59.62$$

**Valuing the Interest Tax Shield:**

We have calculated the unlevered value of the project and now we need to compute the present value of the interest tax shields. To do this we will need to first compute the interest paid in each year.

Interest paid in year 't' =  $r_D \times D_{t-1}$

Interest Tax Shield in year 't' = Interest paid in year 't' x  $T_C$

Year	0	1	2	3	4
Debt Capacity $D_t$	30.62	23.71	16.32	8.43	-
Interest Paid ( $r_D=6\%$ )		1.84	1.42	0.98	0.51
Interest Tax Shield (Tc=40%)		0.73	0.57	0.39	0.20

**Memorization: Concept**

When the firm maintains a target leverage ratio, its future interest tax shields have similar risk to the project's cash flows, so they should be discounted at the project's unlevered cost of capital.

$$PV(\text{Interest Tax Shield}) = \frac{0.73}{1.08} + \frac{0.57}{1.08^2} + \frac{0.39}{1.08^3} + \frac{0.20}{1.08^4} = 1.63$$

Therefore, the Levered Value of the firm using the APV method is:

$$V^L = V^U + PV(\text{Int\_Tax\_Shield}) = 59.62 + 1.63 = 61.25$$

Summary of the APV Method:

1. Determine the investment's value without leverage  $V^U$ , by discounting its free cash flows at the unlevered cost of capital,  $r_U$ . With a constant debt-to-equity ratio,  $r_U$  may be estimated using the Pre-Tax WACC.
2. Determine the present value of the interest tax shield.

- a. Determine the expected interest tax shield: Given the expected debt  $D_t$  on date 't', the interest tax shield on date 't+1' is  $T_c r_D D_t$ .
  - b. Discount the interest tax shield, if a constant debt-to-equity ratio is maintained, using  $r_U$  is appropriate.
3. Add the unlevered value,  $V^U$  to the present value of the interest tax shield to determine the value of the investment with leverage,  $V^L$ .

**Memorization: Advantages/Disadvantages**

**Advantages of APV:**

- Easier to apply when a firm does not hold its Debt-to-Equity ratio constant
- Provides you with a separate number for the interest tax shield.

**Disadvantages of APV:**

- More complicated than WACC as there are two pieces to calculate
- We need the projects debt capacity at each time 't', which requires the projects value at each time 't'.
- Need to calculate the project's debt and value simultaneously.

**18.4 The Flow to Equity Method (FTE)**

The Flow to Equity method calculates the free cash flow available to equity holders after taking into account all payments to and from debt holders. The cash flows are then discounted using the equity cost of capital.

**Calculating the Free Cash Flow to Equity (FCFE):**

The Free Cash Flow to Equity (FCFE) is the free cash flow that remains after adjusting for interest payments, debt issuance, and debt repayment.

$$FCFE = FCF - (1 - T_c) \times (\text{Interest Payments}) + (\text{Net Borrowing})$$

$$FCFE = \text{Sales} - \text{COGS} - \text{Operating Expenses} - \text{Depreciation} - \text{Interest Expense} - \text{Income Tax} + \text{Depreciation} - \text{CapEx} - \text{Increase in NWC} + \text{Net Borrowing}$$

$$\text{Net Borrowing at Date 't'} = D_t - D_{t-1}$$

From the interest tax shield table above we have the Debt in each time period  $D_t$  and the interest paid in each period.

Year	0	1	2	3	4
Debt Capacity $D_t$	30.62	23.71	16.32	8.43	-
Interest Paid ( $r_D = 6\%$ )		1.84	1.42	0.98	0.51

The Net Borrowing at t=0 is 30.62 = (30.62 – 0). The Net Borrowing at t=1 is 23.71 – 30.62 = -6.92 (using the above formula). This is continued for t = 2 to 4. The table below summarizes the cash flows:  
Example:

Year	0	1	2	3	4
Sales	-	60.00	60.00	60.00	60.00
Cost of Goods Sold	-	(25.00)	(25.00)	(25.00)	(25.00)
<b>Gross Profit</b>	-	35.00	35.00	35.00	35.00
Operating Expenses	(6.67)	(9.00)	(9.00)	(9.00)	(9.00)
Depreciation	-	(6.00)	(6.00)	(6.00)	(6.00)
<b>EBIT</b>	(6.67)	20.00	20.00	20.00	20.00
Interest Expense	-	(1.84)	(1.42)	(0.98)	(0.51)
<b>Pretax Income</b>	(6.67)	18.16	18.58	19.02	19.49
Income Tax (40%)	2.67	(7.27)	(7.43)	(7.61)	(7.80)
<b>Net Income</b>	(4.00)	10.90	11.15	11.41	11.70
<b>Free Cash Flow to Equity</b>					
Plus: Depreciation		6.00	6.00	6.00	6.00
Less: Capital Expenditures	(24.00)				
Less: Increase in NWC	-	-	-	-	-
Less: Net Borrowing	30.62	(6.92)	(7.39)	(7.89)	(8.43)
<b>Free Cash Flow to Equity</b>	<b>2.62</b>	<b>9.98</b>	<b>9.76</b>	<b>9.52</b>	<b>9.27</b>

Note: the FCFE at time 0, is the same amount that we calculated previously as the dividend paid to shareholders \$2.62 = \$30.62 - \$28.00.

**Valuing the Equity Cash Flows:**

The value of the discounted FCFE represents the gain to shareholders from the project.

$$NPV(FCFE) = FCFE_0 + \sum \frac{FCFE_t}{(1+r_E)^t}$$

$$NPV(FCFE) = 2.62 + \frac{9.98}{1.10} + \frac{9.76}{1.10^2} + \frac{9.52}{1.10^3} + \frac{9.27}{1.10^4} = \$33.25$$

**Summary of Flow-to-Equity-Method:**

1. Determine the free cash flow to equity of the investment
2. Determine the equity cost of capital  $r_E$
3. Compute the contribution to equity value, E, by discounting the free cash flow to equity using the equity cost of capital.

Using the Free Cash Flow to Equity formula above:

$$FCFE = FCF - (1 - T_C) \times (Interest \_ Payments) + (Net \_ Borrowing)$$

The free cash flow to equity can be calculated with the Free Cash Flow, the Interest Payment and the Net Borrowing from each period.

Year	0	1	2	3	4
Free Cash Flow	(28.00)	18.00	18.00	18.00	18.00
After-tax Interest Expense	-	(1.10)	(0.85)	(0.59)	(0.30)
Net Borrowing	30.62	(6.92)	(7.39)	(7.89)	(8.43)
<b>Free Cash Flow to Equity</b>	<b>2.62</b>	<b>9.98</b>	<b>9.76</b>	<b>9.52</b>	<b>9.27</b>

Using the FTE Method requires that the project’s risk and leverage match the firms, and that the firms equity cost of capital is held constant. **This is only true if the firm maintains a constant debt-to-equity ratio.** If the debt-to-equity ratio changes over time, then the equity cost of capital will change over time.

**Memorization: Advantages/Disadvantages**

**Advantages of FTE Method:**

- Allows for calculation of value of equity for the entire firm when capital structure is complex
- More transparent method for shareholders

**Disadvantages of FTE Method:**

- Need to compute the project’s debt capacity to determine the interest and net borrowing

**18.5 Project Based Costs of Capital**

**Estimate the Unlevered Cost of Capital:**

When a project differs from the firm as a whole and it is not appropriate to use the firms unlevered cost of capital  $r_U$ , you can estimate the unlevered cost of capital using data from similar firms in the market place.

For example, if there are two firms that manufacture the same product as your project and they have the following characteristics:

Firm	Equity Cost of Capital	Debt Cost of Capital	Debt-to-Value (D/D+E)
A	12.0%	6.0%	40%
B	10.7%	5.5%	25%

$$\text{Firm A: } r_U = 0.60 \times 12.0\% + 0.40 \times 6.0\% = 9.6\%$$

$$\text{Firm B: } r_U = 0.75 \times 10.7\% + 0.25 \times 5.5\% = 9.4\%$$

To determine the firm’s  $r_U$  you could simply take the average of the two comparable firms.  $r_U = 9.5\%$ . With this rate we can use the APV method to calculate the value of the investment.



**Project Leverage and the Equity Cost of Capital:**

Suppose we have a different target leverage ratio for a project than the firm as a whole. We can calculate  $r_E$  as follows:

$$r_E = r_U + \frac{D}{E}(r_U - r_D)$$

The project's  $r_E$  depends on the its unlevered cost of capital,  $r_U$ , and the debt-equity ratio of the individual project. Once we have the  $r_E$  we can then calculate the  $r_{WACC}$  using the formula:

$$r_{WACC} = \frac{E}{E+D}r_E + \frac{D}{E+D}r_D(1+T_C)$$

We can develop a new "Project Based WACC" if we define 'd' to be the ratio of Debt-to-Value or D/D+E.

$$\text{Project-Based WACC} = r_{WACC} = r_U - dr_D T_C$$

Proof:

$$\begin{aligned} d &= \frac{D}{E+D} \\ r_{WACC} &= (1-d) \times r_E + d \times r_D(1-T_C) \\ &= (1-d) \times \left( r_U + \frac{D}{E}(r_U - r_D) \right) + d \times r_D(1-T_C) \\ &= r_U + \frac{D}{E}r_U - \frac{D}{E}r_D - dr_U - d \frac{D}{E}r_U + d \frac{D}{E}r_D + dr_D - dr_D T_C \\ &= r_U + r_U \left( \frac{D}{E} - d - d \frac{D}{E} \right) - r_D \left( \frac{D}{E} - d \frac{D}{E} - d \right) - dr_D T_C \\ &= r_U + r_U \left( \frac{D}{E}(1-d) - d \right) - r_D \left( \frac{D}{E}(1-d) - d \right) - dr_D T_C \\ &= r_U + r_U \left( \frac{D}{E} \left( \frac{E}{E+D} \right) - d \right) - r_D \left( \frac{D}{E} \left( \frac{E}{E+D} \right) - d \right) - dr_D T_C \\ &= r_U + r_U \left( \frac{D}{E+D} - \frac{D}{E+D} \right) - r_D \left( \frac{D}{E+D} - \frac{D}{E+D} \right) - dr_D T_C \\ &= r_U - dr_D T_C \end{aligned}$$

Example:

A firm currently has an equity cost of capital of 12.7%, a borrowing cost of 6% and has maintained a debt-to-value ratio of 40%. Suppose the firm enters a new project that is different than the firm as a whole. The firm has surveyed comparable firms and the unlevered cost of capital for those firms is 15%. Also suppose that the firm will finance this project with 10% debt and hold the debt-to-value ratio constant. The debt will have a 6% cost associated with it and the corporate tax rate will be 35%.

Calculate the firm's unlevered cost of capital, and the project's equity cost of capital and project based WACC?

Solution:

The firm's current  $r_E$  is 12.7% and the debt-to-value is 40%.

$$r_{WACC} = 0.60 \times 12.7\% + 0.40 \times 6\% \times (1 - 35\%) = 9.2\%$$

$$r_U = 0.60 \times 12.7\% + 0.40 \times 6\% = 10.0\%$$

The firm's unlevered cost of capital is 10.0%.

The project has an unlevered cost of capital of 15% based on comparable companies. We can estimate the  $r_E$  using the following formula:

$$r_E = r_U^{Est.} + \frac{D}{E}(r_U^{Est.} - r_D) = 15\% + \frac{0.10}{0.90}(15\% - 6\%) = 16\%$$

We can then estimate the project based WACC using the following:

$$r_{WACC}^{Project} = r_U - dr_D T_C = 15\% - 0.10 \times 6\% \times 35\% = 14.8\%$$

**Common Mistake:**

Students often make a mistake with the following question (on previous SOA exams!)

Given a firm's equity cost of capital, debt cost of capital and a debt-to-value, the student is told that the firm increases its current debt-to-value ratio and then asked to determine the new WACC for the firm.

For example, the firm's current equity cost of capital is 12% and the debt cost of capital is 6.67%. The firm currently has a debt-to-value of 25% (tax rate = 40%). Assume that the firm increases its debt-to-value to 50%. What is the new WACC?

It is tempting to just use the WACC formula:  $r_{WACC} = 0.50 * 12\% + 0.50 * 6.67\%(1 - 0.4) = 8\%$  however, this is incorrect. The correct procedure is:

1. Compute the firm's unlevered cost of capital with the firm's old numbers.

$$r_U = 0.75 \times 12\% + 0.25 \times 6.67\% = 10.67\%$$

2. Compute the new WACC

$$r_{WACC} = r_U - d r_D T_C = 10.67\% - 0.50 \times 6.67\% \times 40\% = 9.34\%$$

Assume though, that with the increase in debt (if you increase the debt-to-value ratio you either increase debt or reduce equity), you have a higher debt cost of capital of 7.34%. Then the WACC becomes:

$$r_{WACC} = 10.67\% - 0.50 \times 7.34\% \times 40\% = 9.2\%$$

**Important concepts to remember for a project's incremental financing:**

- Cash is negative debt. Cash can be used to pay off debt immediately. You should use Net Debt in your calculations
- For a firm that is hoarding cash or highly leveraged, the WACC is  $r_U - r_D T_C$
- The optimal leverage depends on the firm and the project
- Safe cash flows can be 100% debt financed, if 100% financed the appropriate discount rate to use is  $r_D(1 - T_C)$  which is just the normal WACC formula with  $D/D+E = 1.00$ .

**18.6 APV with Other Leverage Policies**
**Constant Interest Coverage Ratio:**

If a firm increases its debt level as income rises to shield its income from taxes, it is natural to specify the firm's interest payments as a target fraction of the project's free cash flows. Let 'k' represent this.

$$\text{Interest Paid in Year 't'} = k \times FCF_t$$

**Memorization: Definition**

**Constant Interest Coverage Ratio:** when a firm's interest payments on its debt are a constant percentage of its free cash flow.

We can then determine the value of the project/firm using the APV method. By having a Constant Interest Coverage Ratio it simplifies the calculation for the PV(Interest Tax Shield).

$$PV(\text{Interest Tax Shield}) = PV(T_c \times k \times FCF_t) = T_c k \times PV(FCF_t) = T_c k \times V^U$$

Then the levered value is:

$$V^L = V^U + T_c k V^U = (1 + T_c k) V^U$$

**Example:**

A firm's project will contribute \$3.8M in Free Cash Flow the first year growing by 3% each year thereafter. The firm has an unlevered cost of capital of 8% and a tax rate of 40%. The acquisition cost is \$80M and will be financed by \$50M in new debt at 6%. Compute the value of the project using the APV method. The firm will maintain a constant interest coverage ratio.

**Solution:**

$$V^U = \frac{3.8}{8\% - 3\%} = \$76M$$

With 50M in new debt and a 6% interest rate the first year interest expense is  $50 \times 6\% = \$3M$ . Since the firm maintains a constant interest coverage ratio,  $k = 3.0/3.8 = 78.95\%$  for all years.

$$V^L = (1 + T_c k) V^U = (1 + 0.4 \times 78.95\%) \times 76 = \$100M$$

**Predetermined Debt Levels:**

A firm may choose to set its debt levels to a pre-determined amount.

**Memorization: Concept**

When debt levels are set according to a fixed schedule, we can discount the predetermined interest tax shields using the debt cost of capital  $r_D$ .

A particular simple example is if a firm has permanent fixed debt, maintaining the same level of debt forever. The PV(Interest Tax Shields) becomes:

$$PV(\text{Int.Tax.Shield}) = \frac{T_c r_D D}{r_D} = T_c D \text{ and therefore, } V^L = V^U + T_c D$$

## 18.7 Other Effects of Financing

### Issuance Costs:

Banks that offer debt financing for companies charge fees to accommodate this. These fees must be subtracted from the  $V^L$ .

### Security Mispricing:

Equity may be issued in the public markets at a price that is higher than the true value or lower than the true value. This difference must be incorporated into the levered value calculation. Banks could also give the company an interest rate that does not represent the true risk of the company. This difference needs to be included in the levered value calculation.

### Financial Distress and Agency Costs:

One consequence of debt financing is the possibility of financial distress and agency costs. As debt increases, these potential costs increase. This is best modelled using option techniques.

## 18.8 Advanced Topics In Capital Budgeting

### Periodically Adjusted Debt:

A special case is when the debt is adjusted annually. The expected interest expense for time 't' is known in advance at time 't-1'. Therefore, since we know the next interest payment we can discount at  $r_D$  and then discount it from date 't-1' to 0 at  $r_U$ .

$$PV(T_C \times Int_t) = \frac{T_C \times Int_t}{(1+r_U)^{t-1}(1+r_D)} = \frac{T_C \times Int_t}{(1+r_U)^t} \times \frac{1+r_U}{1+r_D}$$

Similarly, the Project Based WACC for annually adjusted debt is:

$$r_{WACC}^{Project} = r_U - dr_D T_C \frac{1+r_U}{1+r_D}$$

Similarly, the levered value of a firm with a constant interest coverage is:

$$V^L = \left( 1 + T_C k \frac{1+r_U}{1+r_D} \right) V^U$$

**Leverage and the Cost of Capital:**

When debt is set to a predetermined level for some time, the interest tax shields for the debt are relatively safe. The value of these “safe” tax shields should be deducted from the debt the firm has.

We label the amount of predetermined tax shields that will be deducted from debt as  $T^S$ .

Debt Net of Predetermined Tax Shields:  $D^S = D - T^S$

The unlevered cost of capital and equity cost of capital therefore become:

$$r_U = \frac{E}{E + D^S} r_E + \frac{D^S}{E + D^S} r_D$$

$$r_E = r_U + \frac{D^S}{E} (r_U - r_D)$$

Project WACC with a Fixed Debt Schedule becomes:

$$r_{WACC} = r_U - dT_C [r_D + \phi(r_U - r_D)]$$

$$\text{Where, } d = \frac{D}{E + D}, \phi = \frac{T^S}{T_C D}$$

Three cases commonly used in practice:

1. Continuously Adjusted Debt:  $T^S = 0, D^S = D$ , and  $\phi = 0$
2. Annually Adjusted Debt:  $T^S = \frac{T_C r_D D}{1 + r_D}, D^S = D \left( 1 - T_C \frac{r_D}{1 + r_D} \right)$ , and  $\phi = \frac{r_D}{1 + r_D}$
3. Permanent Debt:  $T^S = T_C D, D^S = D(1 - T_C)$ , and  $\phi = 1$

**The WACC or FTE Method with Changing Leverage:**

When a firm does not keep a constant debt-equity ratio, the APV Method is usually the best alternative. However, the WACC and FTE method can still be used after some additional calculations. The equity cost of capital and WACC must be computed for each time period. To calculate the levered value of the project/firm, you would then discount the FCF using a different WACC for each year.

**Personal Taxes:**

Leverage has tax consequences for both investors and corporations. Personal taxes alter the equations for APV. For the WACC method, equity and debt already factor in the cost of investor (personal) taxes. The WACC method does not change in the presence of personal taxes. The APV method does however change.

Let  $T_e$  be the tax investors pay on dividends/equity income and let  $T_i$  be the tax investors pay on interest income.

Define  $r_D^*$  as:

$$r_D^* = r_D \frac{1 - T_i}{1 - T_e}$$

Unlevered Cost of Capital with Personal Taxes is:

$$r_U = \frac{E}{E + D^S} r_E + \frac{D^S}{E + D^S} r_D^*$$

Next, we must compute the interest tax shield using the effective tax advantage of debt,  $T^*$

$$T^* = 1 - \frac{(1 - T_c)(1 - T_e)}{(1 - T_i)}$$

We then calculate the present value of the interest tax shields using  $T^*$  and  $r_D^*$ . We would discount the interest tax shields using  $r_U$  if the firm maintains a target leverage ratio or at  $r_D^*$  if the debt is set according to a predetermined schedule.